Aspects *Old and New, how many Degrees therein.*

Old				Aspects	New			
Names.	*Marks.*	*Signs.*	*Degrees.*		*Names.*	*Marks.*	*Signs.*	*Degrees.*
Sextile	✱	2	60		Semisextile	SS. or ⊻	1	30
Quartile	□	3	90		Semiquintile	♀.	$1\frac{1}{5}$	36
Trine	△	4	120		Semiquartile	♁.	$1\frac{1}{2}$	45
Opposition	☍	6	180		Quintile	ϱ. or ♡	$2\frac{2}{5}$	72
					Sesquiquintile	Td. or ⧖	$3\frac{3}{5}$	108
					Sesquiquadrate	♁.	$4\frac{1}{2}$	135
					Byquintile	Bq. or ∞	$4\frac{4}{5}$	144
					Quincunx	Vc. or ♌	5	150

Late Writers generally use two kinds of Names or Denominations, the one called *Sexagenæ*, and the other *Sexagesimæ*, both from the *Latin* importing Sixties, because of the general Denominator 60. *Denominations of late use.*

Sexagenæ are the collection of Degrees all under 60, standing in the place of the Unit, and having a Cypher to their *Index*, then by Collection all above 60, and under 3600, fall in *Sexagenæ primis*, and have 1 for their *Index*; 2 is the *Index* of *Sexagenæ* Seconds, 3 of *Sexagenæ* Thirds, and so of other Numbers respecting Motion. Sexagenæ *what.*

The Collection of Dayes under 60, resembling the former stand in the place of the Unit, and all above 60, and under 3600, possess the second place from the Unit to the Left Hand, and are called *Sexagenæ Primes*, and 60 times 60 Dayes, which is almost 10 Years, make one *Sexagenæ* Second, and so they are marshalled to Thirds and Fourths, *&c.* And that Hours may yield the place of Unity to Dayes, they are to be reduced to Minutes of a Day, as in the next *Chapter*. Nevertheless where there are no *Sexagenæ*, Hours may keep the place of Unity without Reduction.

Sexagesimæ, both of Motion and Time, are set on the Right Hand of their Integers (Degrees or Dayes) like the Decimals on theirs, and are to the *Sexagenæ*, as Decimals to their integers, and are the division of Minutes, Seconds, Thirds, *&c.* by 60, as aforesaid like the Decimals by Tens. Sexagesimæ *what.*

Both the *Sexagenæ* and *Sexagesimæ* are marked with exponent *Indices*, as the Decimals are, which accordingly shew the power of the Numerator either affirmatively or negatively, that is, either above or below the Unit. *How both marked.*

These *Indices* are found differently expressed, and sometime the one, and sometime the other Form are used, which is not at all material whether. Indices *differently expressed.*

Examples.

	Sexagena's.							*Sexagesima's.*							
Thus,	&c. 6	5	4	3	2	1	0	(1)	(2)	(3)	(4)	(5)	(6)	&c.	Common with Decimals.
	&c. 6	5	4	3	2	1	0	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{5}$	$\overline{6}$	*&c.*	
or															
Thus,	&c vi	v	‵‵‵‵	‵‵‵	‵‵	‵	0	′	″	‴	⁗	v	vi	&c.	Proper to Astronomicals
	&c. æ6	æ5	æ4	æ3	æ2	æ1	0	′	″	‴	⁗	v	vi	*&c.*	

So if I see this Number 3‵‵‵, 1‵‵, 12‵, 15 Dayes, 16′, 10″, 12‴, I read it thus, 3 *Sexagenæ* Thirds, 1 *Sexagenæ* Second, 12 *Sexagenæ* Primes, 15 Dayes, 16 Minutes or Primes, 10 Seconds, and 12 Thirds of a Day; and so of others.

Moreover if the Signs + and — be used, let the Astronomicals be counted Compound, but if not Simple. Astronomicals *Simple and Compound.*

If any intermediate Denominations in Numbers given be omitted, let the places be supplyed with Cyphers, as 2‵‵ and 2′, thus, 2‵‵, 0‵, 0°, 12′. *Cyphers supply intermediate places.*

CHAP. II.

Reduction of Astronomicals.

Reduction, called also Conversion of Astronomicals, is threefold, *viz.* Astronomicals *reduced how many wayes.*

Either to convert { Geodæticals / Decimals } into Astronomicals, or the contrary.
Astronomicals of one sort into another.

The

1. *To reduce Common Signs into Physical and Years into Sexagenæ, or contrary.*

The first sort serveth either to turn Common Signs into *Physical,* and Years or Moneths into *Sexagenas,* or the contrary.

To reduce Common Signs into Physical.

To turn Common Signs into Physical, half them, or reduce Geodætically by 30, the Signs into Degrees, and add the odd Degrees (if any) to them, then divide by 60, what Degrees remain place in the Units place, and the Numbers in the Quotient, if under 60, are Physical Signs, and to be set in the place of *Sexagenæ* Primes; if above 60, divide the Quotient again by 60, and place the Remainers as *Sexagenæ* Primes, and the Quotient of this Division for *Sexagenæ* Seconds, &c.

To reduce Physical Signs into Common.

On the contrary to turn Physical Signs into Common, or *Sexagenæ* of Motion into Circles and Signs; double or multiply the Physical Signs by 2, or reduce Geodætically all the *Sexagenæ* into Degrees, multiplying by 60, and then divide the summe by 30, and the Quotient, if under 12, shall be Signs; if above 12, divide by 12, and the Quotient of this Division shall be Circles.

Example of both.

As if 9 Signs, 12 Degrees, be given to be converted into Physical, reduced into Degrees they are 282, which divided by 60, gives 4 Physical Signs or *Sexagenæ* Primes, and 42 Degrees to be set in the place of Unity; which if on the contrary had been given to have been turned into Common Signs, the 4 Signs multiplyed by 60, make 240, to which the 42 Degrees added make 282 Degrees, which divided by 30 gives 9 Signs in the Quotient, and 12 Degrees remaining.

Common Signs. Degrees.

9 —— 12
30
———
270
12
———
282 Degrees.
———

Degrees. Signs Physical.

|4
282|2(4
60

Degrees. Signs Common.

(1
28(2(9
30

′ °
4 —— 42
60
———
240
42
———
282
———

To reduce Years &c. into Sexagenæ.

To turn Years or Moneths into *Sexagenæ,* reduce all Geodætically into Dayes accompting in, as in Reduction of Geodæticals was observed, the Dayes supernumerary for the Leap-Years, and adding in also the odd Dayes given in the Number, if any be, then divide by 60, and so Quotient after Quotient as far as you can, the Remain of the first Division being Dayes is to be placed as Integers in the Units place, the other Remains and last Quotient in their places orderly to the Left Hand, as *Sexagenæ* Primes, Seconds, Thirds, &c.

To reduce Sexagenæ into Years.

On the contrary to turn *Sexagenæ* of Dayes into Years, multiply Geodætically all the *Sexagenæ* into Dayes by 60, and then divide by the Dayes in one Year, and from the Remain substract the Dayes for the Leap-Years of the Quotienary Number, and the rest of the Remain shall be the odd Dayes, which if occasion be may be turned into Moneths by Division with the Dayes of one Moneth.

Example of both.

As if 1000 Years, 20 Dayes, were to be converted into *Sexagenæ,* the Common Dayes in 1000 Years by Geodætical are found to be 365000, to which 250 Dayes added, because there are so many Leap-Years in 1000, and the 20 Dayes given also added, make the Total 365270; then divided by 60, there is 1‴, 41″, 27′, 50°; that is, 1 *Sexagenæ* Third, 41 *Sexagenæ* Seconds, 27 *Sexagenæ* Primes, 50 Dayes; which if on the contrary were to be turned into Years, after Reduction by 60, and divided by 365, the Quotient will be 1000, and from the Remain 270, if 250 Dayes be substracted for the Leap-Years, there will remain 20 odd Dayes.

Leap
2
1000(250
4
Years

1000 Years.
365
———
365000
250
20
———
365270 Dayes.
———

4|5 °
365270|0(
60

|2 ′
6087|7(
60

|4 ″
101|1(1 ‴
60

‴ ″ ′ °
1 —— 41 —— 27 —— 50
60
———
60
41
———
101 ″
60
———
6060
27
———
6087 ′
60
———
365220
50
———
365270 °
———

Dayes. Years.

365|270(1000
365 250
365 20
365
365

The

2. *To reduce* Astronomicals *into* Decimals, *or the contrary.*

The second sort of Astronomical Reduction is to turn Astronomicals into Decimals, or Decimals into them, as *Sexagenæ* into Integers, and *Sexagesimæ* into Decimals, or the contrary.

Sexagenæ *into* Integers, Decimals *into* Sexagesimæ.

To turn *Sexagenæ* into Integers, or Decimals into *Sexagesimæ*, multiply continually by 6, every time removing the *Seperatrix* one place, but on the contrary to turn Integers into *Sexagenæ*, or *Sexagesimæ* into Decimals, removing the *Seperatrix*, as before, divide continually by 6, proceeding both wayes till you come to the Units place: For Division by 60 removeth the *Seperatrix* one place towards the Left Hand, and divideth by 6; and Multiplication by 60, promoteth the *Seperatrix* one place towards the Right Hand, and multiplyeth by 6; and so the Cypher being cut off by the *Seperatrix*, the work is short and easie.

Variety.

Otherwise by common Geodætical Reduction bring all the Circles and Signs, or *Sexagenæ* thereof, into Degrees, also Years and Moneths, or *Sexagenæ* thereof, into Dayes by Multiplication, and them reserve for the Integers; then reduce by Multiplication with 60, all the *Sexagesimæ* into the lowest Denomination, and the last Product thereof adjoyning Cyphers if need be, divide by 60, Figurated to a Power of the Second, Third, Fourth, &c. Quantity, according to the Right Hand Denomination of the *Sexagesimæ*, and the Quotient shall be the Decimals. And to convert Decimals given into the other, multiply the Decimals as aforesaid, and divide the Integers by their proper Denominators.

Example of both.

As if 6‴, 30‵, 20°, 15′, 45″, were to be reduced into Integers and Decimals; by the first way I place them with the seperating Lines, as at *A.* and beginning at top, multiply the 6‴ by 6, the 36 produced is subscribed under the upper *Seperatrix*, as at *B.* which with the 30‵ multiplyed by 6, produceth 234, adjoyned to the 20°, makes in all 23420 Integers, as at *C.* then beginning with the *Sexagesimæ* at bottom, I divide 45″ by 6, (supposing 00 adjoyned) I superscribe over the lower *Seperatrix* the Quotient 75, as a Decimal to 15′, at *D.* And lastly, I divide 1575 (with 2 Cyphers also supposed to be adjoyned) and superscribe the Quotient 2625, as the Decimals desired to be adjoyned to the Integers; and the whole work stands compleat at *E.*

```
  A.         B.          C.           D.            E.
 6|‴        6‴|         6‴|          6‴|           6‴|  ×6
 30|‵       36          36           36            36
  20°|      30‵|        30‵|         30‵|          30‵|
   ,15|      20°|      23420°|      23420°|    6) 23420,2625 ×6
    ,,45      ,15|        ,15|         ,15|75        ,15|75
               ,,45         ,,45          ,,45    6) ,,45
```

Where there are many Species, this is much the shorter and more ingenuous way; the other for the Integers hath had its like often exemplified in Geodæticals, as at *F.*; for the 15′, 45″, after Reduction into 245″, I adjoyn 0000, and divide by 3600, which is the Square of 60, or the Power of the Second Quantity, the Number being ″ and the Quotient is 2625 at *G.* So is the Number as before 23420,2625.

```
 ‴   ‵   °   ′   ″                        (4)
 6  30  20  15  45   Reduced is 23420,2625.
 60         60
 ---        ---       (1)  60   Root.           2
 360        900       (1)  60                 2298  (6)    (4)
  30         45       ---------               9450000 (2625   G.
F.---       ---       (2) 3600 Square.        36     00
 390        945″      ---------                36
  60        ---                                 36
 ---                                             36
 23400
    20
 -----
 23420 Integers.
```

And if 23420,2625 were given to be converted into Astronomicals this way, I continue Division of 23420, and Multiplication of ,2625 by 60, and obtain my desire.

```
        Sexagenæ.                         Sexagesimæ.
   5    °   |3 ʻ  "                         ,2625
  234|20( 39|0( 6                         |    60
  6    0   6 0                         ---|------
                                       ʻ15| 7500
                                          |   60
                                       ---|------
                                       "45| 0000
```

But by the other way proceed, as before, by 6, to multiply the Decimals, and divide the Integers, cancelling, as some do, the Figures divided or multiplyed.

```
Numbers placed.        6"|  Sexagenæ.          6"|  Sexagenæ.
                       3 ʻ                     3 ʻ
                      390|0                   390|0
        |            23420,|2625             23420,|2625
23420,|2625                                      ʻ15|75
                                        Sexagesimæ  " 45
```

3\. *Astronomicals reduced one into another.* The third sort of Astronomical Reduction is used to convert one Astronomical into another.

As { Degrees into { Decimals of a Day——— / Hours and Decimals of an Hour } or the contrary.
{ Hours into { Decimals of a Day / Minutes of a Day } or the contrary.

To reduce Degrees, &c. into Decimals of a Day. To turn Degrees with or without Decimals annexed to them, into Decimals of a Day, divide by 360, that is 6×60. And contrarily to turn Decimals of a Day into Degrees, multiply by 360, that is 6×60.

Example. As to convert 236,4276 Deg. into Decimals of a Day, I first take the sixth part of the Number, or divide by 6, and subscribe, which is 39,4046, of which the 60th part is 0,6567433 for the Decimals of a Day desired. And if this Number had been given to procure Degrees, aud Decimals of a Degree, I had multiplyed first by 60, and the Number superscribed multiplyed again by 6, as at *H*, but by the common way, the one and the other is at *I.* and *K.* with Cyphers adjoyned, as needful, in the Division at *I*, and supply for the defect of the Decimal at *K*, making the 588(6), to be 6(4), or accompting them so, as most usual in Imperfect Decimals.

```
Divisors.    H.                                  I.                               K.
{6 )    236,|4276    Multiplyers.        2  xxx|1                          0,6567433
{60)     39|4046       × 6 }           20 26522|2       (7)            |       360
          0|6567433    × 60}         236,427600|0( 6567433          ---|----------
                                        36        0                  39|4045980
                                 (7)     36                         197|02299
                                  0       36                        ---|----------
                                 ---       36                       236|427588
                                 (7)        36
                                             36
                                              36
```

To reduce Degrees, &c. into Decimals of an Hour. To turn Degrees with or without Decimals annexed to them, into Hours and Decimals of an Hour, divide by 15, that is 3×5. And on the contrary to turn them into Degrees, multiply by 15, that is 3×5.

Example. As to convert the former Number of Degrees, and Decimals of a Degree 236,4276 into Hours, and Decimals of an Hour, I first divide by 3, and the Number 78,8092 subscribed, divide by 5, and the subscribed 15,76184, as at *L.* is the desire; which if given by Multiplication first by 5, and the Product superscribed multiplyed by 3, would have produced 236,4276, as by Common Division and Multiplication by 15, at *M.* and *N.* appears.

Divisors

Divisors. L. Multiplyers.

(3) 236|4276 × 3
(5) 78|8092 × 5
15|76184

M.

x
8x 9xx (5) (5)
236,42760 (15,76184
x5
x5
(5) x5
0 x5
x5
(5) x5
x5

N.

15,76184
15
78|80920
157|6184
236,4276

To turn Hours with or without Decimals annexed to them, into Decimals of a Day, divide by 24, that is 4 × 6. And to turn them into the other, multiply by 24, that is 4 × 6.

To reduce Hours, &c. into Decimals of a Day.

Example.

As to convert the aforesaid 15 Hours, and 76184 Decimals of an Hour into Decimals of a Day, I first divide by 4, and subscribe the Quotient 3,94046, the which I divide by 6, and the Number subscribed is 0,6567433, the Decimals of a Day desired as at *O*. Contrariwise, if that same Number had been given, I would have multiplyed first by 6, and the Numbers superscribed on the *Seperatrix* would have been 3,94046 as before, which multiplyed by 4, would have produced the 15,76184. And so by Common Division or Multiplication with 24, the Numbers agree as at *P*. and *Q*. allowance being made for the Imperfect Decimal at *Q*.

Divisors. O. Multiplyers.

(4) 15|76184 × 4
(6) 3|94046 × 6
0|6567433

P.

x x
x 3x7088|8 (7)
x5,76x8400 (6567433
x 4
x4
(7) x4
0 x4
x4
(7) x4
x4

Q.

0,6567433
24
2|6269732
13|134866
15,7618392

To turn Hours, with or without Minutes and other smaller parts of an Hour, into Minutes of a Day, that is, into Sixtieth parts of a Day, which Hours are not, and so into other smaller parts of a Day, multiply every Number by 5, and half the Product, that is multiply by $2\frac{1}{2}$. And on the contrary to turn the parts of a Day into Hours, and parts of an Hour, double every Number, and divide by 5, that is divide by $2\frac{1}{2}$, because one Hour answers to 2′ 30″ of a Day, to make the Denominator 60, there being twice 24 and 12 therein.

To reduce Hours, &c. into Minutes of a Day.

Example.

As to convert 12 Hours and 40 Minutes of an Hour into parts of a Day: If I multiply by 5, the Products 60, 200, halfed are 30′, 100″, which 100″, because above 60, I carry a Minute to 30′, and subscribe the 40″ remaining, so the result 31′, 40″, of a Day, as at *R*. which if given to be turned into Hours, I first double them, and they are 62′, 80″; then dividing 62 by 5, I get 12 Hours, and the 2′, 80″, remaining turned into ″ make 200, which divided by 5 give 40′ of an Hour. The Multiplications and Divisions by $2\frac{1}{2}$ are at *S*. and *T*.

R.

	Hours.	′	″	
	12	40		
5 ×	Day	60	200	) 5
2)		30	100	× 2
		31	40	

S.

12 . 40
$2\frac{1}{2}$
24 | 80
6 | 20
30 . 100
31 . 40

T.

′ ″
31 . 40
2
Hours ′
62 . 80 (12 . 40
5

The Reductions of this *Chapter* being reciprocal serve for Proof each to other without further illustration.

Proof of Astronomical Reduction.

CHAP.

CHAP. III.

Addition of Astronomicals.

Astronomicals added. Simple.

ADdition of Astronomicals is either Simple or Compound.

Simple Addition differeth nothing from Geodætical Addition, for you begin at the Right Hand, and add the Numbers of like Denomination to their fellows: And if at any time the Numbers of that Denomination you are adding exceed 60, for every 60 carry an Unit to the next Left Hand Denomination, and subscribe the overplus: And so proceed in order to the Left Hand.

Example.

As to add 13‶, 26‵, 59°, 30′, 45″, to 2‶, 5‵, 17°, 15′, 30″, the Total will be 15‶, 32‵, 16°, 46′, 15″; which is so plain, explanation is needless.

	‶	‵	°	′	″
Addends	13	26	59	30	45
	2	5	17	15	30
Total	15	32	16	46	15

Compound.

Compound Astronomical Addition is like Compound Decimal Addition; for taking the Lesser Numbers out of the Greater, to the Remaining Total subscribe the Sign of the Greater.

Example.

As to add 13‶, 26‵, 59°, 30′, 45″, with —2‶, 5‵, 17°, 15′, 30″; these must be substracted from the other, and the Total remaining will be +11‶, 21‵, 42°, 15′, 15″. And if the Signs be intermixt, as to add 4‵+10° —40′, to 3‵—20° +10′, the Total will be +7‵—10° —30′. As the Examples shew sufficiently without further explanation.

	‶	‵	°	′	″
Addends	+13	26	59	30	45
	—2	5	17	15	30
Totals	+11	21	42	15	15

‵	°	′
+4	+10	—40
+3	—20	+10
+7	—10	—30

When the Signs are intermixed and Units caried to the next.

If where the Signs are intermingled in adding the Numbers of one Denomination together there amount to more than 60, for every of which an Unit be carried over to the next Left Hand Denomination, if the Sign there be changed, then substract an Unit for every 60 so carried, and set down the Total of the rest.

Examples.

As to 3′ —50″, add 2′ —40″, or —3′ +50″ to —2′ +40″; in both cases the 50″ and 40″ make 1′ and 30″, in the first instance —, in the second +, which 1′ carried over to the Left Hand ′, being of a contrary Sign, I therefore substract an Unit from the Total 5′, and subscribe the remaining 4′, as followeth;

	′	″	′	″
Addends	+3	—50	—3	+50
	+2	—40	—2	+40
Totals	+4	—30	—4	+30

Proof of Astronomical Addition.

Astronomical Addition hath the same benefit of being proved by Astronomical Substraction, as other Additions by their respective Substractions. And as Decimals may be also proved by reducing the Numbers into Geodæticals, and comparing the Totals of both Additions together, as equal in value, when the Operations are right after the manner used in Decimals.

As to instance in the 2 last Examples, thus;

′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
+3 —50	+2 —40	+4 —30	—3 +50	—2 +40	—4 +30
60	60	60	60	60	60
+180	+120	+240	—180	—120	—240
— 50	— 40	— 30	+ 50	+ 40	+ 30
+130	+ 80		—130	— 80	

Totals +210 Equal +210 Totals —210 Equal —210

CHAP.

CHAP. IV.

Subſtraction of Aſtronomicals.

Subſtraction of Aſtronomicals is either Simple or Compound. *Aſtronomicals ſubſtracted. Simple.*

Simple Subſtraction differeth nothing from Geodætical Subſtraction; for you begin at the Right Hand, and withdraw the under Number from the uppermoſt of like Denomination, and ſubſcribe the Remain: And if the Number beneath be the greateſt then borrow 60, and ſuppoſing the ſame to be added to the upper, make Subſtraction from the Total, and for every 60 borrowed pay an Unit in the next Left Hand Denomination; except where the Subtrahend is the greateſt, and there, as in Decimals, the difference ſhall be taken with the contrary Signs; or elſe proceeding as before till the Left Hand Denomination, and the Sign there changed to the difference, the Signs of all the other Remains ſhall be as the given Numbers.

Example. As to ſubſtract 13``, 26`, 59°, 30′, 45″; from 15``, 32`, 16°, 46′, 15″, the Remain ſhall be 2``, 5`, 17°, 15′, 30″; which needs no explanation.

	``	`	°	′	″
Greater Number	15	32	16	46	15
Subtrahend	13	26	59	30	45
Remain	2	5	17	15	30

But if 15``, 32`, 16°, 46′, 15″, were to be ſubſtracted from 13``, 26`, 59°, 30′, 45″; here becauſe the Subtrahend is the greateſt Number in the ′, I borrow 60, which I pay again, by counting 16° one more than it is, or 59° an Unit leſs than it is. And in the ` I borrow 60 again, for which I count the 15`` to be 16``, or the 13`` at top but 12``; but now becauſe I cannot take 16`` from 13``, or 15`` from 12``, but ſhall want 3``, I ſet down the difference 3`` with the contrary Sign, as at *A.* Otherwiſe, as in Decimals, I change the Sign to the difference of every Number in the Subtrahend too great to be ſubſtracted from the Numbers that ſtand over him reſpectively at top, as at *B.*

	``	`	°	′	″
	+13	26	59	30	45
A.	+15	32	16	46	15
	—3	+54	+42	+44	+30

	``	`	°	′	″	
	+13	26	59	30	45	Upper Numbers
B.	+15	32	16	46	15	Subtrahends
	—2	—6	+43	—16	+30	Remains

Compound. Compound Aſtronomical Subſtraction is like Compound Decimal Subſtraction; For where Numbers of unlike Signs are to be ſubſtracted one from the other, the Numbers muſt be added together, and their Totals ſhall be the particular Remains, and their Signs ſhall be the upper Numbers Sign.

Example. As if the Remain above at *B.* were to be ſubſtracted from the upper Number from which Subſtraction there is made, then muſt the Subtrahend there be the Remain here; as followeth;

	``	`	°	′	″	
Upper Number	+13	+26	+59	+30	+45	Upper Number
Subtrahend here	—2	—6	+43	—16	+30	Remain above
Remain here	+15	+32	+16	+46	+15	Subtrahend above.

If where the Signs are intermixt in adding up the Numbers of contrary Signs, the ſumme exceed 60, then the overplus is to be ſubſcribed under that Denomination where the ſumme ariſeth, and for every 60 an Unit is to be carried over to the next Left Hand Denomination, which Unit ſhall have the Sign of the upper Number. And if the Number of the next Left Hand Denomination annexed to him be of a like Sign, then this *When in contrary Signs an Unit is carried to the next.*

this Unit or Units so carried over shall be added thereto, but if of a contrary Sign substracted therefrom.

Examples. As if —2′ +50″, be substracted from +3′ —10″, the 50″ and 10″ of contrary Signs added make 60, for which 1, that is —1′, because 10″ the upper Number is — is carried to the +3′, which being contrary is substracted therefrom, and so leaves but +2′ to be added with —2′, which make +4 for the Remain, as at *C.*

But if +2′ —50″ be substracted from —3′ +10″, the 50″ and 10″ added make 60, as before, but + because 10″ the upper Number is +; for which 60 is +1′ carried to the —3′, and being contrary and substracted therefrom, leaves but —2′ to be added with +2′, which make the Remain —4′, as at *D.*

Contrarywise, if —2′ —50″ be substracted from +3′ +10″, or +2′ +50″ from —3′ —10″, in the former the Unit carried over is +1′, and in the latter —1′, and to be added accordingly to the 5′ amounting of —2′ and +3′, or +2′ and —3′, making the Remains +6′ in the one, and —6′ in the other, as at *E.* and *F.*

	C.	*D.*	*E.*	*F.*
	′ ″	′ ″	′ ″	′ ″
Upper Numbers	+3 —10	—3 +10	+3 +10	—3 —10
Subtrahends	—2 +50	+2 —50	—2 —50	+2 +50
Remains	+4 —00	—4 +00	+6 +00	—6 —00

When Signs are intermixt, and the Subtrahend greatest. If among Numbers of intermixt Signs, some of the respective Species or Denominations, both in the Subtrahend and Number from which Substraction is to be made, be of like Signs, and it happen that the Number in the Subtrahend is the greater, then I am left at liberty, whether for the Remain I will change the Sign to the difference, or else as in Simple Substraction borrow 60. But if so, this must be remembred, that the Unit to be paid (for the 60 borrowed) in the next Left Hand Denomination, if the Sign thereof be contrary to the Sign of that Denomination where the 60 was borrowed, this Unit payable must be contrary; that is + with —, and — with +; and although in both Remains the Numbers and Signs differ, they agree in value.

Examples. As to deduct —2′ +50″ from —3′ +10″, there after the former way, the Remain will be —1′ —40″; but after the latter way —2′ +20″; for to take 50″ from 10″, and 60″ borrowed to put thereto, the Remain will be +20″, for which 60, the Unit payable is +1′, and being affirmative therefore lessens the —2′ in the Subtrahend being of a contrary Sign, and makes it but —1′, which deducted from —3′ leaves —2′, as at *G.*

But to take +2′ —50″ from +3′ —10″, there after the former way, the Remain will be +1′ +40″, but after the latter +2′ —20″, because the 60 borrowed there is negative, and so being contrary to the +2′ in the Subtrahend is to be taken therefrom, and the remaining +1′ taken from +3′, leaves for the Remain +2′, as at *H.*

	G.	*H.*	
	′ ″	′ ″	
Upper Numbers, or Numbers from which Substraction is made	—3 +10	+3 —10	
Subtrahends	—2 +50	+2 —50	
Remains	—1 —40	+1 +40 by the former way	Equal.
Remains	—2 +20	+2 —20 by the latter way	

Proof of Astronomical Substraction. Astronomical Substraction will be proved by Astronomical Addition, as well as other Substractions by their respective Additions; and together with Decimals will abide the tryal by being turned into Geodæticals.

As in the last instance at *H.* will be plain.

′ ″	′ ″	′ ″	′ ″
+3 —10	+2 —50	+ 1 +40	+ 2 —20
60	60	60	60
180	120	60	120
— 10	—50	+ 40	— 20
+170 lacking	+70 is by the first Remain	+100 by the other	+100

CHAP.

CHAP. V.

Multiplication of Aſtronomicals.

Multiplication of Aſtronomicals is either Simple or Compound. *Aſtronomicals multiplyed. Simple.*

Simple Multiplication is like the fourth variety of the fourth Caſe of Geodætical Multiplication; for every Number of the Multiplicand is to be multiplyed by every Number of the Multiplyer. And to know the Denomination of the Products add the *Indices* together, as in Decimals, then collect the ſeveral Products or Multiplees into one Total Product. And in collection, if any file of Multiplees exceed 60, for every 60 carry 1 to the next Left Hand Denomination, and ſubſcribe the overplus.

As ſuppoſe *Luna* in her ſwift Motion run in one Day 14 Degrees, 30′, and I would know according to that Diurnal Motion, how far ſhe will run in 3 Dayes, 6 Hours, and 40′. The 6 Hours, 40′, being reduced into Minutes and Seconds of a Day, according to the laſt kind of Aſtronomical Reduction mentioned in the *Second Chapter* before, make 16′, 40″. Then multiplying, as aforeſaid, Number by Number, the ſeveral Multiplees appear as at *A*, and collecting the ſumme, find 47 Degrees, 31′, 40″, or 1 Common Sign, 17 Degrees, 31 Minutes and 40 Seconds. *Example in the Moons Diurnal Motion.*

		°	′	″		
	Multiplicand	14	30			*Index* ′
	Multiplyer	3	16	40		*Index* ″
A.	Multiplees	42	90		‴	
			224	480		
				560	1200	*Index* ‴
	Total Product	47	31	40		

1200/60 (20″
480
560
1060/60 (17′
90
224
331/60 (5°

Compound Aſtronomical Multiplication is like Compound Decimal Multiplication; for every Number of the Multiplicand is to be multiplyed by every Number of the Multiplyer; and the Numbers of like Signs ſhall produce +, and unlike Signs —. And in collection of the Multiplees, by taking the + from the —, or — from the +, the Product may be contracted. *Compound.*

As to multiply 3′—20″, by 2′—30″, the Product ſhall be at large +6″—130‴+600⁗, as at *B*, which may be contracted by taking the 600⁗, that is +10‴ out of the —130‴; the Remain —120‴, that is —2″ taken out of +6″, will leave but 4″ at laſt. *Example.*

		′	″		
	Multiplicand	+3	—20		*Index* ″
	Multiplyer	+2	—30		*Index* ″
			‴	⁗	
B.	Multiplees	+ 6	— 40		
			— 90	+600	*Index* ⁗
	Total Product	+ 6	—130	+600	
	Product contracted	+ 4″			

+600⁗/60 (10‴
—130‴
+ 10
—120/60 (2″
+6″
—2
+4

Aſtronomical Multiplication is to be proved by Aſtronomical Diviſion, as well as other Multiplications by their reſpective Diviſions; and together with Decimals will endure the tryal, if reduced into Geodæticals: As in the laſt Operation thus, *Proof of Aſtronomical Multiplication.*

+ 3′ —20″	+ 2′ —30″		
60	60	160″	
		90″	
+180	+120		
— 20	— 30	14400⁗	14400/60 (240‴/60 (4″
+160″	+ 90″		

CHAP.

CHAP. VI.

Division of Astronomicals.

Astronomicals divided. Simple.

DIvision of Astronomicals is either Simple or Compound.

Simple Division is like the second variety of the fourth Case of Geodætical Division; for by the Number of the highest Denomination of the Divisor, the greater Denomination of the Dividend is to be divided, and thereby an apt Quotient Figure gotten, by which multiply the Divisor, and substract the Product from the Dividend, and set the Remains at top, which when to be brought to the Right Hand multiply by 60, or suppose it so done, and so continue the Division to the end of the work, or till a Quotient be gotten large enough for use. And to know the Denomination of the Quotient, substract the *Indices* as in Decimals.

Simple Division in 3 Cases.

This kind of Division may be throughly understood under the varieties in these three following Cases.

1. Case, When both Dividend and Divisor are single Numbers, though of a different *Index*.

2. Case, When the one of them is Single, and the other Plural.

3. Case, When both the given Numbers are Plural.

1. Data single.

First, Both the given Numbers Single may admit of two varieties; that is, either the Dividend greater than the Divisor, or less.

Dividend greatest and evenly. &c.

If the Dividend be greater, and will be evenly divided by the Divisor, then the Numbers are divided as Integers, and the *Index* found as in Decimals.

Example.

As to divide 45″ by 5′, the Quotient will be 9, and substracting 1 the *Index* of 5, from (2) the *Index* of 45, the Remain (3) is the *Index* of the Quotient 9, as below at *A*.

Dividend greatest and not evenly, &c.

If the Dividend be greater than the Divisor, and will not be evenly divided thereby, then after the first Quotient Figure gotten by dividing, as above, multiply the Remain by 60, and divide the Product by the Divisor, and add this Quotient to the former, and so proceed to the end of the work, or a Quotient large enough for your occasion.

Example.

As to divide 45″ by 12′, the first Quotient Figure gotten will be 3‴, the 9 which is left remaining multiplyed by 60 produceth 540, which divided by 12, giveth 45⁗, as below at *B*.

Divisor greatest.

The other variety of this Case is when the Divisor is greater than the Dividend, and then multiply the Dividend by 60, and divide the Product by the Divisor; and if any thing remain, proceed as last abovementioned.

As to divide 9 Degrees by 10 Degrees. 9 Degrees multiplyed by 60 produce 540′, which divided by 10 give in the Quotient 54′, as at *C*.

A.

5′) 45″ (9‴
(2)
1
Index (3)

B.

9
12′) 45″ 00‴ (3‴ 45⁗
(2) (3)
1 1
Indices (3) (4)

9″
60′
12′) 540‴ (45⁗

C.

10°) 540′ (54′
(1)
0
Index (1)

2. One Single and the other Plural.

2. The one of the given Numbers Single, and the other Plural may also admit of a double variety, that is, either the Divisor Single, or the Dividend.

Divisor Single.

If the Divisor be Single, then thereby divide every Number of the Dividend, and carry over the Product of the Remains, if any be, reduced by 60, to the next Right Denomination, as above at *B*.

Example in the Hourly Motion of the Moon.

As suppose in 1 Day Natural, or 24 Hours, ☽ in her swift Motion runneth 14 Degrees, 58′, 20″, and I would know her Hourly Motion; because 14 is less than 24, I multiply it by 60, and thereto add the 58′, and the Total 898′ divided by 24 giveth in the Quotient 37′, and the 10′ remaining multiplyed by 60, and carried to the 20″, make 620″, which divided by 24 gives 25″ in the Quotient, and there remaineth 20, which multiplyed as before, and the Product divided, addeth to the Quotient 50‴, and the whole Quotient is 37′, 25″, 50‴, for the *Moons* Hourly Motion at the course aforesaid.

Divisor

Divisor Dividend Quotient

24°) 14° 58′ 20″ 00‴ (00° 37′ 25″ 50‴

24

14°	10′	20″
60′	60′	60′
840′	600″	24°) 1200‴ (50‴
58	20	00
24°) 898 (37′	24°) 620 (25″	
178	140	
10	20	

If the Dividend be single, as in the other varieties of this Case, then place Cyphers to the Right Hand as far as shall be needful, and then like the work of the former Examples, inquire with the first Figures of the Divisor for a Quotient Figure out of the Left Hand Numbers of the Dividend, and thereby multiply all the Divisor, and substract the Total of these Products from the Dividend, leaving the Remains at top, and then removing the Divisor inquire for another Quotient Figure, and so repeat this work till the Division be ended, or a Quotient large enough obtained. *Dividend Single.*

As suppose the mean Motion of the *Moon* from the *Sun* daily be 13 Degrees, 10′, 35″, and I would know when, according to that course, she will make her Revolution, or come to that Point of the *Zodiack* where she made her last ☌ with ☉. Then I divide 360 Degrees, which make the whole Circle, having adjoyned a convenient number of Cyphers, by 13°, 10′, 35″; and finding at first 27 times 13 may be had out of 360, I multiply the Divisor by 27, and the Products 355°, 45′, 45″, substracting leave remaining 4°, 14′, 15″; and then removing the Divisor, 13 may be had 19 times out of 4°, 14′, or 254′, the Divisor then multiplyed by 19, the Products to be substracted will be 4°, 10′, 21″, 5‴. And so in like manner proceeding to Thirds which is far enough, the Revolution is 27 Dayes, 19′, 17″, 45‴ of a Day, and reduced into Hours is 27 Dayes, 7 Hours, 43′, 6″ of an Hour. And if occasion were, the Division might be continued, because there remaineth 2‴, 08⁗, 45ᵛ. *Example in the Revolution of Lunæ.*

| 2 8 45

Index 9 55 5

(5) 3 53 55

(2) 4 14 15

(3) 13^Deg. 10′ 35″) 360° 00′ 00″ 00‴ 00⁗ 00ᵛ (27^Dayes 19′ 17″ 45‴ &c.

355 45 45

4 10 21 5

3 43 59 55

9 52 56 15

3. When both Dividend and Divisor are Plural, the Operation is like the last aforegoing; for the Cyphers adjoyned represent the Dividend there to be of Plural Denominations. *Data Plural.*

As in Case ☽ want 2 Degrees, 12′, 40″, 13‴, 20⁗, of *Aldebaran*, or some other fixed *Star*, and her Hourly Motion be 37′, 40″, I would know when she will be in Conjunction with the same *Star*: Then inquiring with 37′, out of 2°, 12′, that is reduced 132′, I find 3 may be taken in the Quotient, and multiplying the Divisor thereby substract the Total Product, and so continue the Division till I get 3 Hours, 31′, 20″, and nothing left remaining. *Example in ☌ of the Moon with the Bulls Eye.*

Index 12 33

(4) 19 40

(2) 37′ 40″) 2° 12′ 40″ 13‴ 20⁗ (3^Hours 31′ 20″

(2) 1 53 00

19 27 40

12 33 20

Compound. Compound Astronomical Division is like Compound Decimal Division; for Number is to be divided by Number. The *Index* of the Quotienary Numbers are got as above; and the Signs, as in Decimals, that is + with like Signs, and — with unlike.

Example. As if $6'' - 130''' + 600''''$, the Product of the Compound Multiplication in the last *Chapter*, were to be divided by $2' - 30''$, there divising with $2'$ out of $6''$, I get $3'$ for the Quotient with the Sign +, because 2 and 6 were both +, by which $3'$, multiplying the Divisor, the Product to be substracted is $6'' - 90'''$, which substracted leaves $-40''' + 600''''$. Then divising again by $+2'$ out of $-40'''$, I get $20''$ for the Quotient with the Sign —, because $2'$ was + but $40'''$ —, and the Divisor multiplyed by this $20''$, makes the Product equal to the Remain before, as is here to be seen.

$$
\begin{array}{lllll}
 & & -\cancel{40} & & \\
2' - 30'' & \big) & +\cancel{6}'' - \cancel{130}''' + \cancel{600}'''' & \big(& 3' - 20'' \\
3 \; - 20 & & +\cancel{6} - \cancel{90} & & \\
\hline
6 \; - 90 & & -\cancel{40} \; + \cancel{600} & & \\
-40 + 600 & & & &
\end{array}
$$

Proof of Astronomical Division. Astronomical Division may be proved by Astronomical Multiplication, like other Divisions by their respective Multiplications; and like Decimals may be turned into Geodæticals, and tryed thereby.

As in the last Example thus;

$$
\begin{array}{l}
+\overset{''}{6} - \overset{'''}{130} + \overset{''''}{600} \\
\quad 60 \\
\hline
+360'' \\
-130 \\
\hline
+230 \\
\quad 60 \\
\hline
+13800'''' \\
+ \quad 600 \\
\hline
+14400 \\
\hline
\end{array}
\qquad
\begin{array}{l}
+\overset{'}{2} - \overset{''}{30} \\
\quad 60 \\
\hline
+120'' \\
- \; 30 \\
\hline
+ \; 90 \\
\hline
\end{array}
\qquad
\begin{array}{l}
+\overset{'}{3} - \overset{''}{20} \\
\quad 60 \\
\hline
+180 \\
- \; 20 \\
\hline
+160 \\
\hline
\end{array}
$$

$$
\begin{array}{l}
\quad \cancel{5} \\
\cancel{144}\cancel{00} \big(\\
\hline
\cancel{9} \;\; \cancel{0}
\end{array}
\begin{array}{l}
|4'' \\
\cancel{16}|0 \big(2' \\
\hline
6\,0
\end{array}
\qquad\qquad
\begin{array}{l}
|4'' \\
\cancel{16}|0 \big(2' \\
\hline
\cancel{6}\,0
\end{array}
$$

CHAP. VII.

Figuration of Astronomicals.

Figurate Astronomicals produced. To produce Figurate Astronomicals, is no other than to multiply any Astronomical Simple or Compound into it self, for the Square and the Square multiplyed by the Root produceth the Cube, *&c.* as other Figural Numbers are produced: And the same being done by Multiplication, the *Indices* and Signs of the Product are found as before declared.

Examples

Examples.

Examples Simple

	°	′	″	‴	⁗	v	vj
Root	1	10	9				
	1	10	9				
	1	10	9				
		10	100	90			
			9	90	81		
Square	1	22	1	1	21		
	1	10	9				
	1	22	1	1	21		
		10	220	10	10	210	
			9	198	9	9	189
Cube	1	35	53	29	43	42	9

Compound

	′	″	‴	⁗	v	vj
Root	1	—10				
	1	—10				
		1	—10			
			—10	+100		
Square		1	—20	+100		
	1	—10				
			1	—20	+100	
				—10	+200	—1000
Cube			1	—30	+300	—1000

To extract the Root of a Simple Astronomical, prick the Number, and proceed in the same as in Extraction of Roots before taught, only observing, as before in Decimals, that when the Right Hand Denomination will not be parted evenly by the *Index* of the Quantity whose Root you would extract, you must adjoyn one or more Cyphers, that so it may be divisible accordingly, that is by 2 for the Square, 3 for the Cube, 4 for the Squared Square, *&c.* *Roots of Simple Astronomicals extracted.*

As to extract the Square Root of 1°, 22′, 1″, 1‴, 21⁗, the Numbers pricked will be 21⁗, 1″, 1°, the Greatest Square in 1° is 1°, and the Root thereof 1°, the double whereof is 2, the Divisor to 22′, by which 10′ gotten for the next Quotient Figure, and 10 times 2 taken from 22, leaves 2′ behind. Then the Square of 10′ is 100″, that is 1′, 40″, which substracted in order from the next pricked Number, or added into a Gnomon, with the Multiplication of the Divisor, and substracted leaves 21″. Then doubling the Quotient 1°, 10′, the next Divisor will be 2°, 20′, by which 21″, 1‴, divided, 9″ will be gotten for the Quotient, and the Gnomon to be substracted cut off all the Remain. *Example in the Square.*

Square) 1° 22′ 1″ 1‴ 21⁗ (1° 10′ 9″ Root

21

1

Gnomon { 20 / 1 40

Gnomon { 21 / 1 21

Also to extract the Cube Root of 1°, 35′, 53″, 29‴, 43⁗, 42v, 9vj, the pricked Numbers will be 9vj, 29‴, 1°, the Greatest Cube in 1° is 1°, and the Root thereof 1°, the treble whereof, because 1 doth not multiply, is the Divisor to 35′, and thereby 10′ is gotten for the next Quotient Number, multiplyed by 3 makes 30, which with the Square of 10 increased by the triple of 1, and the Cube added into a Total, make the Gnomon 35′, 16″, 40‴; then will the next Divisor be 4°, 5′, 0″, and the Gnomon to be substracted cut off all the Remain. *Example in the Cube.*

Cube

36 49

Cube) 1° $35'$ $53''$ $29'''$ $43''''$ 42^{v} 9^{vi} (1° $10'$ $9''$ Root

1

Gnomon { 30
5 00
16 40

Gnomon { 36 45 00
4 43 30
12 9

Roots of Compound Astronomicals *extracted.*

To extract the Root of a Compound Astronomical, prick the Number as before according to the Quantity, and out of the pricked Numbers to the Left Hand, having taken the greatest Figurate Number, whose Root you would extract, and placed the Root in the Quotient, you get the Divisor as before, and differ in nothing from the Extractions of the Simple, save as Compound Multiplication or Division differs from the Simple.

Example in the

As to extract the Square Root of $1''$ —$20'''$ +$100''''$, and the Cube Root of $1'''$ —$30''''$ +300^{v} —1000^{vi}, the Numbers pricked in the first are $100''''$ and $1''$, in the other 1000^{vi} and $1'''$, in both the Greatest Square and Cube of 1 is but 1, and the Root of both is but 1, which in the Square will be ′ the half of ″, and in the Cube′ the third part of ‴, this 1 doubled is the Divisor to $20'''$ in the Square, and tripled, because 1 doth not multiply, makes 3 the Divisor to $30''''$ in the Cube, whereby $10''$ is gotten for the Quotient to both, and so proceeding the rest of the work is plain in the Operations following.

Square.

Square) $1''$ —$20'''$ +$100''''$ ($1'$—$10''$ Root

1

—20

+100

+ 2 Divisor — 10
—10 — 10

—20 +100

Cube.

Cube) $1'''$ —$30''''$ +300^{v} —1000^{vi} ($1'$—$10''$ Root

1

—30

+300

—1000

— 10 — 10
+ 3 Divisor — 10 — 10
—10

+100 +100
—30 + 3 — 10

+300 —1000

These Numbers above Exemplary being of such Denominations as will be equally divided by the *Indices* of their Quantities need no Cyphers to be adjoyned, such as do having no other difference in their Extraction than to continue the Extraction to the end of the adjoyned Cyphers, need not be made exemplary here.

Proof of Astronomical Figuration.

Besides the Proof of Production by Extraction, and Extraction by Production, as in other Numbers; if the Astronomicals be reduced into Geodæticals of the Lowest Denomination, and the Compound turned into Simple, and their Figurations compared with the Figurations of the Numbers so reduced as Integers, the works will equally agree.

Examples in the Numbers above-mentioned in this *Chapter.*

Simple.

Root 1° 10' 9''
60

70'
60

Reduced 4209''
4209

37881
84180
16836

Square 17715681''''
4209''

159441129
354313620
70862724

Cube 74565301329^vi

Square 1° 22' 1'' 1''' 21''''
60

82'
60

4921''
60

295261'''
60

17715681''''

Cube 1° 35' 53'' 29''' 43'''' 42^v 9^vi
60

95'
60

5753''
60

345209'''
60

20712583''''
60

1242755022^v
60

74565301329^vi

Square 17715681 | Root 4209
16
Gnomon 164
Gnomon 000
Gnomon 75681

Cube 74565301329 | Root 4209
64
Gnomon 10088
Gnomon 0000
Gnomon 477301329

17715681 | 1 | 295261 | 1 | 4921 | 1 | 82 | 2 | 1 Square

74565301329 | 9 | 1242755022 | 2 | 20712583 | 3 | 345209 | 9 | 5753 | 53 | 95 | 5 | 1 Cube

Compound.

Root 1' — 10''
60

60''
—10

Reduced +50''
50''

Square 2500''''
50''

Cube 125000^vi

Square 1'' — 20''' + 100''''
60

60'''
—20

+40'''
60

2400''''
+ 100

2500''''

Cube 1''' — 30'''' + 300^v — 1000^vi
60

60''''
—30

+30''''
60

1800^v
+ 300

2100^v
60

126000^vi
— 1000

125000^vi

Square 2500 | 50 Root
25
00

Cube 125000 | 50 Root
125
000

1|4 '''' '''
250|0 |41 Square $= 1'' - 20''' + 100'''' = 1'' - 19''' + 40''''$
6 0

2|2 vj 2|4 v ''''
125000|0 |208|3 |34 Cube $= 1''' - 30'''' + 300^{v} - 1000^{vj} = 1''' - 26'''' + 44^{v} - 40^{vj}$
6 0 6 0

CHAP. VIII.

Of the Sexagenary Table.

Sexagenary Table, its use, and why so called.

ENough hath been said of the Simple Elements of *Astronomicals*, to understand them; yet it is convenient to say something of the *Sexagenary Table*, because in Multiplication, Division and Extraction of Roots, before shewed, adding up the several Multiplees in Multiplication, and the Remains in Division and Extraction oftentimes need Reduction by 60, whereby the work is more tedious; therefore is the *Table* made to contain all the Products of any two Numbers under 60, multiplyed together and reduced into the next Denomination, where the Products will bear the same, and from thence called *The Sexagenary Table.*

Triangular Form sufficient.

The *Table* consists of a Quadrangular Form, yet if it were Triangular, as some make it, were sufficient: As may be seen by the upper part thereof above the Black Scale; for that 9 times 10, and 10 times 9, make the Product alike, *viz.* both 90.

Table explained.

The *Table* on each side hath 60 Columns, the outermost on the Left Hand, and the other on the Head, signifie sometimes Multiplicands and Multiplyers, sometimes Quotients and Roots. The other Columns serve to find out the Products of any two Numbers under 60, being multiplyed one by another, and signifie Dividends and Square Numbers, as occasion requireth, among which the Square Numbers are easily discernable, being only those that stand next above the Black Scale.

Those Angles that have 2 Numbers in them, imply that next the Right Hand to be the Number of the lowest Denomination the Product will afford, and the Number next the Left Hand one place higher.

As if I enter with 10'' at the Left Hand, and 20'' at the Head, the Common Angle is 3, 20, which 20 shall be '''', and the 3 shall be one place higher, that is '''.

And if I had entred with 10'' and 20''', then should the Angular 3, 20, be 3'''', 20^{v}, the like is to be understood of all others in finding the true *Index*.

The

Place the *Sexagenary Table* here.

The Use of the Table.

Use of the Table in Multiplication.

In Multiplication, enter the *Table* with the two Factors or multiplying Numbers, the one at the Head, and the other at the Left Side of the *Table*, and the Numbers or Number in the Common Angle is the desired Product. And if there be 2 Numbers, subscribe the Right Hand Number, and carry the Left Hand Number in mind to be added to the next Product, and so proceed till the Multiplicand be run through; and last of all set down the Left Hand Number, if there be any, ordering the *Indices* as above was directed, and adding together all the Multiplees, you have the Total Product.

In Division.

In Division enter with the Divisor at the Head, or the Left Side of the *Table*, and run along with your Eye in the same Column, till you espy a Number equal, or near to your Numbers in the Dividend standing over the first Left Hand Figures of your Divisor entred with: If they be not found exactly, take the lesser, alwayes having respect to the greatness or smallness of the other Figures in the Divisor, and the other outermost Number answering to the Common Angle, shall be the Quotient Figure. For if the Divisor be entred with at the Side, the Quotient is found at the Head, and if at the Head, the contrary. The Quotient Figure found, the Divisor is to be multiplyed thereby, and the Product substracted from the Dividend, as before; and then inquire for another Quotient Figure, and so proceed to the end of the work.

Example in Multiplication

As if 06\`, 05, 20, be multiplyed by 18\`, 15°, 10'. I enter the *Table* with 18 and 20, and find in the Common Angle 6, 0; subscribing the 0, I carry the 6 in mind. Then entring the *Table* with 18 and 5, I find 1, 30; to which 30 I add the 6 in mind, and subscribe the amounting 36, and bear the 1 in mind. Again, I enter the *Table* with 18 and 6, and find 1, 48; to which 1 added is 1, 49, to be set down; and so I have done with the first Multiplyer. And thus proceeding with the rest, and adding all the Multiplees together, the Product at last will be 1\`\`\`, 51\`\`, 8\`, 20°, 53', 20'', as at *A*.

Example in Division.

And if I divide this Product by one of the Factors, as suppose by 18\`, 15°, 10', first I enter the *Table* with 18, and running along in the Column, I look for the Dividend Numbers 1, 51, which I find not in the Column, but the next lesser Number is 1, 48, and over against the same, in the other outermost Column, is 6, which is to be set in the Quotient, and multiplying all the Divisor thereby, the Product 1, 49, 31, 0, I substract and leave remaining 1, 37 for the next inquiry. Then against the divising 18, I look in the *Table* for 1, 37, and find the next lesser Number to be 1, 30, and in the other outermost Column, answering to 18, is found 5, which set in the Quotient, and the Divisor multiplyed thereby, produceth 1, 31, 15, 50'; this substracted leaveth 6, 5, for the next inquiry, which in the Column against 18 is not found in the *Table*, but 6, 0, the next lesser, which hath 20 for the Quotient answering thereto, the Product of which multiplyed into the Divisor, and substracted leaveth 0 behind, as at *B*.

				\`	°	'			
	Multiplicand			6	5	20			(1)
				\`	°	'			
	Multiplyer			18	15	10			(1)
			1	49	36	00			
A	Multiplees			1	31	20	00		
					1	00	53	20	(2)
		\`\`\`	\`\`	\`	°	'	''		
	Total Product	1	51	08	20	53	20		

			6					
Divisor	1	37	5	3			Quotient
18\` 15° 10')	1\`\`\`	51\`\`	08\`	20°	53'	20''	(6\` 5° 20'
B	1	49	31	00			
		1	31	15	50		(2)
			6	5	3	20	(1)
	1	51	08	20	53	20	(1)

Table *useful in Extraction of the* Square Root.
What to be done for Higher Roots.

In Extraction of Roots, forasmuch as Division and Multiplication is used therein, the *Table* is not a little helpful to Astronomical Extraction : Only if a Root higher than the Square be extracted, those higher Quantities substracted at first out of the Numbers belonging to the Left Hand Prick, must be gotten by ordinary Multiplication, no other Figural Numbers but Squares being in the *Table*.

Partis Secundæ Libri Tertii

FINIS.

THE

THE THIRD PART OF THE THIRD BOOK.

CHAP. I.

Of LOGARITHMES.

I Have with all possible brevity transited *Decimals* and *Astronomicals*, and shall now apply my self to overlook *Logarithmes*, martialled in the beginning of this *Book*, in the third rank of Numbers specially Contract.

Logarithmes next ranked in Numbers specially Contract.

Logarithmes, are Numbers artificially prepared for other Numbers, first invented by the Honourable *John Nepeir* Baron of *Marchiston* in *Scotland*, and afterward transformed, and their foundation and use illustrated by the truly Ingenuous Mr. *Henry Briggs*, (from whose Labours I acquainted my self with them) but need say the less of them here, because their excellent use in the *Mathematicks* hath made them familiar to many; for by them, and with much expedition, all troublesome Multiplications and Divisions in *Arithmetick* are avoided, and performed only by Addition instead of Multiplication, and by Substraction instead of Division, The Curious and Laborious Extractions of Roots, are also performed with great Ease, as hereafter shall be shewed. Proportions Disjunct and Continued, Double, Triple, and what else, are thereby made more facil than otherwise can be possible. All Triangles, of what kind soever, with facility resolved. Also not only in *Arithmetick*, but generally in *Geometry*, *Geography*, *Navigation*, *Astronomy*, *&c.* their use is such, as a Volumn of it self is little enough to give Example.

Logarithmes *by whom first invented and illustrated.*

How excellently useful.

They are called *Logarithmes* from the *Greek* word λόγος, which signifieth Reason or Proportion, and ἀριθμὸς, another *Greek* word signifying Numbers. So as the word *Logarithmes* implyeth Rational or Proportional Numbers.

Whence the word, and what implyed thereby.

They have the same Foundation with Decimal and Astronomical *Arithmetick*, as the *Table* in the *First Chapter* of *Decimals* well understood will clearly testifie, because as was there hinted, the uppermost Numbers are in Arithmetical Proportion, and are *Indices* or *Logarithmes*, and the lower in Geometrical Progression or Proportion, and do perform by Multiplication and Division, what the other by Addition and Substraction. For if $3 + 2 = 5$. *Ergo* $1000 \times 100 = 100000$; 3 being the *Index* of 1000, and 2 of 100, the Total of both 5, shall be the *Index* of the Product of 100 multiplyed into 1000, that is 100000; and so of others.

Their foundation one with the Decimals, &c.

The *Indices* of Numbers being thus useful, gave rise to the Invention of Logarithmes, which are indeed nothing else but the *Indices* or Numbers of places in the Higher Powers of Figural Numbers. For if all Integers be advanced into one and the same Quantity of a very High Power, as suppose to the Ten Thousandth Million, then the number of places contained in those Figural Quantities shall be the several Logarithmes for those Integers so advanced.

Their rise from Decimal Indices.

Logarithmes *what they are, and how made.*

To verifie this, you may make tryal with any Number, as suppose 100, the *Index* of which is 2: Let then 100 be multiplyed Figurately to the 10^{th} Power, the Figural Number thereof will be 100000000000000000000, that is an Unit and 20 Cyphers. Then shall the 100^{th} Power be 1 and 200 Cyphers; the 1000^{th} Power 1, and 2000 Cyphers: And so consequently the 10000000000^{th} Power 1, and 20000000000 Cyphers. The Number of places then being so many lacking 1, because the *Index* of

To make the Log. *of* 100.

the Units place is 0, that Number of Cyphers shall be the Logarithme of 100, *viz.* 20000000000. All this is plain by the following Operation.

Example.

		Number of Quantities.	
Root	100	1	2 Decimal Indices
Square	10000	2	4
Cube	1000000	3	6
&c.	100000000	4	8
	10000000000	5	10
	1000000000000	6	12
	100000000000000	7	14
	10000000000000000	8	16
	1000000000000000000	9	18
	100000000000000000000	10	20
1.	40	20	40
1.	80	40	80
1.	120	60	120
1.	160	80	160
1.	200	100	200
1.	2000	1000	2000
1.	20000	10000	20000
1.	200000	100000	200000
1.	2000000	1000000	2000000
1.	20000000	10000000	20000000
1.	200000000	100000000	200000000
1.	2000000000	1000000000	2000000000
1.	20000000000	10000000000	20000000000
	Figural Quantities.	Figural Indices.	Logarithmes.

What being certain is omitted in Logarithmes.

In the use of Logarithmes both the Figural Numbers themselves, and the Figural Indices which declare the Number of their Quantities or Figurate Multiplications, are omitted, and only the Decimal Indices or number of places in such Figural Numbers used. And because these Indices used for Logarithmes have reference to some certain Quantity or Power to which all absolute Integers are to be contracted, therefore they are rightly placed among Contract Numbers. And though the Figural Quantities, and their Figural Indices be omitted, yet they are certain, and may certainly be known by the number of places in the Logarithme.

Explained.

As because the Logarithme of 100 is 20000000000, in which there is 11 places; I know that 100 was multiplyed Figurately 10 Thousand Million of times; for that 10 Thousand of Millions is the 11[th] place in Numeration. And if I use the Logarithme of 100 multiplyed to such an High Quantity, all the other Logarithmes I use must be equivalent; that is, the other Numbers must be multiplyed to the same Power or Quantity of 10 Thousand of Millions, and the number of places, or Decimal Indices, taken for their several Logarithmes.

Tables of Logarithmes *to be gotten ready for use.* *What called the* Canon of Logarithmes. *Tables of Mr.* Briggs *large.* *Large* Tables *best.* *The* Table *following transcribed from his.* *The* Table *explained.*

To have Logarithmes to seek when they should be used, is inconvenient; to make them as above is more tedious than to resolve the Question otherwise without them. To ease the Artist therefore in his work with Logarithmes, *Tables* are to be prepared, which some call, *The Canon of Logarithmes.* Mr. *Briggs* hath fitted Logarithmes for all Whole Numbers from 1 to 100000, which are sufficient enough to serve for any use, seeing thereby the Logarithme of any Number betwixt 100000 and 10000000000 may be found out. And every Practitioner will find a large *Table* most beneficial. But lest those *Tables* may not be in hand, that the Learner may not be altogether unfurnished wherewith to make experiment, and prove the truth of the Exemplary Operations following, I have therefore in the end of this *Chapter* transcribed from Mr. *Briggs*, a *Table* of Logarithmes for all Integers, from an Unit to 1000, which will be sufficient to give Example by.

The *Table* consists of two Columns, in that towards the Left Hand under the Title Numbers, you have the Absolute Numbers from 1 to 1000, in the Right Hand Column, over against the several Numbers you have their Logarithmes.

The Logarithme of 1 is 0; of 10 is 1, with Cyphers; of 100 is 2, with Cyphers; of 1000 is 3, with Cyphers, and so the *Table* may be increased; for the Logarithme of 10000 is 4, with Cyphers; of 100000 is 5, with Cyphers, and so infinitely.

The Left Hand Figure or Cypher of every Logarithme, may fitly be called the *Characteristique* or *Index* of the Logarithme; for it shews how many places the Absolute Number doth consist of, because it is alwayes less by an Unit than the places of the Absolute Number. As the *Index* of all Absolute Numbers under 10 shall be 0, for they have but 1 place, and Unit substracted from Unit leaves 0. But from 10 to 100, the *Index* shall be 1, because the Numbers are but 1 place distant from the Unit. And for the like Reason Numbers from 100 to 1000, shall have 2 for their *Index*; and Numbers that have 4 places, as 1000 hath, and all Numbers from 1000 to 10000 shall have 3 for their *Index*, and so infinitely; as in the *Tables* in the *First Chapter* of *Decimals* is plainly to be seen: Wherefore in some *Tables* the *Characteristique* being so generally known is omitted. Characteristique *or* Index *of the* Logarithme *what, and how made.*

The like is to be observed in Decimal Fractions; from 1 to Tenth Parts the *Index* is 0; from Tenth Parts to 100 Parts, it's 1; from Hundred Parts to 1000 Parts, it's 2. And so decreasing infinitely.

The *Characteristique* is commonly seperated from the rest of the Logarithme by a Cōma, as also the next 5 places from the Remainder by another Cōma, and every 10 Logarithmes in the *Table* from the ensuing by a Line. All which is for no other service, than to help the Eye more readily to discern and find them out. Some omit 2 or 3 of the Right Hand Figures, which breeds no material Error. Cōma, *of what use in* Logarithmes.

For distinction between the Logarithme of Integers, and the Logarithme of Fractions; it is to be noted, that as the Logarithme of a Proper Fraction is defective or negative, so it is to be marked with — the sign of Substraction, either over the Head, or at the Left Hand of the *Index*. And yet the Logarithme of an Improper Fraction, or Mixt Number, that consists of an Whole Number and a Fraction, is not defective, but affirmative or abundant. Logarithme *of a* Fraction *how known from the* Logarithme *of an* Integer.

Numbers

Nu.	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.
1	0,00000,00000	51	1,70757,01761	101	2,00432,13738	151	2,17897,69473
2	0,30102,99957	52	1,71600,33436	102	2,00860,01718	152	2,18184,35879
3	0,47712,12547	53	1,72427,58696	103	2,01283,72247	153	2,18469,14308
4	0,60205,99913	54	1,73239,37598	104	2,01703,33393	154	2,18752,07208
5	0,69897,00043	55	1,74036,26895	105	2,02118,92991	155	2,19033,16982
6	0,77815,12504	56	1,74818,80270	106	2,02530,58653	156	2,19312,45984
7	0,84509,80400	57	1,75587,48557	107	2,02938,37777	157	2,19589,96524
8	0,90308,99870	58	1,76342,79936	108	2,03342,37555	158	2,19865,70870
9	0,95424,25094	59	1,77085,20116	109	2,03742,64979	159	2,20139,71243
10	1,00000,00000	60	1,77815,12504	110	2,04139,26852	160	2,20411,99827
11	1,04139,26852	61	1,78532,98350	111	2,04532,29788	161	2,20682,58760
12	1,07918,12460	62	1,79239,16895	112	2,04921,80227	162	2,20951,50145
13	1,11394,33523	63	1,79934,05495	113	2,05307,84435	163	2,21218,76044
14	1,14612,80357	64	1,80617,99740	114	2,05690,48513	164	2,21484,38480
15	1,17609,12591	65	1,81291,33566	115	2,06069,78404	165	2,21748,39442
16	1,20411,99827	66	1,81954,39355	116	2,06445,79892	166	2,22010,80880
17	1,23044,89214	67	1,82607,48027	117	2,06818,58617	167	2,22271,64711
18	1,25527,25051	68	1,83250,89127	118	2,07188,20073	168	2,22530,92817
19	1,27875,36010	69	1,83884,90907	119	2,07554,69614	169	2,22788,67046
20	1,30102,99957	70	1,84509,80400	120	2,07918,12460	170	2,23044,89214
21	1,32221,92947	71	1,85125,83487	121	2,08278,53703	171	2,23299,61104
22	1,34242,26808	72	1,85733,24964	122	2,08635,98307	172	2,23552,84469
23	1,36172,78360	73	1,86332,28601	123	2,08990,51114	173	2,23804,61031
24	1,38021,12417	74	1,86923,17197	124	2,09342,16852	174	2,24054,92483
25	1,39794,00087	75	1,87506,12634	125	2,09691,00130	175	2,24303,80487
26	1,41497,33480	76	1,88081,35923	126	2,10037,05451	176	2,24551,26678
27	1,43136,37642	77	1,88649,07252	127	2,10380,37210	177	2,24797,32664
28	1,44715,80313	78	1,89209,46027	128	2,10720,99696	178	2,25042,00023
29	1,46239,79979	79	1,89762,70913	129	2,11058,97103	179	2,25285,30310
30	1,47712,12547	80	1,90308,99870	130	2,11394,33523	180	2,25527,25051
31	1,49136,16938	81	1,90848,50189	131	2,11727,12957	181	2,25767,85749
32	1,50514,99783	82	1,91381,38524	132	2,12057,39312	182	2,26007,13880
33	1,51851,39399	83	1,91907,80924	133	2,12385,16410	183	2,26245,10897
34	1,53147,89170	84	1,92427,92861	134	2,12710,47984	184	2,26481,78230
35	1,54406,80444	85	1,92941,89257	135	2,13033,37685	185	2,26717,17284
36	1,55630,25008	86	1,93449,84512	136	2,13353,89084	186	2,26951,29442
37	1,56820,17241	87	1,93951,92526	137	2,13672,05672	187	2,27184,16065
38	1,57978,35966	88	1,94448,26722	138	2,13987,90864	188	2,27415,78493
39	1,59106,46070	89	1,94939,00066	139	2,14301,48003	189	2,27646,18042
40	1,60205,99913	90	1,95424,25094	140	2,14612,80357	190	2,27875,36010
41	1,61278,38567	91	1,95904,13923	141	2,14921,91127	191	2,28103,33672
42	1,62324,92904	92	1,96378,78273	142	2,15228,83444	192	2,28330,12287
43	1,63346,84556	93	1,96848,29486	143	2,15533,60375	193	2,28555,73090
44	1,64345,26765	94	1,97312,78536	144	2,15836,24921	194	2,28780,17299
45	1,65321,25138	95	1,97772,36053	145	2,16136,80022	195	2,29003,46114
46	1,66275,58317	96	1,98227,12330	146	2,16435,28558	196	2,29225,60714
47	1,67209,78579	97	1,98677,17343	147	2,16731,73347	197	2,29446,62262
48	1,68124,12374	98	1,99122,60757	148	2,17026,17154	198	2,29666,51903
49	1,69019,60800	99	1,99563,51946	149	2,17318,62684	199	2,29885,30764
50	1,69897,00043	100	2,00000,00000	150	2,17609,12591	200	2,30102,99957

Nu	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.
201	2,30319,60574	251	2,39967,37215	301	2,47856,64956	351	2,54530,71165
202	2,30535,13694	252	2,40140,05408	302	2,48000,69430	352	2,54654,26635
203	2,30749,60379	253	2,40312,05212	303	2,48144,26285	353	2,54777,47054
204	2,30963,01674	254	2,40483,37166	304	2,48287,35836	354	2,54900,32620
205	2,31175,38611	255	2,40654,01804	305	2,48429,98393	355	2,55022,83531
206	2,31386,72204	256	2,40823,99653	306	2,48572,14265	356	2,55144,99980
207	2,31597,03455	257	2,40993,31233	307	2,48713,83755	357	2,55266,82161
208	2,31806,33350	258	2,41161,97060	308	2,48855,07165	358	2,55388,30266
209	2,32014,62861	259	2,41329,97641	309	2,48995,84794	359	2,55509,44486
210	2,32221,92947	260	2,41497,33480	310	2,49136,16938	360	2,55630,25008
211	2,32428,24553	261	2,41664,05073	311	2,49276,03890	361	2,55750,72019
212	2,32633,58609	262	2,41830,12913	312	2,49415,45940	362	2,55870,85705
213	2,32837,96034	263	2,41995,57485	313	2,49554,43376	363	2,55990,66250
214	2,33041,37733	264	2,42160,39269	314	2,49692,96481	364	2,56110,13836
215	2,33243,84599	265	2,42324,58739	315	2,49831,05538	365	2,56229,28645
216	2,33445,37512	266	2,42488,16366	316	2,49968,70826	366	2,56348,10854
217	2,33645,97338	267	2,42651,12614	317	2,50105,92622	367	2,56466,60643
218	2,33845,64936	268	2,42813,47940	318	2,50242,71200	368	2,56584,78187
219	2,34044,41148	269	2,42975,22800	319	2,50379,06831	369	2,56702,63662
220	2,34242,26808	270	2,43136,37642	320	2,50514,99783	370	2,56820,17241
221	2,34439,22737	271	2,43296,92909	321	2,50650,50324	371	2,56937,39096
222	2,34635,29745	272	2,43456,89040	322	2,50785,58717	372	2,57054,29399
223	2,34830,48630	273	2,43616,26470	323	2,50920,25223	373	2,57170,88318
224	2,35024,80183	274	2,43775,05628	324	2,51054,50102	374	2,57287,16022
225	2,35218,25181	275	2,43933,26938	325	2,51188,33610	375	2,57403,12677
226	2,35410,84391	276	2,44090,90821	326	2,51321,76001	376	2,57518,78449
227	2,35602,48572	277	2,44247,97691	327	2,51454,77527	377	2,57634,13502
228	2,35793,48470	278	2,44404,47959	328	2,51587,38437	378	2,57749,17998
229	2,35983,54823	279	2,44560,42033	329	2,51719,58979	379	2,57863,92100
230	2,36172,78360	280	2,44715,80313	330	2,51851,39399	380	2,57978,35966
231	2,36361,19799	281	2,44870,63199	331	2,51982,79938	381	2,58092,49757
232	2,36548,79849	282	2,45024,91083	332	2,52113,80837	382	2,58206,33629
233	2,36735,59210	283	2,45178,64355	333	2,52244,42335	383	2,58319,87740
234	2,36921,58574	284	2,45331,83400	334	2,52374,64668	384	2,58433,12244
235	2,37106,78623	285	2,45484,48600	335	2,52504,48070	385	2,58546,07295
236	2,37291,20030	286	2,45636,60331	336	2,52633,92774	386	2,58658,73047
237	2,37474,83460	287	2,45788,18967	337	2,52762,99009	387	2,58771,09650
238	2,37657,69571	288	2,45939,24878	338	2,52891,67003	388	2,58883,17256
239	2,37839,79009	289	2,46089,78428	339	2,53019,96982	389	2,58994,96013
240	2,38021,12417	290	2,46239,79979	340	2,53147,89170	390	2,59106,46070
241	2,38201,70426	291	2,46389,29890	341	2,53275,43790	391	2,59217,67574
242	2,38381,53660	292	2,46538,28514	342	2,53402,61061	392	2,59328,60670
243	2,38560,62736	293	2,46686,76204	343	2,53529,41200	393	2,59439,25504
244	2,38738,98263	294	2,46834,73304	344	2,53655,84426	394	2,59549,62218
245	2,38916,60844	295	2,46982,20160	345	2,53781,90951	395	2,59659,70956
246	2,39093,51071	296	2,47129,17111	346	2,53907,60988	396	2,59769,51859
247	2,39269,69533	297	2,47275,64493	347	2,54032,94748	397	2,59879,05068
248	2,39445,16808	298	2,47421,62641	348	2,54157,92439	398	2,59988,30721
249	2,39619,93471	299	2,47567,11883	349	2,54282,54270	399	2,60097,28957
250	2,39794,00087	300	2,47712,12547	350	2,54406,80444	400	2,60205,99913

Nu.	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.
401	2,60314,43726	451	2,65417,65419	501	2,69983,77259	551	2,74115,15989
402	2,60422,60531	452	2,65513,84348	502	2,70970,37171	552	2,74193,90777
403	2,60530,50461	453	2,65609,82020	503	2,70156,79851	553	2,74272,51313
404	3,60638,13651	454	2,65705,58529	504	2,70243,05364	554	2,74350,97647
405	2,60745,50232	455	2,65801,13957	505	2,70329,13781	555	2,74429,29831
406	2,60852,60336	456	2,65896,48427	506	2,70415,05168	556	2,74507,47916
407	2,60959,44092	457	2,65991,62001	507	2,70500,79593	557	2,74585,51952
408	2,61066,01631	458	2,66086,54780	508	2,70586,37123	558	2,74663,41990
409	2,61172,33080	459	2,66181,26855	509	2,70671,77823	559	2,74741,18079
410	2,61278,38567	460	2,66275,78317	510	2,70757,01761	560	2,74818,80270
411	2,61384,18219	461	2,66370,09254	511	2,70842,09001	561	2,74896,28613
412	2,61489,72160	462	2,66464,19756	512	2,70926,99610	562	2,74973,63156
413	2,61595,00517	463	2,66558,09910	513	2,71011,73651	563	2,75050,83949
414	2,61700,03411	464	2,66651,79806	514	2,71096,31190	564	2,75127,91040
415	2,61804,80967	465	2,66745,[illegible]9529	515	2,71180,72290	565	2,75204,84478
416	2,61909,33306	466	2,66838,59167	516	2,71264,97016	566	2,75281,64312
417	2,62013,60550	467	2,66931,68806	517	2,71349,05431	567	2,75358,30589
418	2,62117,62818	468	2,67024,5853[illegible]	518	2,71432,97597	568	2,75434,83357
419	2,62221,40230	469	1,67117,28427	519	2,71516,73578	569	2,75511,22664
420	2,62324,92904	470	2,67209,78579	520	2,71600,33436	570	2,75587,48557
421	2,62428,20958	471	2,67302,09071	521	2,71683,77233	571	2,75663,61082
422	2,62531,24510	472	2,67394,19986	522	2,71767,05030	572	2,75739,60288
423	2,62634,03674	573	2,67486,11407	523	2,71850,16889	573	2,75815,46220
424	2,62736,58506	474	2,67577,83417	524	2,71933,12870	574	2,75891,18924
425	2,62838,89301	475	2,67669,36096	525	2,72015,93034	575	2,75966,78447
426	2,62940,95991	476	2,67760,69527	526	2,72098,57442	576	2,76042,24834
427	2,63042,78750	477	2,67851,83750	527	2,72181,06152	577	2,76117,58132
428	2,63144,37690	478	2,67942,78966	528	2,72263,39225	578	2,76192,78384
429	2,63245,72922	479	2,68033,55134	529	2,72345,56720	579	2,76267,85637
430	2,63346,84556	480	2,68124,12374	530	2,72427,58696	580	2,76342,79936
431	2,63447,72702	481	2,68214,50764	531	2,72509,45211	581	2,76417,61324
432	2,63548,37468	482	2,68304,70382	532	2,72591,16323	582	2,76492,29846
433	2,63648,78964	483	2,68394,71308	533	2,72672,72090	583	2,76566,85548
434	2,63748,97295	484	2,68484,53616	534	2,72754,12570	584	2,76641,28471
435	2,63848,92570	485	2,68574,17386	535	2,72835,37820	585	2,76715,58661
436	2,63948,64893	486	2,68663,62693	536	2,72916,47897	586	2,76789,76160
437	2,64048,14370	487	2,68752,89612	537	2,72997,42857	587	2,76863,81012
438	2,64147,41105	488	2,68841,98220	538	2,73078,22757	588	2,76937,73261
439	2,64246,45202	489	2,68930,88591	539	2,73158,87652	589	2,77011,52948
440	2,64345,26765	490	2,69019,60800	540	2,73239,37598	590	2,77085,20116
441	2,64443,85895	491	2,69108,14921	541	2,73319,72651	591	2,77158,74809
442	2,64542,22693	492	2,69196,51028	542	2,73399,92865	592	2,77232,17067
443	2,64640,37262	493	2,69284,69193	543	2,73479,98296	593	2,77305,46934
444	2,64738,27901	494	2,69372,69489	544	2,73559,88997	594	2,77378,64450
445	2,64836,00110	495	2,69460,51989	545	2,73639,65023	595	2,77451,69657
446	2,64933,48587	496	2,69548,16765	546	2,73719,26427	596	2,77524,62597
447	2,65030,75231	497	2,69635,63887	547	2,73798,73263	597	2,77597,43311
448	2,65127,80140	498	2,69722,93428	548	2,73878,05585	598	2,77670,11840
449	2,65224,63410	499	2,69810,05456	549	2,73957,23445	599	2,77742,68224
450	2,65321,25138	500	2,69897,00043	550	2,74036,26895	600	2,77815,12504

Nu	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.
601	2,77887,44720	651	2,81358,09886	701	2,84571,80180	751	2,87563,99370
602	2,77959,64913	652	2,81424,75957	702	2,84633,71121	752	2,87621,78406
603	2,78031,73121	653	2,81491,31813	703	2,84695,53250	753	2,87679,49762
604	2,78103,69386	654	2,81557,77483	704	2,84757,26591	754	2,87737,13459
605	2,78175,53747	655	2,81624,13000	705	2,84818,91170	755	2,87794,69516
606	2,78247,26242	656	2,81690,38394	706	2,84880,47011	756	2,87852,17955
607	2,78318,86911	657	2,81756,53696	707	2,84941,94138	757	2,87909,58795
608	2,78390,35793	658	2,81822,58936	708	2,85003,32577	758	2,87966,92056
609	2,78461,72926	659	2,81888,54146	709	2,85064,62352	759	2,88024,17759
610	2,78532,98350	660	2,81954,39355	710	2,85125,83487	760	2,88081,35923
611	2,78604,12102	661	2,82020,14595	711	2,85186,96007	761	2,88138,46568
612	2,78675,14221	662	2,82085,79894	712	2,85247,99936	762	2,88195,49713
613	2,78746,04745	663	2,82151,35284	713	2,85308,95299	763	2,88252,45380
614	2,78816,83711	664	2,82216,80794	714	2,85369,82118	764	2,88309,33586
615	2,78887,51158	665	2,82282,16453	715	2,85430,60418	765	2,88366,14352
616	2,78958,07122	666	2,82347,42292	716	2,85491,30223	766	2,88422,87696
617	2,79028,51640	667	2,82412,58339	717	2,85551,91557	767	2,88479,53639
618	2,79098,84751	668	2,82477,64625	718	2,85612,44442	768	2,88536,12200
619	2,79169,05490	669	2,82542,61178	719	2,85672,88904	769	2,88592,63398
620	2,79239,16895	670	2,82607,48027	720	2,85733,24964	770	2,88649,07252
621	2,79309,16002	671	2,82672,25202	721	2,85793,52647	771	2,88705,43781
622	2,79379,03847	672	2,82736,92731	722	2,85853,71976	772	2,88761,73003
623	2,79448,80467	673	2,82801,50642	723	2,85913,82973	773	2,88817,94939
624	2,79518,45897	674	2,82865,98965	724	2,85973,85662	774	2,88874,09607
625	2,79588,00173	675	2,82930,37728	725	2,86033,80066	775	2,88930,17025
626	2,79657,43332	676	2,82994,66959	726	2,86093,66207	776	2,88986,17213
627	2,79726,75408	677	2,83058,86687	727	2,86153,44109	777	2,89042,10188
628	2,79795,96437	678	2,83122,96939	728	2,86213,13793	778	2,89097,95970
629	2,79865,06454	679	2,83186,97743	729	2,86272,75283	779	2,89153,74577
630	2,79934,05495	680	2,83250,89127	730	2,86332,28601	780	2,89209,46027
631	2,80002,93592	681	2,83314,71119	731	2,86391,73770	781	2,89265,10339
632	2,80071,70783	682	2,83378,43747	732	2,86451,10811	782	2,89320,67531
633	2,80140,37100	683	2,83442,07037	733	2,86510,39746	783	2,89376,17621
634	2,80208,92579	684	2,83505,61017	734	2,86569,60599	784	2,89431,60627
635	2,80277,37253	685	2,83569,05715	735	2,86628,73391	785	2,89486,96567
636	2,80345,71156	686	2,83632,41157	736	2,86687,78143	786	2,89542,25460
637	2,80413,94323	687	2,83695,67371	737	2,86746,74879	787	2,89597,47324
638	2,80482,06787	688	2,83758,84382	738	2,86805,63618	788	2,89652,62175
639	2,80550,08582	689	2,83821,92219	739	2,86864,44384	789	2,89707,70032
640	2,80617,99740	690	2,83884,90907	740	2,86923,17197	790	2,89762,70913
641	2,80685,80295	691	2,83947,80474	741	2,86981,82080	791	2,89817,64835
642	2,80753,50281	692	2,84010,60945	742	2,87040,39053	792	2,89872,51816
643	2,80821,09729	693	2,84073,32346	743	2,87098,88138	793	2,89927,31873
644	2,80888,58674	694	2,84135,94705	744	2,87157,29355	794	2,89982,05024
645	2,80955,97146	695	2,84198,48046	745	2,87215,62727	795	2,90036,71287
646	2,81023,25180	696	2,84260,92396	746	2,87273,88275	796	2,90091,30677
647	2,81090,42807	697	2,84323,27781	747	2,87332,06018	797	2,90145,83214
648	2,81157,50059	698	2,84385,54226	748	2,87390,15979	798	2,90200,28914
649	2,81224,46968	699	2,84447,71757	749	2,17448,18177	799	2,90254,67793
650	2,81291,33566	700	2,84509,80400	750	2,87506,12634	800	2,90308,99870

Numb.

Nu.	Logarithmes.	Nu	Logarithmes.	Nu.	Logarithmes.	Nu.	Logarithmes.
801	2,90363,25161	851	2,92992,95601	901	2,95472,47910	951	2,97818,05169
802	2,90417,43683	852	2,93043,95948	902	2,95520,65375	952	2,97863,69484
803	2,90471,55453	853	2,93094,90312	903	2,95568,77503	953	2,97909,29006
804	2,90525,60487	854	2,93145,78707	904	2,95616,84305	954	2,97954,83747
805	2,90579,58804	855	2,93196,61147	905	2,95664,85792	955	2,98000,33716
806	2,90633,50418	856	2,93247,37647	906	2,95712,81977	956	2,98045,78923
807	2,90687,35347	857	2,93298,08219	907	2,95760,72871	957	2,98091,19378
808	2,90741,13608	858	2,93348,72878	908	2,95808,58485	958	2,98136,55091
809	2,90794,85216	859	2,93399,31638	909	2,95856,38832	959	2,98181,86072
810	2,90848,50189	860	2,93449,84512	910	2,95904,13923	960	2,98227,12330
811	2,90902,08542	861	2,93500,31515	911	2,95951,83770	961	2,98272,33877
812	2,90955,60292	862	2,93550,72658	912	2,95999,48383	962	2,98317,50720
813	2,91009,05456	863	2,93601,07958	913	2,96047,07775	963	2,98362,62871
814	2,91062,44049	864	2,93651,37425	914	2,96094,61957	964	2,98407,70339
815	2,91115,76087	865	2,93701,61075	915	2,96142,10941	965	2,98452,73133
816	2,91169,01588	866	2,93751,78920	916	2,96189,54737	966	2,98497,71264
817	2,91222,20565	867	2,93801,90975	917	2,96236,93357	967	2,98542,64741
818	2,91275,33037	868	2,93851,97252	918	2,96284,26812	968	2,98587,53573
819	2,91328,39018	869	2,93901,97765	919	2,96331,55114	969	2,98632,37771
820	2,91381,38524	870	2,93951,92526	920	2,96378,78273	970	2,98677,17343
821	2,91434,31571	871	2,94001,81550	921	2,96425,96302	971	2,98721,92299
822	2,91487,18175	872	2,94051,64849	922	2,96473,09211	972	2,98766,62649
823	2,91539,98352	873	2,94101,42437	923	2,96520,17010	973	2,98811,28403
824	2,91592,72117	874	2,94151,14326	924	2,96567,19712	974	2,98855,89569
825	2,91645,39485	875	2,94200,80530	925	2,96614,17327	975	2,98900,46157
826	2,91698,00473	876	2,94250,41062	926	2,96661,09867	976	2,98944,98177
827	2,91750,55096	877	2,94299,95934	927	2,96707,97341	977	2,98989,45637
828	2,91803,03368	878	2,94349,45159	928	2,96754,79762	978	2,99033,88548
829	2,91855,45306	879	2,94399,88751	929	2,96801,57140	979	2,99078,26918
830	2,91907,80924	880	2,94448,26722	930	2,96848,29486	980	2,99122,60757
831	2,91960,10238	881	2,94497,59084	931	2,96894,96810	981	2,99166,90074
832	2,92012,33263	882	2,94546,85851	932	2,96941,59124	982	2,99211,14878
833	2,92064,50014	883	2,94596,07036	933	2,96988,16437	983	2,99255,35178
834	2,92116,60506	884	2,94645,22650	934	2,97034,68762	984	2,99299,50984
835	2,92168,64755	885	2,94694,32707	935	2,97081,16109	985	2,99343,62305
836	2,92220,62774	886	2,94743,37219	936	2,97127,58487	986	2,99387,69149
837	2,92272,54580	887	2,94792,36198	937	2,97173,95909	987	2,99431,71527
838	2,92324,40186	888	2,94841,29658	938	2,97220,28384	988	2,99475,69446
839	2,92376,19608	889	2,94890,17610	939	2,97266,55923	989	2,99519,62916
840	2,92427,92861	890	2,94939,00066	940	2,97312,78536	990	2,99563,51946
841	2,92479,59958	891	2,94987,77040	941	2,97358,96234	991	2,99607,36545
842	2,92531,20915	892	2,95036,48544	942	2,97405,09028	992	2,99651,16722
843	2,92582,75746	893	2,95085,14589	943	2,97451,16927	993	2,99694,92485
844	2,92634,24466	894	2,95133,75188	944	2,97497,19943	994	2,99738,63844
845	2,92685,67089	895	2,95182,30353	945	2,97543,18085	995	2,99782,30807
846	2,92737,03630	896	2,95230,80097	946	2,97589,11364	996	2,99825,93384
847	2,92788,34103	897	2,95279,24430	947	2,97634,99790	997	2,99869,51583
848	2,92839,58523	898	2,95327,63367	948	2,97680,83373	998	2,99913,05413
849	2,92890,76902	899	2,95375,96917	949	2,97726,62124	999	2,99956,54882
850	2,92941,89257	900	2,95424,25094	950	2,97772,36053	1000	3,00000,00000

CHAP.

CHAP. II.

Reduction of Logarithmes.

TO operate by Logarithmes, two things are necessary. *What necessary to work with Logarithmes.*

1\. *First*, That for every Integer from an Unit upward, and for every Fraction from an Unit downward, *ad infinitum*, we know how to fit a Logarithme thereto; only an Unit, which neither multiplyeth nor divideth, needeth no Logarithme.

2\. *Secondly*, That for every Logarithme we be able to find out the Integer, or Fraction, Common or Decimal, that answers thereto. Both these are to be found under the *Propositions* following in this *Chapter* of *Reduction*, called therefore by some, *Invention of Logarithmes*. Reduction *sometime called* Invention of Logarithmes.

Proposition I. *To find a Logarithme for any Absolute Number under* 1000, *expressed in the* Table. Prop. 1. *To find a* Logarithme *for a Number under* 1000.

Seek in the Left Hand Column of the *Table*, under the Title Numbers, for the Number whose Logarithme is desired; and over against the same, in the Right Hand Column, you shall find the Logarithme answering thereto.

As if the Logarithme of 12 be desired; over against 12 stands in the *Table*, 1,07918,12460, which is the Logarithme thereof. So the Logarithme of 340 is found to be 2,53147,89170. *Example.*

Prop. II. *To find a Logarithme for any Absolute Number above* 1000. Prop. 2. *To find a* Logarithme *for a Number above* 1000.

Where the Tables of Logarithmes are large enough, the Logarithme is to be found, as the Logarithme of any Number under 1000 is to be found in the *Table* foregoing.

But where the Tables are too short, proceed thus;

1. According to the first Proposition find the first 3, or more Left Hand Figures of your given Number, as far as your Tables will serve.
2. Instead of the *Index* of the Logarithme found, place another which shall fit the Number given.
3. Take the difference between the Logarithme found, and the next ensuing, and take the rest of the Figures that remain to the Number given, after the 3 or more Left Hand Figures be cut off.
4. By these get a proportional part thus: As an Unit and so many Cyphers, as there be Figures remaining, to the same Figures, so shall the difference between the Logarithme found, and that which follows, be to another Number, which found is to be added to the Logarithme before found; and the summe you may take for the quesited Logarithme. And though there will be some little alteration from the true Logarithme, yet the difference being inconsiderable makes the Error immaterial.

Example. If the Logarithme of 99945, be sought, then by the *Table* the Logarithme of 999, the first 3 Left Hand Figures is seen to be 2,99956,54882. The *Index* of 99945, must be 4, therefore I alter the *Index*, and make the Logarithme 4,99956,54882. The difference between the Logarithme of 999, and the Logarithme of 1000, the next is 43,45118. The Figures that remain besides 999 are 45. Then the Analogy is thus: As 100. 45 :: 4345118. 1955303; for by multiplying 45 into 4345118, and dividing the Product by 100, that Number of 1955303 is gotten, which added to the Logarithme before found makes 4,99976,10185 the Logarithme desired; and differs but little with the true Logarithme in the Tables of Mr. *Briggs*, found there to be 4,99976,10723. *Example.*

Prop. III. *To find the Logarithme of a Common Fraction, or Integer and Fraction Mixt.* Prop. 3. *To find the* Logarithme *of a Common Fraction.*

Vulgar Fractions in the *Second Part* of the *First Book*, were considered, as Proper, Equal, and Improper.

Equal Fractions being alwayes an Unit, have 0 for their Logarithme. *Equal.*

The Logarithme of Proper or Improper Fractions are found by substracting the Logarithme of the Lesser Term out of the Logarithme of the Greater. The remain shall be the Logarithme of the Fraction, which shall be affirmative if the Fraction be Improper, but negative if the Fraction be Proper. *Proper or Improper.*

As to find the Logarithme of $\frac{3}{4}$ or $\frac{4}{3}$, the Logarithme of each is 0,12493,87366, but of the Proper Fraction negative, of the other affirmative. *Examples.*

Proper Fraction $\frac{3}{4}$.	Improper Fraction $\frac{4}{3}$.
0,60205,99913 Log. of 4.	0,60205,99913 Log. of 4.
0,47712,12547 Log. of 3.	0,47712,12547 Log. of 3.
—0,12493,87366 Log. of $\frac{3}{4}$ differ.	0,12493,87366 Log. of $\frac{4}{3}$.

Mixt Numbers firſt to be reduced into an Improper Fraction. Example.

If the Logarithme of a Mixt Number be ſought, reduce the ſame into an Improper Fraction, and ſeek the Logarithme thereof as before.

As to get the Logarithme of $2\frac{1}{2}$, reduce it into $\frac{5}{2}$, and taking the Logarithme of 2 from the Logarithme of 5, find the Log. deſired of $\frac{5}{2}$ or $2\frac{1}{2}$ to be 0,39794,00086.

Arithmetical Complement what. Logarithme of the Decimal Fraction how got. Example.

If the Logarithme of the Denominator of any Common Fraction be ſubſtracted out of the Logarithme of the Numerator, the Remain will differ nothing, ſave in the *Index*, from the Arithmetical Complement (which is the Remainder of any Logarithme ſubſtracted out of 10 with Cyphers); and this Complement, with the true *Index*, may be taken for the Logarithme of the Fraction, but is properly the Logarithme of the Decimal Fraction; for ſo the Logarithme of a Decimal Fraction may be gotten.

As in the former inſtance of $\frac{3}{4}$, taking the Logarithme of 4 from the Logarithme of 3, in the *Index* there will lack an Unit, therefore marked negative. The Logarithme by this way is found to be —1,87506,12634, which Logarithme in the *Table* anſwers to 75, being here negative ſhall be the Logarithme of '75, or $\frac{75}{100}$, which is $\frac{3}{4}$: For 75 is $\frac{3}{4}$ of 100. And this Logarithme and the former make up the Logarithme of 1, as at *A.* may be ſeen, by adding them together, and differs not from the Arithmetical Complement, except in the *Index*, as following may be ſeen at *B.*

Proper Fraction $\frac{3}{4}$.
0,47712,12547 Log. of 3.
0,60205,99913 Log. of 4.

—1,87506,12634 Log. of $\frac{3}{4}$ of 100, or '75.

A.	*B.*
—1,87506,12634 Log. of '75.	10,00000,00000 10, and Cyphers.
—0,12493,87366 Log of $\frac{3}{4}$.	0,12493,87366 Log. of $\frac{3}{4}$.
0,00000,00000 Log. of 1.	9,87506,12634 Arith. Compl.

Notice to be taken how the Logarithme of the Fraction is gotten.

But great notice is to be taken, whether the Logarithme of the Fraction be gotten this or the other way: For that in Addition and Subſtraction, Multiplication and Diviſion of Logarithmes hereafter, the Caſe will differ between the Logarithme of the Decimal, and the Logarithme of the Common Fraction.

Prop. 4. To find the Logarithme of a Decimal Pure or Mixt.

Prop. IV. *To find the Logarithme of a Decimal, or Integers mixt with Decimals.*

Beſides the way of getting the Decimal Logarithme of a Fraction laſt mentioned, take this General Rule.

Suppoſe the Numbers whoſe Logarithme is ſought to be an Integer, and find the Logarithme thereof accordingly, then prefix an *Index* thereto according to the diſtance of the firſt Left Hand Figure of the given Number from unity: For the Characteriſtique alwayes differs according to the nature of the Number, though the reſt of the Logarithme may be the ſame. Examples.

Examples.

	Numbers.	Logarithmes.
Integers	485	2,68574,17386.
Mixt	48,5	1,68574,17386.
	4,85	0,68574,17386.
Decimals	,485	—1,68574,17386.
	,0485	—2,68574,17386.

Prop. 5. To find the Number for a Logarithme given.

Prop. V. *To find the Abſolute Number correſponding to a Logarithme given, be it integral or mixed.*

Thoſe Logarithmes that anſwer to Integers or Mixt Numbers may be found in the *Tables.*

1. Either

1. Either the Logarithme with the *Index*. Or,
2. The Logarithme with another *Index* Greater or Lesser. Or,
3. The *Index* with another Logarithme. Or else,
4. Neither *Index* nor Logarithme exactly. And therefore,

If the Logarithme be in the Tables.

If the Logarithme be expressed in the Tables, then by the orderly increase or decrease of the Logarithmes, seeking in the Column under the Title Logarithmes, you will soon find the Logarithme sought, and just against it under the Title Numbers in the Left Hand Column, you find the Absolute Number that answers thereto.

Example.

As if the Logarithme 2,15836,24921 be given, and the Absolute Number belonging thereto be desired, I look in the *Table* and find the Logarithme, and in the Left Hand Column 141 to be the corresponding Number.

If the Logarithme have an Index Greater or Lesser.

When found with a Lesser.

If the Logarithme given have a greater *Index* than is to be found in the *Tables*, then considering that the Logarithme of 2 is the Logarithme of 20, only altering the *Index*; and so the Logarithme of 3 the same with 30, of 4 with 40, *&c.* the Logarithme of 11 the same with 110; the Logarithme of 12 with 120, *&c.* It is easie having found the Logarithme with the least *Index* in the *Tables* to produce the true Number corresponding to the Greatest *Index*, by adding to the Right Hand of the Number answering the Logarithme found with the least *Index*, so many Cyphers as there are Units in the *Index* of the given Logarithme more than in the *Index* of the Logarithme found.

Example.

As if 4,30102,99957, be the Logarithme given, neglecting the *Index* I look in the *Tables*, and find the Logarithme against 2, 20, 200, *&c.* but all of a different *Index*. I take that of 2, being the least, and adjoyn to 2 the Number corresponding 4 Cyphers, because the *Index* of the Logarithme given was 4, and the *Index* of 2 was 0. So is 20000 the Number answering to the Logarithme 4,30102,99957.

When found with a Greater Index.

If the Logarithme given be not precisely found in the *Table*, in the proper place, according to the *Index* thereof, or with a lesser *Index*, then the same may be sought for among the Logarithmes of a greater *Index*. And if found there you shall have the absolute Number thereof in more Figures than the *Index* of the given Logarithme requires. Wherefore cut off so many of the Right Hand Figures as are superfluous; for the Numerator of a Fraction, whose Denominator shall be an Unit with so many Cyphers as there be Figures in the Numerator, or it may be set as a Decimal.

Example.

Example. Let the Logarithme given be 1,09691,00130, which sought for among the Logarithmes whose *Index* is 1, cannot be found exactly, but is found among the Logarithmes that have 2 for their *Index*, and over against the same the absolute Number 125, which consists of 3 Figures, whereas the *Index* of the given Logarithme being but 1, required but 2 Figures in the absolute Number, therefore I cut off the last Right Hand Figure of 125, and leave 2, *viz.* 12, and the 5 is Numerator of a Fraction to 10 the Denominator; so shall the absolute Number be $12\frac{5}{10}$, or 12,5.

☞ And if the given Logarithme had been found with a greater *Index* than 2, as it happeneth oftentimes in large Tables, such as those of Mr. *Briggs* are: As suppose 1,04328,36656, the Logarithme given and found under the *Index* 4, and the corresponding Number 11048, then should the Number be 11,048, or $11\frac{048}{1000}$. And if the *Index* of the given Logarithme had been 2, then $110\frac{48}{100}$. If 3, then $1104\frac{8}{10}$, *&c.*

If the Index be found with another Logarithme.

If the *Index* be found with another Logarithme than that given, and the *Table* not large enough to find it with a greater *Index*, then enter the *Table* with the *Index* of the Logarithme given, and find the next lesser Logarithme to the given Logarithme, and you have the Integer answering thereto, to which a Fraction is to be adjoyned, which is thus gotten.

Substract the Logarithme found from the Logarithme given, the Remain shall be the Numerator, and the difference between the Logarithme found, and that which next follows in the *Table* shall be the Denominator of the Fraction.

Example.

As in the former instance; if 1,09691,00130, be the given Logarithme, the next lesser Logarithme found in the *Table* with the *Index* 1, is 1,07918,12460, and the absolute Number answering thereto is 12; then substracting the Logarithme of 12 from the Logarithme given, the Remain is 1772,87670, which shall be the Numerator to 3476,21063, the difference between the Logarithme of 12 and the Logarithme of 13. And being near $\frac{1}{2}$, by cutting off many of the Right Hand Figures, (without sensible Error) may be reduced to $\frac{17}{34}$ equivalent to $\frac{5}{10}$ or $\frac{1}{2}$, as before.

Best to be turned into a Decimal.

Or rather, turn it into a Decimal Fraction, which is thus done; adjoyn Cyphers to the Right Hand of the Difference or Remain after the next lesser Logarithme in the *Table* is substracted from the given Logarithme: And divide this Number with the Cyphers so adjoyned by the Difference between the next Lesser and next Greater Logarithmes found in the *Table*; wherefore if to 1772,87670, there be but 2 Cy- *Example.*

phers

phers adjoyned, and the ſame be divided by 3476,21063, the Quotient will be 51, to be added to 12 as a Decimal, and ſo the Logarithme given, ſhall be the Logarithme for 12,51, and by adjoyning more Cyphers, and continuing the Diviſion, the Decimal will be greater.

9278
347 1385 7
1772,8767000 (51
3476,21063
3476,2106

If Index *nor* Logarithme *be found.*

If neither *Index* nor Logarithme be found exactly in the *Table*, proceed thus; under the Greateſt *Index* your Tables will afford, find the next leſſer Logarithme to the Logarithme given, neglecting the *Indices* of both, and reſerve the Number anſwering to the Logarithme found apart, and note the true *Index* of that Number. Then ſubſtract the Logarithme found from the Logarithme given, and with the Difference between the Logarithme found and that which next follows in the *Table*, you may get a proportional part by this Analogy. As the Difference between the Logarithme found and the following, to the Difference between the Logarithme found, and the Logarithme given; ſo is an Unit with ſo many Cyphers as there are Abſolute Numbers wanting in the Number found to make up the *Index* of the Logarithme given, to the ſame Numbers wanting, which Numbers gotten adjoyn to the Right Hand.

Example.

As if the Logarithme given be 4,99976,10185, under the greateſt *Index* in the foregoing *Table* 2, I find 99956,54882, the next Logarithme to the Logarithme given, and the correſponding Number to be 999, the true *Index* whereof is 2, the Logarithme of 999 taken from the given Logarithme leaves 19,55303, the Difference between the Logarithme of 999, and the Logarithme of 1000 is 43,45118, the *Index* of the given Logarithme is 4, and of 999 but 2, therefore 2 places are wanting: Then I ſay, As 43,45118, to 19,55303, ſo is 100 to 45, almoſt; which 45 adjoyned to 999, makes 99945, for the Number correſponding to the given Logarithme.

434510
2 172558 8
19,55303000 (44 and above.
43,451188
43,4511

And in like manner the Abſolute Number anſwering to Logarithmes given that have 6, 7, 8, or more Units in their *Index* may be had exact enough for Operation.

Prop. 6. To find a Fraction for a Logarithme.

Prop. VI. *To find a Fraction for a given Logarithme.*

Fractions deſired for a given Logarithme may be either Common or Decimal. The Decimal much the eaſier to be found, and therefore in common uſe the Logarithmes of them are beſt to be choſen; yet ſometime the Logarithme of a Vulgar Fraction may be neceſſary: The way to get the Logarithmes of both, ſee in the *Sections* following.

Logarithme of a Common Fraction found with the Index *and deſired Common.*

§. 1. When a Common Fraction is deſired for the Logarithme thereof given. If the given Logarithme with the proper *Index* be found in the *Table*, then the abſolute Number correſponding ſhall be the Denominator of the Fraction, to which 1 ſhall be Numerator.

Example.

As if —0,30102,99957, be the Logarithme given, the abſolute Number anſwering thereto is 2, which ſhall be the Denominator; ſo ſhall $\frac{1}{2}$ be the Fraction of the given Logarithme.

Logarithme of a Common Fraction found, and deſired to be a Decimal.

§. 2. When the Logarithme of a Common Fraction with the given *Index* is found in the *Table*, and the Fraction is deſired to be a Decimal; ſubſtract the given Logarithme from the Logarithme of the Decimal Denominator given, the correſponding Number to the remaining Logarithme ſhall be the Numerator to the given Denominator.

Example.

As if —0,30102,99957, be given, and the Decimal thereof be ſought in Primes or Tenths, then I take the Logarithme given from the Logarithme of 10, the Remain is

is 0,69897,00043, which is the Logarithme of 5; so shall 5 be the Numerator to 10, and the Decimal Fraction $\frac{5}{10}$.

If the Denominator given had been 100, and so the Decimal set in Seconds, then the Logarithme given taken from the Logarithme of 100, leaves 1,69897,00043, the Logarithme of 50, which shall be the Numerator to 100, and the Fraction $\frac{50}{100}$, or $\frac{5}{10}$ as before

The like is to be done with others: Neverthelesse it often happens, the greater the Denominator given, the more easie to find the corresponding Number to the Logarithme of the Remain, to be an Integer. *The Greater the Denominator the more easie to find.*

§. 3. When the given Logarithme of a Common Fraction is not found in the *Table*, yet if the Fraction be desired to be a Decimal, the work is the same with the last above-mentioned. Logarithme *of a Common Fraction not found, and desired to be a Decimal.*

As if —0,12493,87366, be the Logarithme given, and I desire to know the Decimal Fraction signified thereby, whose Denominator shall be 100; or Seconds, after Substraction of the given Logarithme from the Logarithme of 100, the Remain is 1,87506,12634, the Logarithme of 75, which shall be the Numerator to 100, as was seen before. *Example.*

Here may be noted what was mentioned above, that the greater the Denominator, the more exact the Whole Number may be found. For if to this Logarithme 0,12493,87366, the Denominator had been given 10, then would the Remain have been 0,87506,12634, the nearest Integer answering which is 7, which is 7 Primes or Tenths; but then for that the Remain is greater than the Logarithme of 7, the Numerator is not exact, but I lose the 5 Seconds, unless I work for them by some of the varieties in the *Fifth Proposition.* *The Greater the Denominator the nearer the truth.*

§. 4. When the Logarithme of a Common Fraction is given, which is not to be found in the *Table*, and the Fraction thereof desired Vulgar and not Decimal: Then after the Decimal Fraction thereof is found as above, reduce the same to the least Termes. Logarithme *of a Common Fraction not found, and desired Common.*

As if $\frac{75}{100}$ be found as above, for the Logarithme of —1,87506,12634, I abbreviate $\frac{75}{100}$ to $\frac{3}{4}$ the Common Fraction, whose Log. as above was —0,12493,87366. *Example.*

§. 5. When the Logarithme of a Decimal Fraction is given, look out the corresponding Number: As if the given Logarithme were the Logarithme of an Integer, and then place the *Seperatrix* according to the Units contained in the *Index*, or which is all one, prefix before the Number found so many Cyphers lacking one, as there be Units in the *Index* of the Logarithme given. *Decimal found for the* Logarithme *thereof.*

As if —2,68574,17386, be the Logarithme of a Decimal, which I find in the *Table* against the Integers 485, then I place Unity two places before the Left Hand Figure thereof, for that the *Index* here is 2 defective, so is the Decimal 0,0485. *Examples.*

But if the Logarithme given were —1,68574,17386, then should the Decimal be 0,485. If the Logarithme were 0,68574,17386, the Integers and Decimal would be thus 4,85, *&c.* as was seen in the *Fourth Proposition* of this Chapter, and the like is to be understood of others.

Nothing need be added to prove the Invention of Logarithmes for Numbers, or Numbers for Logarithmes, seeing both are to be examined and proved by the foregoing *Table* of *Logarithmes.* *Proof by the* Tables.

CHAP. III.

Addition of Logarithmes.

Addition of Logarithmes is equivalent to Multiplication of other Numbers. If therefore the Logarithme of the Multiplyer be added to the Logarithme of the Multiplicand, the Total shall be the Logarithme of the Product. Logarithmes *added what equivalent to.*

To add two Logarithmes, consider whether the given Logarithmes be both of one Nature or not, that is, Affirmative, Negative or Mixt; for Affirmative will produce Affirmative, and Negative Negative; but if they be Mixt, the Product will be some- *The Key of* Addition *of* Logarithmes.

 time

time the one, and ſometime the other; for one of them is in nature ſubſtractive from the other.

As if $4 + 2 = 6$, then is $10000 \times 100 = 1000000$. And as $\overline{4} + \overline{2} = \overline{6}$, ſo is $,0001 \times ,01 = ,000001$. But $3 + \overline{2} = 1$, or $1000 \times ,01 = 10$, is as $1000 \div 100 = 10$. And as $\overline{3} + 2 = \overline{1}$, or $,001 \times 100 = 0,1$, ſo is $100 \div 1000 = 0,1$. This Key may unlock Addition of Logarithmes, but the particular Caſes following make all plain.

1. *If both be Affirmative.* 1. *Caſe.* If the given Logarithmes be both affirmative, then add the Numbers, as if they were Integers, and the ſumme ſhall be a Logarithme of the ſame kind, and his Abſolute Number the Product.

Example. As to add the Logarithme of 90 to the Logarithme of 9, the Total is the Logarithme of 810, the Product of both.

Multiplicand	1,95424,25094	Log. of	90
Multiplyer	0,95424,25094	Log. of	9
Product	2,90848,50188	Log. of	810

2. *If both be Negative.* 2. *Caſe.* If the given Logarithmes be both Negative, as before, add the Numbers as Integers, and the ſumme ſhall be a Logarithme of the ſame kind; only in the Logarithmes of Decimals, if from the Left Hand place next the *Index*, any Tens be carried onward to the *Index*, they are alwayes Affirmative, although the *Index* be Negative: And then the Affirmative muſt be ſubſtracted out of the Negative *Indices*, as was taught in Addition of Decimal *Indices*.

Example in a Fraction. As to add —0,30102,99957, the Logarithme of $\frac{1}{2}$ to it ſelf, the Total ſhall be —0,60205,99914, the Logarithme of $\frac{1}{4}$, which is the Product of $\frac{1}{2}$ multiplyed by it ſelf, and herein is no difficulty.

Example in a Decimal. But if the Fraction had been a Decimal, that is 0,5, or $\frac{50}{100}$, the Logarithme of which is —1,69897,00043, then in adding them, when I come to the Figures next the *Indices*, an Unit is to be carried over to the *Indices* for the 10 which is there, and this Affirmative I take from the ſumme of both the Negative *Indices*, and the Remain is 1 Negative; ſo is the Total —1,39794,00086, which is the Logarithme of 0,25, that is the Product of 5′ by 5′, and agrees to the other in value, though in other Terms, becauſe 25 is $\frac{1}{4}$ of 100.

Multiplicand	—0,30102,99957	Log. of	$\frac{1}{2}$	—1,69897,00043	Log. of	0,5′
Multiplyer	—0,30102,99957	Log. of	$\frac{1}{2}$	—1,69897,00043	Log. of	0,5′
Product	—0,60205,99914	Log. of	$\frac{1}{4}$	—1,39794,00086	Log. of	,25″

3. *Data of divers kinds, and the Logarithme defective be the Logarithme of a Fraction.* 3. *Caſe.* If the given Logarithmes be of different kinds, that is, the one Affirmative and the other Negative, and the defective Logarithme be the Logarithme of a Common Fraction; then ſubſtract the Leſſer Logarithme out of the Greater, the Remainder ſhall be the Logarithme of the Product required, and ſhall alwayes be of the ſame kind with the Greater Logarithme.

Examples. As if 5 were to be multiplyed by $\frac{1}{2}$, then —0,30102,99957, the Logarithme of $\frac{1}{2}$ taken from 0,69897,00043, the Logarithme of 5, leaveth 0,39794,00086, the Logarithme of $\frac{5}{2}$ or $2\frac{1}{2}$, the Product not defective becauſe the Logarithme of 5 the Greater Logarithme was abundant.

But if 20 be multiplyed by $\frac{1}{40}$, then 1,30102,99957, the Logarithme of 20 being the Leſſer, is to be ſubſtracted from —1,60205,99913, the Logarithme of $\frac{1}{40}$, and the Remain will be —0,30102,99956, the Logarithme of the Product $\frac{20}{40}$ or $\frac{1}{2}$ defective, becauſe the Greater Logarithme is Negative.

Multiplicand	0,69897,00043	Log. of	5	1,30102,99957	Log. of	20
Multiplyer	—0,30102,99957	Log. of	$\frac{1}{2}$	—1,60205,99913	Log. of	$\frac{1}{40}$
Product	0,39794,00086	Log. of	$\frac{5}{2}$	—0,30102,99956	Log. of	$\frac{20}{40}$

4 *Caſe.*

4. *Case.* If the given Logarithmes be of divers kinds, and the Negative Logarithme be the Logarithme of a Decimal Fraction, then add the Logarithmes together till you come to the *Index*, and there substract the Lesser *Index* from the Greater, remembring the 10 carried over, if any be, is Affirmative, as before noted in the *Second Case*, and to be ordered accordingly.

4. Data *of divers kinds, and the* Logarithme *defective be the* Logarithme *of a Decimal. Examples.*

As if —1,69897,00043, the Logarithme of $\frac{50}{100}$, or 0,5, which is $\frac{1}{2}$, be added to the Logarithme of 5, that is 0,69897,00043, the Total Product is 0,39794,00086, the Logarithme of 2,5 and Affirmative; for there the Unit carried for the 10 next the *Index* being Affirmative, substracted from the Negative *Index* 1, leaves 0 to the Product abundant.

But if the defective Logarithme of the Decimal 0,025, that is —2,39794,00087 be added to the Logarithme of 20, the Total will be —1,69897,00044, the Logarithme of 5 Primes or ,50″; for the *Index* 1 Affirmative taken from 2 Negative, leaves the Remain defective, because the *Index* of that kind was the Greater.

Multiplicand	—1,69897,00043	Log. of 0,5′	1,30102,99957	Log. of 20
Multiplyer	0,69897,00043	Log. of 5	—2,39794,00087	Log. of ,025‴
Product	0,39794,00086	Log. of 2,5′	—1,69897,00044	Log. of ,5′

5. *Case.* If several Numbers be given to be multiplyed one into another, add all their Logarithmes together, according to the Directions foregoing, and the Total thereof shall be the Product desired, whereby in dispatch of great Multiplications, a wonderful expedition is attained. View the Examples following.

5. Data *many Numbers.*

Examples.

Integers.

0,47712,12547 Log. of 3		0,30102,99957 Log. of 2	Numbers multiplyed one into another.
1,04139,26852 Log. of 11		0,69897,00043 Log. of 5	
1,23044,89214 Log. of 17		1,90308,99870 Log. of 80	
2,74896,28613 Log. of 561		2,90308,99870 Log. of 800	Product.

Common Fractions.

—0,60205,99913 Log. of $\frac{1}{4}$	—0,17609,12590 Log. of $\frac{2}{3}$	Fractions multiplyed together.
—0,60205,99913 Log. of $\frac{1}{4}$	—0,12493,87366 Log. of $\frac{3}{4}$	
—0,30102,99957 Log. of $\frac{1}{2}$	—0,07918,12461 Log. of $\frac{5}{6}$	
—1,50514,99783 Log. of $\frac{1}{32}$	—0,38021,12417 Log. of $\frac{5}{12}$	Product.

Decimals.

—1,00000,00000 Log. of 0,1′	—2,47712,12547 Log. of 0,03″
—1,17609,12591 Log. of 0,15″	—3,90308,99870 Log. of 0,008‴
—1,95424,25094 Log. of 0,9′	—3,60205,99913 Log. of 0,004‴
—2,13033,37685 Log. of 0,0135⁗	—7,98227,12330 Log. of $0,00000096^{viii}$

Mixt Numbers.

0,30102,99957 Log. of 2	1,22530,92817 Log. of 16,8′
2,15836,24921 Log. of 144	—1,87506,12634 Log. of 0,75″
—1,87506,12634 Log. of 0,75″	—2,69897,00043 Log. of 0,05″
2,33445,37512 Log. of 216,00″	—1,79934,05494 Log. of $0,63000^{v}$

The Product of the Numbers multiplyed answering to the Number corresponding to the Total Logarithme of the Logarithmes added, is Proof sufficient of Logarithmical Addition.

Proof of Addition *of* Logarithmes.

CHAP.

CHAP. IV.

Subſtraction of Logarithmes.

Logarithmes *ſubſtracted what it performeth.* *The Key of* Subſtraction *of* Logarithmes.

Subſtraction of Logarithmes performeth as much as Diviſion of other Numbers, and therefore to ſubſtract the Logarithme of the Diviſor from the Logarithme of the Dividend, the Remain ſhall be the Logarithme of the Quotient.

But as before in Addition, the Logarithmes being ſome defective, and others abundant, conſideration muſt be had in the Subſtraction, that the Remain or Quotient be rightly Denominate, Affirmative or Negative, according as the Natures of the given Logarithmes will admit.

For as in *Indices* $4 - 2 = 2$: And in multiplyed Numbers correſponding $10000 \mathfrak{D} 100 = 100$. And $\overline{4} - \overline{2} = \overline{2}$; that is in Decimals thus, $,0001 \mathfrak{D} ,01 = ,01$. So if $0 - \overline{2}$ be taken from $0 + 3$, the Remain ſhall be $= 5$, that is in Numbers. If $1 \mathfrak{D} ,01$ be taken from 1×1000, the Remain ſhall be $= 100000$. For there as the *Indices* are added, 1000 ſhall be multiplyed by 100, the Quotient of the Diviſion. But if $0 + 2$ be taken from $0 - \overline{3}$, the Remain ſhall be $= \overline{5}$, which in Numbers is as 1×100 taken from $1 \mathfrak{D} ,001$, the Remain ſhall be $= ,00001$. This is the Key of Subſtraction, and may be fully underſtood in the Caſes following.

1\. *If both be Affirmative.*

1\. *Caſe.* If the given Logarithmes be both affirmative, and that of the Diviſor leſs than that of the Dividend, then ſubſtract the leſſer Logarithme from the greater, as if they were Integers; and the Remain ſhall be a Logarithme of the ſame kind, and the abſolute Number anſwering thereto, the Quotient.

Example.

As to divide 810 by 9, the Quotient is 90; ſo to ſubſtract 0,95424,25094, the Logarithme of 9 from 2,90848,50189, the Logarithme of 810, the Remain is 1,95424,25095, the Logarithme of 90, the Quotient.

Dividend	2,90848,50189	Log. of	810
Diviſor	0,95424,25094	Log. of	9
Quotient	1,95424,25095	Log. of	90

2\. *If both be Affirmative, and the Diviſor greateſt.*

2\. *Caſe.* If both the given Logarithmes are affirmative, and that of the Diviſor greater than that of the Dividend, then after Subſtraction of all the reſt of the Logarithme of the Diviſor from the Logarithme of the Dividend, except the *Index*, take the leſſer *Index* from the greater, and change the Sign of the Remain. And in ſubſtracting, if in the next Figure to the *Index* you have occaſion to borrow 10, then accompt the *Index* of the Dividend 1 leſs than it is; as if 3, accompt it 2, if 2 but 1, &c. Examples of both.

Examples.

As if 9 be divided by 90, the Quotient will be $\frac{1}{10}$ or 0,1; ſo to ſubſtract 1,95424,25094 the Logarithme of 90, from 0,95424,25094, the Logarithme of 9, the Remain will be —1,00000,00000, the Logarithme of $\frac{1}{10}$ or 0,1, as at *A.*

And if 36 be divided by 90, the Quotient by abbreviation will be $\frac{2}{5}$: So 1,95424,25094 the Logarithme of 90, taken from the Logarithme of 36, which is 1,55630,25008, the Remain will be the Logarithme of 0,4′, or abbreviated $\frac{2}{5}$; where 1 for the 10 borrowed next the *Index* abates the Characteriſtique 1 of the upper Logarithme; and ſo 0 being left for the leſſer *Index* taken from the lower, changes the Sign, as at *B.*

	A.			*B.*		
Dividend	0,95424,25094	Log. of	9	1,55630,25008	Log. of	36
Diviſor	1,95424,25094	Log. of	90	1,95424,25094	Log. of	90
Quotient	—1,00000,00000	Log. of	0,1′	—1,60205,99914	Log. of	0,4′

4\. *Caſe.*

3. *Case.* If the given Logarithmes are both Negative, of Vulgar Fractions, and that of the Divisor the lesser; then as if they were Integers, substract the Logarithme of the Divisor from the Logarithme of the Dividend, and the Remain shall be a defective Logarithme of the same kind. But if the Logarithme of the Divisor be the greater, take the lesser Logarithme from the greater, and change the Sign of the Remain, for in this case it shall be abundant. 3. *If both be Negative of Fractions.*

As if $\frac{1}{32}$ be divided by $\frac{1}{4}$, the Quotient will be $\frac{1}{8}$; so if the Logarithme of $\frac{1}{4}$, that is —0,60205,99913, be taken from —1,50514,99783, the Logarithme of $\frac{1}{32}$, the Remain will be —0,90308,99870, the Logarithme of $\frac{1}{8}$, as at *C*. *Examples.*

But if $\frac{1}{2}$ be divided by $\frac{1}{4}$ the Quotient will be 2; for the Divisor having the greater Logarithme, the *Index* which was negative is now changed, and the Remain affirmative, as at *D*.

	C.		*D.*	
Dividend	—1,50514,99783	Log. of $\frac{1}{32}$	—0,30102,99957	Log. of $\frac{1}{2}$
Divisor	—0,60205,99913	Log. of $\frac{1}{4}$	—0,60205,99913	Log. of $\frac{1}{4}$
	—0,90308,99870	Log. of $\frac{1}{8}$	0,30102,99956	Log. of 2

4 *Case.* If the Logarithmes given are both of Decimals, then substract them as Integers till you come to the *Index*; and there if in the next place to the *Index* you borrow 10, accompt the *Index* of the Dividend 1 less than it is; as if —1, accompt it —2, if —2 then —3, &c. and then take the lesser *Index* from the greater: And if the upper *Index*, which is of the Dividend, be the least, then change the Sign of the Remain, which shall be the Logarithme of the Quotient. 4. *Data Decimals.*

As to divide 0,0135'''' by 0,1', the Quotient will be 0,135''': So if the Logarithme of 0,1' be taken from —2,13033,37685, the Logarithme of 0,0135'''', the Remain will be —1,13033,37685, the Logarithme of 0,135''', as at *E*. *Examples.*

And if 0,0135'''' be divided by 0,9', the Quotient will be 0,015''': So if the Logarithme of 0,9', which is —1,95424,25094, be taken from —2,13033,37685, the Logarithme of 0,0135'''', there will be left —2,17609,12591, the Logarithme of 0,015''', as at *F.*; where by borrowing the 10, next the *Index*, the upper *Index* is accompted —3.

	E.		*F.*	
Dividend	—2,13033,37685	Log. of 0,0135''''	—2,13033,37685	Log. of 0,0135''''
Divisor	—1,00000,00000	Log. of 0,1'	—1,95424,25094	Log. of 0,9'
Quotient	—1,13033,37685	Log. of 0,135'''	—2,17609,12591	Log. of 0,015'''

Other Examples of the varieties that may happen under this Case here follow.

Dividend	—1,87506,12634	Log. of 0,75''	—1,65321,25138	Log. of 0,45''
Divisor	—2,69897,00043	Log. of 0,05''	—1,95424,25094	Log. of 0,9'
Quotient	—1,17609,12591	Log. of 15,0'	—1,69897,00044	Log. of 0,5'
Dividend	—2,13033,37685	Log. of 0,0135''''	—1,30102,99957	Log. of 0,2'
Divisor	—2,69897,00043	Log. of 0,05''	—2,60205,99913	Log. of 0,04''
Quotient	—1,43136,37642	Log. of 0,27''	0,69897,00044	Log. of 5,0'
Dividend	—1,95424,25094	Log. of 0,9'	—2,07918,12460	Log. of 0,012'''
Divisor	—1,17609,12591	Log. of 0,15''	—1,17609,12591	Log. of 0,15''
Quotient	—0,77815,12503	Log. of 6,0	—2,90308,99869	Log. of 0,08''

5. *Data of divers kinds.*

5. *Case.* If the Logarithmes given are of divers kinds; that is one affirmative and the other negative, and the defective Logarithme be the Logarithme of a Common Fraction; add them together, and the summe shall be the Logarithme of the Quotient, and of the same kind with the Dividend. But if the defective Logarithme be the Logarithme of a Decimal Fraction, then substract the Logarithme of the Divisor out of the Logarithme of the Dividend till you come to the *Index*, and then add the *Indices* together: And if 10 be borrowed, accompt the *Index* of the Dividend one less, as before.

Examples. Examples of both sorts follow so plainly that illustration thereof is needless.

Integers with Common Fractions.

Dividend	0,77815,12504	Log. of	6	—0,12493,87366	Log. of	$\frac{1}{4}$
Divisor	—0,12493,87366	Log. of	$\frac{1}{4}$	0,77815,12504	Log. of	6
Quotient	0,90308,99870	Log. of	8	—0,90308,99870	Log. of	$\frac{1}{8}$
Dividend	1,20411,99827	Log. of	16	—1,90308,99870	Log. of	$\frac{1}{80}$
Divisor	—1,90308,99870	Log. of	$\frac{1}{80}$	1,20411,99827	Log. of	16
Quotient	3,10720,99697	Log. of	1280	—3,10720,99697	Log. of	$\frac{1}{1280}$

Integers with Decimals.

Dividend	0,60205,99913	Log. of	4	—3,90308,99870	Log. of	$0,008'''$
Divisor	—3,90308,99870	Log. of	$0,008'''$	0,60205,99913	Log. of	4
Quotient	2,69897,00043	Log. of	500	—3,30102,99957	Log. of	$0,002'''$
Dividend	0,69897,00043	Log. of	5	—1,39794,00087	Log. of	$0,25''$
Divisor	—1,39794,00087	Log. of	$0,25''$	0,69897,00043	Log. of	5
Quotient	1,30102,99956	Log. of	20	—2,69897,00044	Log. of	$0,05''$

Mixt Numbers.

Dividend	0,68574,17386	Log. of	$4,85''$	0,69897,00043	Log. of	5
Divisor	—1,79588,00173	Log. of	$0,625'''$	—2,90308,99870	Log. of	$0,08''$
Quotient	0,88986,17213	Log. of	$7,76''$	1,79588,00173	Log. of	$62,5'$

Proof of Substraction of Logarithmes. The Quotient of the Numbers divided, answering to the Number of the Remaining Logarithme of the Logarithmes substracted, is Proof enough of the truth of Logarithmical Substraction.

CHAP. V.

Multiplication of Logarithmes.

Logarithmes multiplyed produceth Figurals. MUltiplication of Logarithmes resembleth Multiplication of Ratio's, hereafter treated of; for that it maketh the Product a Figurate Number. As to multiply a Logarithme by 2, produceth the Logarithme of the Square. And to multiply by 3 the Logarithme of the Cube, *&c.* according to the Figural *Index* of the Quantity used for Multiplyer.

1. *Case.*

1. *Case.* There is no difficulty therein, if the *Index* of the given Logarithme be affirmative, or the defective Logarithme be the Logarithme of a Common Fraction; for then the Multiplication is as in Integers.

1. *If the* Index *be affirmative, or the defective* Logarithme *be the* Logarithme *of a Fraction. Examples.*

For as 9 multiplyed by 9 giveth 81; so the Logarithme of 9 multiplyed by 2 produceth the Logarithme of 81. And the Fraction $\frac{2}{9}$ squared is $\frac{4}{81}$; so the Logarithme of $\frac{2}{9}$ multiplyed by 2 shall give the Logarithme of $\frac{4}{81}$. And the like may be done for any other power.

	Integers.		Common Fractions.	
Root	0,95424,25094	Log. of 9	—0,65321,25137	Log. of $\frac{2}{9}$
	2		2	
Square	1,90848,50188	Log. of 81	—1,30642,50274	Log. of $\frac{4}{81}$
Root	0,69897,00043	Log. of 5	—0,17609,12590	Log. of $\frac{2}{3}$
	3		3	
Cube	2,09691,00129	Log. of 125	—0,52827,37770	Log. of $\frac{8}{27}$
Root	0,60205,99913	Log. of 4	—0,69897,00043	Log. of $\frac{1}{5}$
	4		4	
Sq Squa.	2,40823,99652	Log. of 256	—2,79588,000172	Log. of $\frac{1}{625}$

2. *Case.* If the given Logarithme be the Logarithme of a Decimal, there is no difference between the Multiplication thereof and others, save when in multiplying the Figure next the *Index* if any Tens arise, the Units carried over for them are affirmative, and to be substracted from the Product of the negative *Index*.

2. *If the* Logarithme *be the* Logarithme *of a Decimal. Examples.*

As in squaring 0,05″, the 10 carried from the Multiplication of 6 in the Logarithme, shall abate 1 from the Product of the *Index*, and leave but —3. The like also happeneth in other Examples.

Root	—2,69897,00043	Log. of 0,05″	—1,30102,99957	Log. of 0,2′
	2		2	
Square	—3,39794,00086	Log. of 0,0025″″	—2,60205,99914	Log. of 0,04″
Root	—2,69897,00043	Log. of 0,05″	—2,30102,99957	Log. of 0,02″
	3		3	
Cube	—4,09691,000129	Log. of 0,000125^{vi}	—6,90308,99871	Log. of 0,000008^{vi}
Root	—1,69897,00043	Log. of 0,5′	—1,30102,99957	Log. of 0,2′
	4		4	
Sq.Squ.	—2,79588,00172	Log. of 0,0625″″	—3,20411,99828	Log. of 0,0016″″

The Product of the Numbers multiplyed Figurately answering to the Number of the produced Logarithmes, serveth for a sufficient Proof of the truth of Logarithmical Multiplication.

Proof of Multiplication *of* Logarithmes.

CHAP.

CHAP. VI.

Division of Logarithmes.

Logarithmes divided, is Extraction of Roots.

AS Multiplication of Logarithmes produceth Figural Numbers, so Division of Logarithmes answereth to Extraction of Roots, and is much like Division of Ratio's hereafter spoken to in the *Fourth Book* : For the Quotient is the Ratio of the Figurate Number whose Logarithme is the Dividend, according to the Ratio in the Divisor. So as if the Divisor be 2, the Quotient is the Logarithme of the Square Root ; if 3, the Logarithme of the Cube Root, *&c.* according to the Figural *Index* of the Quantity used for Divisor.

1. *If the* Index *be affirmative, or the defective* Logarithme *be the* Logarithme *of a Fraction.*

1. *Case.* Division is easie, if the *Index* of the given Logarithme be affirmative, or the defective Logarithme be the Logarithme of a Common Fraction ; for then the Division is performed as in Integers. As in the Examples of the former *Chapter* may be seen.

Examples.

	Integers.		Fractions.	
Square	1,90848,50188	Log. of 81	—1,30642,50274	Log. of $\frac{4}{81}$
	2		2	
Root	0,95424,25094	Log. of 9	—0,65321,25137	Log. of $\frac{2}{9}$
Cube	2,09691,00129	Log. of 125	—0,52827,37770	Log. of $\frac{8}{27}$
	3		3	
Root	0,69897,00043	Log. of 5	—0,17609,12590	Log. of $\frac{2}{3}$

2. *If the* Logarithme *be the* Logarithme *of a Decimal.*

2. *Case.* If the Logarithme given to be divided be the Logarithme of a Decimal, and the *Index* will be evenly divided by the Divisor, then there is no difference between the Division thereof and others : But when the *Index* of the Logarithme will not be evenly divided by the Divisor, then add to the *Index* so many Units till it may be evenly divided thereby, and setting the Quotient down for a new *Index*, keep the Units added in mind, multiply them by 10, and add the Product thereof to the next Right Hand Figure of the Logarithme, and then divide the rest of the Logarithme, as others, where nothing was borrowed.

Examples.

As among the former Examples in the last *Chapter* foregoing, where —6,90308,99871, was found to be the Logarithme of the Cube 0,000008^{vi} ; because —6 the Characteristique will be evenly divided by 3, the Division is as in Integers, at *A*. But in dividing —4,09691,00129, the Logarithme of the Cube 0,000125^{vi}, because —4 the *Index*, will not be evenly divided by 3, I add in mind 2 Units to 4, to make it so divisible, and so get —2 for the new *Index* ; which added 2, multiplyed by, make 20, and to this should have been added the next Figure of the Logarithme, but there being 0, leaves 20, only to be divided by 3, which gives 6, and so continuing the Division, the Logarithme of 0,05$''$ is gotten, as at *B*.

	A.		*B.*	
Cube	—6,90308,99871	Log. of 0,000008^{vi}	—4,09691,00129	Log. of 0,000125^{vi}
	3		3	
Root	—2,30102,99957	Log. of 0,02$''$	—2,69897,00043	Log. of 0,05$''$

For the better understanding of the Division of Decimal Logarithmes whose Characteristiques will not be evenly divided, Mr. *Oughtred* at the end of the *Resolution of Affected Æquations*, hath presented us with part of a *Table* to be increased at pleasure ; which though the foregoing Rule may supply the use of, yet because some by

Occular

Occular inſpection in a *Table* receive a firmer impreſſion in their Minds of the Numbers uſed therein, I have here tranſcribed it a little altered, with the Explanation following.

A Table for Diviſion of Decimal Logarithmes.

Divisors							Indices
2)					1	2	1
					3	4	2
					5	6	3
					7	8	4
					9	10	5
						&c.	
3)				1	2	3	1
				4	5	6	2
				7	8	9	3
						&c.	
4)			1	2	3	4	1
			5	6	7	8	2
						&c.	
5)		1	2	3	4	5	1
		6	7	8	9	10	2
						&c.	
	&c.	40	30	20	10	0	

In this *Table* the Numbers that ſtand to the Left Hand within a bowed Line are Diviſors, or the Numbers of Quantity, as 2, 3, 4, 5, and may be increaſed as the *&c.* denotes. *The Table explained.*

The Numbers next the Right Hand are the *Indices* of the Quotient, anſwering collaterally to ſuch Diviſors with ſuch *Indices* of the Dividends as ſtand along with them in the ſame Line from the Right Hand to the Left.

The Numbers next the *Indices* of the Quotient are ſuch *Indices* of the Dividends as will be evenly divided by the Diviſors ſtanding againſt them, and ſo need no Units to be added to them. Theſe may alſo be increaſed at pleaſure, as the ſeveral *&c.* ſignifie.

But the Numbers next them, if they happen to be the Characteriſtiques of the Logarithmes given to be divided, muſt have 1 Unit added, and the next File of Numbers muſt have 2 Units added, the next File 3, and the next 4; and ſo of others, if the *Table* be enlarged. Every of which multiplyed by 10, as before noted in the Rule, makes the 10, 20, 30, 40, at the bottom of the *Table*, to be added to the Figure next the *Index* of the Logarithme to be divided.

So if I were to divide a Logarithme whoſe *Index* is —7 by 2, I need add but 1 to it, and the *Index* of the Quotient ſhall be —4; but if a Logarithme whoſe *Index* is —7 by 3, I muſt add 2, and the *Index* of the Quotient ſhall be —3. The like is to be ſeen of others, and all to be underſtood of Decimal Logarithmes.

The Root of any Number extracted agreeing with the Number anſwering to the Quotient of the divided Logarithme, will be Proof enough of the truth of Logarithmical Diviſion. *Proof of Diviſion of Logarithmes.*

Partis Tertiæ Libri Tertii

FINIS.

THE FOURTH PART OF THE THIRD BOOK.

CHAP. I.

Of COSSICKS.

Numbers specially contract whose Denominators are uncertain.

NUmbers ſpecially Contract, that have their Denominators implyed, and becauſe of their certainty omitted, are already handled in the three former Parts of this *Third Book*. The next in order are thoſe ſpecial Contract Numbers, whoſe Denominations are uncertain, and therefore needful to be expreſſed : As *Coſſicks*, *Surdes*, and *Species*.

Coſſicks *what.*

Coſſicks, are Figural-Numbers-Compound, either *per ſe*, or *inter ſe*.

Compound per ſe.

Thoſe Compound *per ſe*, or by themſelves, are ſuch Figural Numbers of ſome ſingle Species as have annexed to them ſome Abſolute Number. As 3 Roots, 8 Squares, 5 Cubes, or the like.

Compound inter ſe.

Thoſe Compound *inter ſe*, or among themſelves, are ſuch Figural Numbers of different Species as are annexed one to another, and have to each of them ſome Abſolute Numbers alſo adjoyned. As 2 Roots and 3 Squares, or 4 Cubes lacking 2 Roots, or the like.

Abſtract Numbers uſed with Coſſicks *are marked diſtinct*

So as Coſſicks are a Number of Figural Numbers, or a Quantity of Quantities ; and in both ſorts the Numbers are Contract to the Figural Quantities whereto they are adjoyned. And when any Abſtract Number is uſed with them beſides the Number of the Quantities, this Abſtract Number is kept diſtinct and marked accordingly.

Characters uſed for what end.

Whence the Name.

Characters uncertain.

The Figural Names or Denominations of the Quantities, as well to avoid prolixity in the often reſcription, as for conveniency in working, are uſually expreſſed by Marks or Characters, under the cover whereof the Numbers got the Name of *Coſſicks*, derived, as is thought, from the *Hebrew* כָּסָה, ſignifying to cover, hide or conceal : And from thence both the *Latin*, *Coſſa* and *Coſſicus*, and the *Italian*, *Coſſica*, whence it ſeems the Name came to us. But as the Denominations are various, and therefore muſt be expreſt ; ſo the Stenographical Mantles in which they are wrapt up, are not certain, but arbitrary at the pleaſure of the Operator, as he conceiveth moſt expeditious or commodious for his uſe, and therefore are to be ſought in the reſpective *Nomenclatura* of every Author. In the enſuing Operations let the following Characters be thus underſtood.

A Table *of the* Cossical *Characters used in this Book.*

Indices.	*Characters.*	*Signification of the Characters.*
0	N	An Abſolute Number, as if it had no Mark.
1	ℨ	The Root of any Number.
2	ʒ	A Square.
3	φ	A Cube.
4	ʒʒ	A Squared Square, or Zenzizenzike.
5	ʃβ	A Surſolide.
6	ʒφ	A Squared Cube, or Zenzicube.
7	Bʃβ	A Second Surſolide.
8	ʒʒʒ	A Zenzizenzizenzike, or Square of Squared Square.
9	φφ	A Cubed Cube.
10	ʒʃβ	A Square of Surſolids.
11	Cʃβ	A Third Surſolide.
12	ʒʒφ	A Zenzizenzicube, or Square of Squared Cubes.
13	Dʃβ	A Fourth Surſolide.
14	ʒBʃβ	A Square of Second Surſolids.
15	φʃβ	A Cube of Surſolids.
16	ʒʒʒʒ	A Zenzizenzizenzizenzike, or Square of Squares Squaredly (Squared.
17	Eʃβ	A Fifth Surſolide.
18	ʒφφ	A Zenzicubicube, or Square of Cubick Cubes.
19	Fʃβ	A Sixth Surſolide.
20	ʒʒʃβ	A Square of Squared Surſolids.
21	φBʃβ	A Cube of Second Surſolids.
22	ʒCʃβ	A Square of Third Surſolids.
23	Gʃβ	A Seventh Surſolide.
24	ʒʒʒφ	A Square of Squares of Squared Cubes, or a Zenzizenzizenzicube
&c.	&c.	

What Indices *the* Indices *of* Coſſicks *are.*

The Reader may not take theſe *Indices* either for Decimal *Indices*, or the *Indices* of Logarithmes, nor confound the one with the other, for theſe are no other than the Figural *Indices*, or Numbers of the Quantities as they exceed in Power, mentioned before in the *Second Part* of the *Second Book* among Simple Figural Numbers.

How to increaſe the Characters of Coſſicks.

As the Powers may be increaſed, ſo may the Coſſical Characters be formed higher, though it will be rare that ſo many as above ſet down need be uſed. But if any deſire to know how to form the Characters of Higher Powers than theſe; let him firſt ſet down the Figural *Indices* of the Powers in their Natural Order or Progreſſion as far as he pleaſeth, then obſerve, that all the Characters after that of the Root, are but of 3 ſorts, ʒ, φ, ʃβ, *viz.* thoſe for the Square, Cube and Surſolide. For whereas B. C. D. *&c.* are uſed with the Surſolids of different Powers, it is but Numerically according as they ſtand in the Alphabet, to ſhew they are the ſecond, third, fourth, *&c.* of that ſort; ſo ſhall the Eighth Surſolide be Hʃβ, the Ninth Iʃβ, the Tenth Kʃβ, and ſo on. This being obſerved it remains then, that as the Figural *Indices* are Numbers Uncompound or Compound, ſo are the Characters, and conſequently the Arithmetical Names of theſe Higher Powers ſignified by the Coſſical Characters.

For Indices *Uncompound.*

In the *Firſt Book*, *Chap.* 2. it was ſhewed that Uncompound Numbers are procreated by collection of Units, but cannot be made by Multiplication, as 5, 7, 11, 13, 17, 19, 23, 29, 31, *&c.* under all which Uncompound *Indices* after 3, fall Surſolids; therefore under the firſt of them is to be placed the Surſolide Character; under the ſecond, the ſame Character, only with B. the ſecond Letter in the Alphabet added to denote it is a Second Surſolide. Wherefore (as above) againſt 11, the third of theſe uncompound *Indices* is Cʃβ, betokening the Third Surſolide, againſt 13, Dʃβ, againſt 17, Eʃβ, *&c.* And if the *Table* were enlarged, againſt 29 would be Hʃβ, and 31, Iʃβ, *&c.*

For Indices *Compound.*

Numbers Compound muſt be compound of 2 or 3, or of 2 and 3, or of 2 or 3, and ſome other Number.

Of 2.

If the *Index* be compound of 2, ſet down ʒ ſo often as 2 is in the compoſition. As 16 being compound of 2 four times, ſhall have ʒʒʒʒ for its Character. So 32 having 2 five times in the compoſition, ſhall have ʒ five times; and 64, ſix times, *&c.*

Of 3.

If the *Index* be compound of 3 only, then ſet down the Character of the Cube as often as 3 is uſed to compound the *Index*. As 9, compounded of 3, twice ſhall be φφ; ſo 27, becauſe it is compounded of 3, thrice ſhall have φφφ for the Character, *&c.*

If the *Index* be compounded of 2 and 3, then for every time that 2 is multiplyed in the compoſition, ſet down ʒ; and for every time 3 is multiplyed ſet down φ, minding ſtill to ſet ʒ before φ to the Left Hand, becauſe ʒ is the foremoſt Power. Thus againſt 6, made of 2 and 3, is ʒφ; and againſt 12, compounded of 2 by 2, which makes 4, and then by 3, the Character ʒʒφ; ſo 18 hath ʒφφ, becauſe twice 3 is 6, and thrice 6 is 18. *Of 2 and 3.*

If the *Index* be compound of 2 or 3, and ſome other Number, then joyn their Characters together in ſuch order, as the Leſſer Figural Powers may precede the Greater. As 10, compounded of 2 and 5, is ʒſβ, where the Mark of the ſecond Quantity being the Leſſer, ſtands before ſβ, the Mark of the fifth Quantity; 15 likewiſe compounded of 3 and 5, is φſβ. *Of 2 or 3, and ſome other.*

In like manner for Higher *Indices*; as 20, compounded of 4 and 5, ſhall be a ʒʒſβ; 50, of 5 and 10, ſhall be a ʒſβſβ, *&c.*

According to the Characters ſo are their Arithmetical Names, ſave that ſome to ſhorten and facilitate the long Names of ſuch Higher Powers, as have the Square or Cube often ingeminated, borrow ſome Names bordering on the *Latin*, and call ʒʒ a Biquadrate, ʒʒʒ a Triquadrate, ʒʒʒʒ a Quaquadrate, ʒʒʒʒʒ a Quinquadrate, *&c.* Alſo φφ a Bicube, φφφ a Tricube, *&c.* and their Compounds accordingly; as ʒʒφ a Biquadratcube, ʒʒʒφ a Triquadratcube, *&c.* But having kept the older Names in the former Treatiſe of Figural Numbers, I have here retained them to their Coſſical Characters. *How ſome ſhorten the Names.*

As to the Nature of Coſſicks, they are either Whole or Broken, and both either Simple or Compound: For though all Coſſicks are Compound Figural Numbers (as aforeſaid), yet are none counted Compound Coſſicks, unleſs they admit of various Denominations, and have their Characters connexed by the Signs + or —. *Nature of Coſſicks.*

Of Broken Coſſicks, ſee the 6, 7, 8, 9, 10 and 11th *Chapters* following.

Simple Coſſicks then that are Whole or Integral, are Homogeneal, or Quantities of one ſort: As 4 ℨ, or 10 ʒ, or 8 φ, *&c.* to be read, 4 Roots, 10 Squares, 8 Cubes, and ſhall be underſtood with no more relation one to another, than 4 Inches, 10 Planks, 8 Trees, becauſe they are not knit together by any Sign to bring them under an obligation of relation to one and the ſame Root, from whence they ſhould all ſpring: Yet if Simple Coſſicks are to be added or ſubſtracted, multiplyed or divided with other Simple Coſſicks, then they are underſtood to have all one Root. *Simple Whole Coſſicks.*

Compound Whole Coſſicks, connexing together divers Simple Coſſicks, are Heterogeneal. As 8 φ + 4 ℨ, read thus, 8 Cubes more [or and] 4 Roots: So 10 ʒ — 4 ℨ, that is 10 Squares lacking 4 Roots, whoſe Quantities are ever conſidered in ſumme, according to the mixture of the Signs + or —: For they are increaſed by +, and diminiſhed by —. *Compound Whole Coſſicks*

Compound Coſſicks are of 3 ſorts. *Compound of 3 ſorts.*

Thoſe connexed by the Additional Sign +, are called *Binomials.* Binomials.

Thoſe connexed by the Sign of Subſtraction —, are called *Reſiduals*, and ſometime *Apotomes.* Reſiduals *or* Apotomes.

Thoſe knit together by both Signs are called *Medials*, and by ſome *Multinomials*, or *Polynomials*, that is, many named. And yet more Coſſicks than 2 joyned with + or —, deſerve as well as others the name of a *Polynomial*, and are improperly called *Binomials* or *Reſiduals.* Medials, *how otherwiſe called.*

Examples.

Examples of Binomials.

6 ʒ + 4 ℨ that is, 6 Squares and 4 Roots.
5 φ + 3 ʒ + 10 ℨ is 5 Cubes and 3 Squares and 10 Roots.

Examples of Reſiduals.

12 φ — 5 ʒ that is, 12 Cubes lacking 5 Squares.
10 ſβ — 3 φ — 2 ʒ is 10 Surſolids lacking 3 Cubes and 2 Squares.

Examples of Medials or Polynomials.

8 ʒʒ + 2 φ — 4 ʒ that is, 8 Squared Squares and 2 Cubes, lacking 4 Squares.
5 φ — 4 ʒ + 5 ℨ is 5 Cubes and 5 Roots, lacking 4 Squares.

Difference of Simple and Compound Coſſicks.

Coſſicks Simple and Compound, being all Rooted Numbers, admit of this difference, that the Simple having not relation one to another, may have different Roots indefi-

 nitely;

nitely; but Compound Cossicks in one Question alwayes imply one and the same definite Root to all the Quantities joyned together. Neverthelesse several Compound Cossicks, or the same Cossicks in several Questions or Operations may have several Roots. For 3 ℨ, 10 ʒ, the Simple Cossicks; suppose one Root be 3, the 3 ℨ shall be 9, in Absolute Number: But the 10 ʒ are not tyed to the Root 3, but may have any other Number for the Root: Except it were 10 ʒ + 3 ℨ, or 10 ʒ — 3 ℨ; then if the ℨ be 3, the Root of the ʒ shall also be 3, and the 10 ʒ in Absolute Numbers 90. Yet in another Operation the 10 ʒ + 3 ℨ are capable of any other Absolute Number for the Root thereof.

Sign + where understood. Cossicks how placed.

In Cossicks, where the sign — is not, the sign + is understood, and on the outmost to the Left Hand of Compound Cossicks commonly omitted. And in all Polynomials it is most usual, though not essential except in Multiplication and Division, to set all the Cossicks connexed with + to the Left Hand of those connexed with —, though these are Quantities of Higher Powers than the other. Some promiscuously use the word Sign, as well for the Cossick Quantity, as for the sign + or —. But it is most orderly to reserve the name of Sign only to these. And this is to be remembred both in *Surdes* and *Species*. And also that these signs + and — be used, where a Simple Quantity or Magnitude is affirmative or negative to another Simple Quantity or Magnitude: But when an Asterisque is set over either of them, then the Quantity or Magnitude-Compound is affirmed or denyed of a Simple, or a Simple of a Compound. As $5\varphi \overset{*}{—} 4ʒ + 5ℨ$ shall signifie, that the 5 Cubes shall want 4 ʒ and 5 ℨ, which without the Asterisque, the 5 ℨ affirmative and the 5 φ, shall be added together, and from the summe only the 4 ʒ shall be deducted.

Word Sign how used.

Signs used with or without Asterisques what noted thereby.

Absolute Numbers used with Cossicks, sometime not marked.

Cossicks Simple and Compound, Whole and Broken, take into their society Absolute Numbers, marked as before with N. and sometime used without any Character.

CHAP. II.

Addition of Whole Cossicks.

Addition of Whole Cossicks Simple and Homogeneal.

TO add Simple Cossicks, if they be Homogeneal, add the Numbers together as Integers, and to the Total adjoyn the Cossical Character common to the given Numbers.

As 5 ℨ added to 10 ℨ, make 15 ℨ. So 15 ʒ to 40 ʒ are 55 ʒ.

Examples.

	Homogeneal.			
Simple	5 ℨ 10 ℨ	15 ʒ 40 ʒ	30 φ 10 φ	Addends.
	15 ℨ	55 ʒ	40 φ	Totals.

Simple and Heterogeneal.

If the Simple Cossicks to be added are Heterogeneal, then place the Highest Power to the Left Hand, and connex the other thereto by the sign of Addition +.

As to add 14 ʒ to 10 φ; or 30 φ to 4 ʒ and 15 ʒʒ; they are set thus,

Examples.

	Heterogeneal.		
Simple	10 φ 14 ʒ	15 ʒʒ 30 φ 4 ʒ	Addends.
	10 φ + 14 ʒ	15 ʒʒ + 30 φ + 4 ʒ	Totals

Compound of like Signs.

To add Compound Cossicks, add together like Cossicks with like, as ℨ with ℨ, and ʒ with ʒ, &c. also + with +, and — with —. But if any Cossick be odd, or Heterogeneal to the others given to be added, adjoyn him to the Total with his proper Sign.

As

As to add 12ʒ + 10ℨ to 5ʒ + 6ℨ: Or 12ℨ — 10ℨ to 5ʒ — 6ℨ; the Total of the Binomials will be 17ʒ + 16ℨ, and of the Residuals 17ʒ — 16ℨ. But if 10φ + 10ʒ were added to 4φ + 3ʒ + 2ℨ; or to 4φ + 3ʒ — 2ℨ, there the Totals shall be 14φ + 13ʒ + 2ℨ, or 14φ + 13ʒ — 2ℨ. *Examples.*

Binomials.	Residuals.	Medials or Polynomials.	
12ʒ + 10ℨ	12ʒ — 10ℨ	10φ + 10ʒ	10φ + 10ʒ
5ʒ + 6ℨ	5ʒ — 6ℨ	4φ + 3ʒ + 2ℨ	4φ + 3ʒ — 2ℨ
17ʒ + 16ℨ	17ʒ — 16ℨ	14φ + 13ʒ + 2ℨ	14φ + 13ʒ — 2ℨ

If the Compound Cossicks have their Signs unlike, then take the Lesser Number out of the Greater, and to the Remain, which shall be the Total, subscribe the Sign that belongeth to the Greater Number, whether it be + or — accordingly. *Compound of unlike Signs.*

As to add 12ʒ + 10ℨ to 5ʒ — 6ℨ, or 12ʒ — 10ℨ with 5ʒ + 6ℨ, the Total of the former will be 17ʒ + 4ℨ, of the latter 17ʒ — 4ℨ, as at *A.* and *B.* Other Examples of Polynomials follow at *C. D, E.* *Examples.*

Binomials and Residuals.

	A.		*B.*
	12ʒ + 10ℨ		12ʒ — 10ℨ
	5ʒ — 6ℨ		5ʒ + 6ℨ
	17ʒ + 4ℨ		17ʒ — 4ℨ

D.	
	18φ + 16ʒ — 9N.
	4φ — 10ʒ + 4N.
	22φ + 6ʒ — 5N.

Polynomials.

C.	
	46ℨℨ + 10φ + 2ʒ
	16ℨℨ + 8φ — 5ʒ
	62ℨℨ + 18φ — 3ʒ

E.	
	4ſβ + 16ℨℨ + 19ʒ
	3ſβ — 13φ — 14ʒ
	7ſβ + 16ℨℨ — 13φ + 5ʒ

To prove Cossical Addition, besides the tryal by Cossical Substraction, resolve your Cossicks into Abstract Numbers, by supposing 2, 3, or some other Number for a Root, and so accordingly getting the summe of the ʒ, φ, *&c.* of the other Cossicks, and then compare the Numbers to be added with the Total of the Addition, and the summes will be parallel when the Operation is right. *Proof of Cossical Addition.*

As in the former instance at *A*, suppose 2 be a Root, then is 10ℨ 20, and 4 being the Square of 2, the 12ʒ shall be 48; which 48 and 20 make 68; then 5ʒ more is 20 lacking 6ℨ which is 12, leave 8 remaining, this added to 68 makes the Total 76. And so much is 17ʒ and 4ℨ, supposing 2 for a Root.

12ʒ + 10ℨ	48 + 20 = 68
5ʒ — 6ℨ	20 — 12 = 8
17ʒ + 4ℨ	68 + 8 = 76

CHAP. III.

Substraction of Whole Cossicks.

TO substract Simple Cossicks, if they be Homogeneal, take the Lesser Number out of the Greater, and to the Remain subscribe the Cossical Character common to both the given Numbers, when the Subtrahend is the least, but if it be the greatest of the two, then change the Sign to the Remain. Substraction *of Whole* Cossicks. *Simple and Homogeneal.*

As to abate 3φ from 10φ, there will remain 7φ: But if 10φ were to be taken from 3φ, there will lack 7φ, therefore the Sign shall be — to the Remain. *Examples.*

Homogeneal.

		Homogeneal.		
Simple	10 φ	3 φ	Numbers from which Substraction is made.
	3 φ	10 φ	Subtrahends.
	7 φ	— 7 φ	Remains.

Simple and Heterogeneal.

If the Simple Cossicks to be substracted are Heterogeneal, then place the Highest Power to the Left Hand, and connex the other thereto with the Sign of Substraction —.

Examples.

As to take 4 ℨ out of 10 ʒ, or 5 φ from 20 ʒʒ + 2 ℨ, they are set thus ;

	Heterogeneal.		
Simple	10 ʒ	20 ʒʒ + 2 ℨ	Numb. from which Substraction is made.
	4 ℨ	5 φ	Subtrahends.
	10 ʒ — 4 ℨ	20 ʒʒ — 5 φ + 2 ℨ	Remain.

Compound of like Signs.

To substract Compound Cossicks, take like Cossicks from like, as ℨ from ℨ, and ʒ from ʒ, *&c.* also + from +, and — from —, and to the Remain subscribe the same Sign, except the Number to be substracted be the greater, then change the Sign, and set down thereto the difference of the Numbers. And if any single Cossick have none to fellow him, adjoyn him to the Remain with the contrary Sign if he belonged to the Subtrahend, but with the same Sign he hath if he belonged to the Number from which Substraction is made.

Examples.

As to take 5 ʒ + 6 ℨ from 12 ʒ + 10 ℨ, the Remain shall be 7 ʒ + 4 ℨ, as at *A.* So 5 ʒ — 6 ℨ from 12 ʒ — 10 ℨ, shall leave 7 ʒ — 4 ℨ, as at *B.* But to substract 5 ʒ + 10 ℨ from 12 ʒ + 6 ℨ, the Remain shall be 7 ʒ — 4 ℨ, as at *C.* And 5 ʒ — 10 ℨ from 12 ʒ — 6 ℨ, shall leave 7 ʒ + 4 ℨ, as at *D.* For in both these latter the Roots to be substracted being the Greater Number changeth the Sign to the difference of the Numbers.

Also if 3 φ + 4 ʒ + 10 N, be taken from 16 φ + 18 ʒ, the Remain shall change the Sign to N, as at *E.* But if N had not been in the Subtrahend, he shall keep his Sign as at *F.*

	Binomials.		Residuals.		Medials or Polynomials.
A.	12 ʒ + 10 ℨ	*B.*	12 ʒ — 10 ℨ	*E.*	16 φ + 18 ʒ
	5 ʒ + 6 ℨ		5 ʒ — 6 ℨ		3 φ + 4 ʒ + 10 N
	7 ʒ + 4 ℨ		7 ʒ — 4 ℨ		13 φ + 14 ʒ — 10 N
C.	12 ʒ + 6 ℨ	*D.*	12 ʒ — 6 ℨ	*F.*	16 φ + 18 ʒ + 10 N
	5 ʒ + 10 ℨ		5 ʒ — 10 ℨ		3 φ + 4 ʒ
	7 ʒ — 4 ℨ		7 ʒ + 4 ℨ		14 φ + 14 ʒ + 10 N

Compound of unlike Signs.

If the Compound Cossicks have contrary Signs, add the Cossicks with their fellows of unlike Signs, and to the Total, which is the Remain, adjoyn the Sign of the upper Number, that is that from which Substraction is to be made.

Examples.

As to take 5 ʒ — 6 ℨ from 12 ʒ + 10 ℨ, or 5 ʒ — 10 ℨ from 12 ʒ + 6 ℨ, in both the Remain shall be 7 ʒ + 16 ℨ, as at *G.* and *H.* which 16 ℨ is the summe of — 6 ℨ and + 10 ℨ, or — 10 ℨ and + 6 ℨ. But if 5 ʒ + 6 ℨ be taken from 12 ʒ — 10 ℨ, or 5 ʒ + 10 ℨ from 12 ʒ — 6 ℨ, in both these Cases the Remain shall be 7 ʒ — 16 ℨ, as at *I.* and *K.* because the Sign of the upper Number was —. Other Examples of Polynomials follow at *L.* and *M.*

Binomials

Binomials and Refiduals.		Polynomials.
G. 12ʒ + 10ℨ	*I.* 12ʒ — 10ℨ	*L.* 3φ + 10ʒ + 100N
5ʒ — 6ℨ	5ʒ + 6ℨ	9φ + 10ʒ — 30N
7ʒ + 16ℨ	7ʒ — 16ℨ	130N — 6φ
H. 12ʒ + 6ℨ	*K.* 12ʒ — 6ℨ	*M* 3ʒ + 19ℨ — 10N
5ʒ — 10ℨ	5ʒ + 10ℨ	18ℨ — 3ʒ + 4N
7ʒ + 16ℨ	7ʒ — 16ℨ	6ʒ + 1ℨ — 14N

To prove Coſſical Subſtraction, as before in Addition, by ſome fit Root convert the Coſſicks into Abſtract Numbers, and making Subſtraction as in Integers, the Remains will be left equal, if the Operations be right. *Proof of Coſſical Subſtraction.*

As in the Example above at *G*, ſuppoſing the Root 2, then ſhall 10ℨ be 20, and 12ʒ be 48, in all 68. And the Subtrahend 5ʒ ſhall be 20 lacking 12, which is 6ℨ, that is 8, which 8 taken from 68, leaves 60 equal to 7ʒ, that is 28, and 16ℨ which is 32.

12ʒ + 10ℨ	48 + 20 = 68
5ʒ — 6ℨ	20 — 12 = 8
7ʒ + 16ℨ	28 + 32 = 60

Beſides this Proof, if you add 7ʒ + 16ℨ to 5ʒ — 6ℨ, the Total will be 12ʒ + 10ℨ, as before, and ſhew the alternate Proof of Coſſical Addition by Subſtraction, and Subſtraction by Addition.

CHAP. IV.

Multiplication of Whole Coſſicks.

TO multiply Simple Coſſicks, multiply Number by Number as in Integers, and add the reſpective *Indices* of the Coſſical Quantities, and the Total ſhall be the *Index* of the Product, whoſe Character is to be affixed thereto. *Multiplication of Whole Coſſicks. Simple. Examples.*

As to multiply 3φ by 9ʒ, the Numbers 3 and 9 multiplyed produce 27, and 2 the *Index* of ʒ, added to 3 the *Index* of φ, make 5, which is the *Index* of ſß; ſo is the Product 27ſß.

So 10ʒʒ multiplyed by 8φ, produce 80Bſß; for the *Index* of φ 3, added to 4 the *Index* of ʒʒ, make together 7, which hath Bſß for his Character.

Muitiplicands	3φ	3	10ʒʒ	4	30ſß	5	
Multiplyers	9ʒ	2	8φ	3	10ʒʒ	4	Indices.
Product	27ſß	5	80Bſß	7	300φφ	9	

If Compound Coſſicks be multiplyed, let the Coſſicks be placed according to their Quantities, and every Number in the Multiplicand be multiplyed by every Number in the Multiplyer, and the reſpective *Indices* thereof gotten, as if they were Simple Coſſicks. And for the Signs, as before in other Contract Numbers, obſerve like Signs multiplyed together produce +, and unlike —. *Compound.*

As to multiply 3φ + 2ʒ — 4ℨ by 2ʒ + 3ℨ — 8N, the Product will be 6ſß + 13ʒʒ — 26φ — 28ʒ + 32ℨ, as by the Operation it ſelf beginning at the Left Hand, and after Multiplication adding the ſeveral Lines of Production, or Multiplees together, appeareth plainly thus, *Example.*

Multiplicand	3φ + 2ʒ — 4ℨ
Multiplyer	2ʒ + 3ℨ — 8N
Multiplees	6ß + 4ʒʒ — 8φ
	9ʒʒ + 6φ — 12ʒ
	— 24φ — 16ʒ + 32ℨ
Product	6ß + 13ʒʒ — 26φ — 28ʒ + 32ℨ

Proof of Cossical Multiplication.

To prove Cossical Multiplication, besides the tryal by Cossical Division, turn the Cossicks into Abstract Numbers, taking at pleasure some fit Number for a Root, and after Multiplication of the Numbers as Integers, compare the Product with the summe of the Cossical Product, for without Error they exactly agree.

As if 3ʒ + 4ℨ be multiplyed by 2ʒ — 3ℨ, the Product will be 6ʒʒ — 1φ — 12ʒ. To prove which I suppose 2 the ℨ, then shall 4ℨ be 8, and 3ʒ, 12, in summe 20. And 2ʒ, 8, lacking 3ℨ, 6, makes the summe of the Multiplyer but 2, which multiplying 20, produceth 40, and so much is the summe of 6ʒʒ — 1φ — 12ʒ.

3ʒ + 4ℨ	12 + 8	=	20
2ʒ — 3ℨ	8 — 6	=	2
6ʒʒ + 8φ	96 + 64	=	160
— 9φ — 12ʒ	— 72 — 48	=	— 120
6ʒʒ — 1φ — 12ʒ	96 — 8 — 48	=	40

CHAP. V.

Division of Whole Cossicks.

Division of Whole Cossicks

Simple.

TO divide Simple Cossicks, divide Number by Number, as in Integers, and substract the *Index* of the Divisor from the *Index* of the Dividend, and the Character belonging to the Remaining *Index* adjoyn to the Quotient. And if the Numbers will not evenly be divided without a Remainer, or the Divisor have the Greater Cossick, then are they to be set, as Cossical Fractions.

Examples.

As to divide 40ʒʒ by 10φ, the Quotient will be 4ℨ: For if 3, the *Index* of φ, be abated from 4, the ʒʒ *Index*, the Remain will be 1, the *Index* of ℨ.

So 60ß, divided by 5ʒ, gives in the Quotient 12φ.

	Indices.			*Indices.*			
Dividends	4	40ʒʒ (	4ℨ	5	60ß (	12φ	Quotients
Divisors	3	10φ		2	5ʒ		
	1			3			

Compound.

If Compound Cossicks be divided, then as before in Compound Decimal and Astronomical Division, let the Numbers in the Dividend be divided by the Numbers in the Divisor. The *Indices* of the Quotienary Numbers are got as in Division of Simple Cossicks. And the Signs alike give +, and unlike —, as in Multiplication of Compound Cossicks and other Contract Numbers. If any Cossical Quantity be omitted in the given Numbers, express the same with Cyphers, and the Sign +, and place all Cossicks according to their Powers, whether their Signs be + or —. And if the Numbers will not be evenly divided, or the Divisor have the Greater Cossicks, then the Divisor is placed beneath to represent it as a Fraction.

Example.

As to divide 6ß + 13ʒʒ — 26φ — 28ʒ + 32ℨ by 2ʒ + 3ℨ — 8N, after by the first application of the Divisor to the Dividend 3φ is gotten in the Quotient, the Divisor is multiplyed thereby, and the Product 6ß + 9ʒʒ — 24φ substracted from the

the Dividend; by the second application of the Divisor $2\mathfrak{z}$ is gotten in the Quotient, the Product of the Divisor multiplyed thereby is $4\mathfrak{z}\mathfrak{z} + 6\varphi - 16\mathfrak{z}$. And after this substracted the last application of the Divisor gives $4\,ᴢ$ in the Quotient, and the Divisor multiplyed thereby produceth a Cossick Quantity equal to what was left on the Dividend after the former Substractions.

Divisor	Dividend	Quotient
	$-\ 8\varphi$	
	$4\mathfrak{z}\mathfrak{z} - 2\varphi - 12\mathfrak{z}$	
$2\mathfrak{z} + 3ᴢ - 8N$)	$6\beta + 13\mathfrak{z}\mathfrak{z} - 26\varphi - 28\mathfrak{z} + 32ᴢ$	($3\varphi + 2\mathfrak{z} - 4ᴢ$
	$6\beta + 9\mathfrak{z}\mathfrak{z} - 24\varphi$	
	$4\mathfrak{z}\mathfrak{z} + 6\varphi - 16\mathfrak{z}$	
	$-\ 8\varphi - 12\mathfrak{z} + 32ᴢ$	

To prove Cossical Division as before in the other Elementary Operations, let the Cossicks be exchanged for Abstract Numbers, at pleasure taking some fit Number for a Root, and the Division as in Integers performed, the Quotient will parallel the summe of the Cossical Quotient so exchanged, when the Division is well wrought. *Proof of Cossical Division.*

As if $8\varphi + 64N$, be divided by $2ᴢ + 4N$, the Quotient will be thus, $4\mathfrak{z} - 8ᴢ + 16N$. I then suppose the Root 2, the Dividend then shall be 128, for so much is 8φ and 64 N. The Divisor is 8, that is $2ᴢ$ which are 4, and 4 Numbers over. And the Quotient of 128 divided by 8 is 16, which agreeth exactly with the Cossical Quotient: For $4\mathfrak{z}$ is 16, and 16 N make 32, from which $8ᴢ$ taken which are 16, there is left but 16 for the summe of the Quotient.

	$-16\mathfrak{z} + 32ᴢ$	
$2ᴢ + 4N$)	$8\varphi + 0\mathfrak{z} + 0ᴢ + 64N$	($4\mathfrak{z} - 8ᴢ + 16N$
	$8\varphi + 16\mathfrak{z}$	
	$-16\mathfrak{z} - 32ᴢ$	
	$+32ᴢ + 64N$	

	$-64 + 64$		128
$4 + 4$)	$64 + 0 + 0 + 64$	($16 - 16 + 16$	8 (16
	$64 + 64$		
	$-64 - 64$		
	$+64 + 64$		

Besides this Proof, if you multiply $4\mathfrak{z} - 8ᴢ + 16N$ by $2ᴢ + 4N$, the Product will be the Dividend $8\varphi + 0\mathfrak{z} + 0ᴢ + 64N$, as before, and thereby shew the alternate Proof of Multiplication by Division, and Division by Multiplication in Cossicks, as well as other Numbers.

CHAP. VI.

Of Broken Cossicks.

As Whole Cossicks (of which the foregoing Five *Chapters* have sufficiently spoken) are of two sorts, *viz.* Simple and Compound; so are their Fractions. *Broken Cossicks.*

Simple Cossical Fractions are some Broken Parts, or a part only of some Simgle Cossick, and are expressed like Vulgar Fractions with the Cossical Character or Denomination annexed. As $\frac{2}{3}ᴢ$ signifieth $\frac{2}{3}$ of a Root, let the Root be what it will. So $\frac{3}{5}\mathfrak{z}$ importeth $\frac{3}{5}$ of a Square, &c. And hereby is understood that the Cossical Quantity is divided into so many parts as the Denominator denoteth, and a certain number of those parts to be taken as the Numerator denoteth. And hence it may happen *Simple how expressed. Examples.*

happen sometime that the Cossical Fraction may in value be an Integer, and no Fraction: For if the Cossical Fraction be $\frac{3}{4}$ʒ, if the Square be 16, then shall $\frac{3}{4}$ of that Square be 12 Integers. Also $\frac{2}{3}$φ shall be 18 Integers, if the Cube be 27. All these Simple Cossical Fractions may be set as Compound, by placing the Cossical Denomination to the Numerator, and N to the Denominator; or else leaving the Denominator as an Integer: For $\frac{2}{3}$ʒ is as $\frac{2ʒ}{3N}$ or $\frac{2ʒ}{3}$.

Compound of 2 sorts. Compound Cossical Fractions are of two sorts, Dual or Plural.

Dual. Examples. Dual, when the Fraction consisteth only of two Cossical Denominations, and look like Simple Fractions. As $\frac{3ʒ}{2φ}$ which is as much as 3ʒ to be divided by 2φ. So $\frac{4ſß}{9ʒφ}$ import that 9ʒφ must divide 4ſß, &c.

Plural. Plural Cossical Fractions are when the Numerator or Denominator, or both, consist

Example. of more than two Cossical Quantities. As $\frac{4φ + 3ℨ - 10N}{3ℨ + 12N}$ that implyeth that 4 Cubes and 3 Roots lacking 10 Numbers are to be divided by 3 Roots and 12 Numbers added together; and so of others.

Proper and Improper how known. Cossical Fractions are also Proper and Improper, when the Denominator is the Greater Cossick, it is a Proper Fraction, but Improper when the contrary.

CHAP. VII.

Reduction of Broken Cossicks.

Broken Cossicks reduced. THE Reduction of Cossical Fractions, is either to reduce them to their least Terms, or to like Denominators.

To their least Termes. The first sort, as in Vulgar Fractions, may be called Abbreviation; for the Simple Cossical Fractions may be reduced lower sometime in their Numbers, and the Compound sometime both in their Numbers and Quantities. But if the Numbers be incommensurable in either, or any one Quantity of the Compound be N, then each of them respectively must be kept as they happen, unaltered, unless alterable by the subsequent Proposition.

Examples. As $\frac{24}{28}$ʒ may be reduced in its Numbers, by the Rules of abbreviating Fractions seen before in the *Second Part* of the *First Book*, *Chap.* 2. to $\frac{6}{7}$ʒ, as at *A*.

So $\frac{30φ}{36ʒʒ}$ may be abbreviated in its Numbers to $\frac{5φ}{6ʒʒ}$, and in its Quantities to $\frac{5N}{6ℨ}$, there being alike Quantities abated from the Cossical Numerator and Denominator; ℨ being as far distant from ʒʒ, as N from φ. See below at *B*.

Also in Plural Fractions, as $\frac{3ʒʒ + 6ʒ}{3ſß + 12φ}$ the Numbers being all commensurable by 3, may be reduced first in its Numbers to $\frac{1ʒʒ + 2ʒ}{1ſß + 4φ}$, and in the Quantities till one of them be an Absolute Number, thus, $\frac{1ʒ + 2N}{1φ + 4ℨ}$, as at *C*.

		Abbreviated,				
Simple Cossical Fraction		$\frac{24}{28}$ʒ	$\frac{12}{14}$ʒ	$\frac{6}{7}$ʒ		*A.*
Compound Cossical	Dual Fraction	$\frac{30φ}{36ʒʒ}$	$\frac{15ʒ}{18φ}$	$\frac{5ℨ}{6ʒ}$	$\frac{5N}{6ℨ}$	*B.*
			in Numbers		in Quantities	
	Plural Fraction	$\frac{3ʒʒ + 6ʒ}{3ſß + 12φ}$	$\frac{1ʒʒ + 2ʒ}{1ſß + 4φ}$		$\frac{1ʒ + 2N}{1φ + 4ℨ}$	*C.*

The

The Proposition above-mentioned.

Sometimes, and but sometimes, it happeneth, when yet the Numbers are incommensurable, and the Quantities reduced as low as N, that some part of the composition may be abated, and yet the Remain be in like proportion. *Cossicks in their least Terms sometime shortned.*

As $\frac{4\varphi \;+ 12ʒ}{8ʒ\varphi + 24ß}$ reduced in its Numbers is $\frac{1\varphi \;+ 3ʒ}{2ʒ\varphi + 6ß}$ then in its Quantities is $\frac{1ℨ \;+ 3N}{2ʒʒ + 6\varphi}$. But now if I abate the Quantities that follow the Sign of Composition +, yet the remaining Fraction will retain the same proportion, and $\frac{1ℨ}{2ʒʒ}$ or $\frac{1N}{2\varphi}$ shall be equal. *Examples.*

Also Residual Numbers, as $\frac{4\varphi \;- 12ʒ}{8ʒ\varphi - 24ß}$ may be reduced to $\frac{1ℨ - 3N}{2ʒʒ - 6\varphi}$, and consequently to $\frac{1N}{2\varphi}$ as before.

For the better understanding of this kind of Reduction, observe; *When this happeneth what must be.*

1. That the Signs must be *Synonima's*, that is both + or both —; for if one be more, and the other less, it will not suffer this Reduction.

2. The Numbers taken away must be in like proportion to them that remain. As in the former Examples, 3 to 6, is as 1 to 2, or 4 to 8, as 12 to 24.

3. As the Numbers, so the Cossical Quantities must also be in like proportion. For in the former Examples, N to φ is as ℨ to ʒʒ, the difference 3, being between their several *Indices*.

So that if the difference of the abatement unto abatement, be as the whole is in proportion to the whole, then shall the residue be in like proportion to the residue, as the whole is to the whole, by *Euclid.* 5 *Lib.* 19 *Prop.* *The Reason thereof.*

The second sort of Cossical Reduction, to bring Cossicks of different Denominations into one, comprehendeth, *To reduce Cossicks to one Denominator.*

1. To reduce Simple Fractions to one Denomination, which is effected as Abstract Fractions, without altering the Cossical Quantities. *Simple.*

And so $\frac{2}{3}ʒ + \frac{3}{4}\varphi$ reduced, shall stand as at *D*, and be $\frac{8ʒ \quad 9\varphi}{12}$. *Example.*

$$D.\quad \begin{array}{c|c|c} 8ʒ & \times & 9\varphi \\ \frac{2}{3}ʒ & & \frac{3}{4}\varphi \\ & 12 & \end{array}$$

2. To reduce Dual Fractions to one Denomination. And this differs nothing from the other, but in increasing the Cossical Quantities according to the nature of Cossical Multiplication. *Dual.*

And so $\frac{3ʒ}{4\varphi}$ reduced with $\frac{1ℨ}{3N}$ shall stand as at *E*, and be $\frac{9ʒ \quad 4ʒʒ}{12\varphi}$. *Example.*

$$E.\quad \begin{array}{c|c|c} 9ʒ & \times & 4ʒʒ \\ 3ʒ & & 1ℨ \\ 4\varphi & & 3N \\ & 12\varphi & \end{array}$$

3. To reduce Plural Fractions to one Denomination, which *mutatis mutandis* is like the last. *Plural.*

And so $\frac{1ʒ + 2ℨ}{2\varphi + 3N}$ reduced with $\frac{2\varphi \;- 4N}{3ʒʒ - 5ℨ}$, shall stand as at *F.*; and be $\frac{3ʒ\varphi + 6ß \;- 5\varphi - 10ʒ \qquad 4ʒ\varphi - 2\varphi - 12N}{6Bß - 1ʒʒ - 15ℨ}$, as by the respective Multiplications of Denominator by Denominator, and then alternately the Denominator of each into the others Numerator will appear. *Example.*

$$F.\quad \begin{array}{c|c|c} 3ʒ\varphi + 6ß - 5\varphi - 10ʒ & \times & 4ʒ\varphi - 2\varphi - 12N \\ 1ʒ + 2ℨ & & 2\varphi - 4N \\ 2\varphi + 3N & & 3ʒʒ - 5ℨ \\ & 6Bß - 1ʒʒ - 15ℨ & \end{array}$$

Mixt turned into Improper Fractions.

4. To reduce Mixt Numbers, *viz.* Whole and Broken Cossicks into an Improper Fraction, or to set an Whole Cossick in form of a Fraction, both like as was shewed in Abstract Fractions.

Examples.

And so $2\text{ʒ}\,\frac{2N}{3\mathfrak{z}}$ shall be reduced into $\frac{6\varphi + 2N}{3\mathfrak{z}}$ by multiplying 2ʒ, the Whole Cossick, into $3\mathfrak{z}$ the Denominator of the Fraction, which make 6φ, and then adding thereto $2N$, makes the Numerator $6\varphi + 2N$, to which the $3\mathfrak{z}$ shall be the Denominator.

Absurd Numbers what.

And if $\frac{2N}{3\mathfrak{z}} - 2\text{ʒ}$, which indeed is an absurd Number, or less than nothing were to be reduced, the Reduction shall be $\frac{2N - 6\varphi}{3\mathfrak{z}}$.

And if $2\text{ʒ} + 3N\,\frac{3\varphi - 2N}{4\text{ʒʒ}}$ be reduced, the Improper Fraction will stand thus, $\frac{8\text{ʒ}\varphi + 12\text{ʒʒ} + 3\varphi - 2N}{4\text{ʒʒ}}$.

Whole Coffick set as a Fraction.

But if any Whole Cossick be set Fraction wise, there is only $1N$ to be subscribed. And so $3\varphi + 4\text{ʒ} - 2\mathfrak{z}$ shall be set thus, $\frac{3\varphi + 4\text{ʒ} - 2\mathfrak{z}}{1N}$.

Improper turned back to Integral.

5. To reduce Improper Cossical Fractions back into Whole Cossicks, or an Whole and Broken Cossick, divide the Numerator by the Denominator, whereby sometime the Denominator is wholly discharged, when nothing remains upon the Division, or the Integer turned into a Mixt Number.

Examples.

And so $\frac{63\varphi}{21\text{ʒ}}$ will be reduced into $3\mathfrak{z}$. And $\frac{30\text{ſβ} + 24\text{ʒʒ} + 18\varphi}{20\text{ʒ}}$ into $1\frac{1}{2}\varphi + 1\frac{1}{5}\text{ʒ} + \frac{9}{10}\mathfrak{z}$.

Many Fractions to one Denominator.

6. To reduce several sorts of Fractions, or many of one sort, into one Denomination, by the Reduction proper to each of them.

Examples.

As $\frac{3}{4}\varphi$ reduced with $\frac{2\text{ʒ}}{3\varphi}$ shall be $\frac{9\text{ʒ}\varphi \quad 8\text{ʒ}}{12\varphi}$.

And so $\frac{1\mathfrak{z}}{2N}$ and $\frac{2N}{3\mathfrak{z}}$ reduced with $\frac{3\text{ʒ} - 2N}{5\varphi}$ shall be $\frac{15\text{ſβ} \quad 20\varphi \quad 18\varphi - 12\mathfrak{z}}{30\text{ʒʒ}}$ and abbreviated, $\frac{15\text{ʒʒ} \quad 20\text{ʒ} \quad 18\text{ʒ} - 12N}{30\varphi}$.

Proof of Cossical Reduction.

One part of Cossical Reduction hath the same faculty with other Reductions, to prove the other part reciprocal thereto, as may easily be discerned without Example. And moreover, by resolving the Cossical Fractions into abstract Numbers, as before in the works of Whole Cossicks, every part of Reduction may be fully proved.

As in the last Example, supposing 2 be a Root, then shall each of the Cossical Fractions be as at *G*, and the reduced Fractions found to agree in value without and with abbreviation.

	$15\text{ſβ} = 480$	$20\varphi = 160$	$18\varphi - 12\mathfrak{z} = 144 - 24$
G.	$\frac{1\mathfrak{z}}{2N} = \frac{2}{2}$	$\frac{2N}{3\mathfrak{z}} = \frac{2}{6}$	$\frac{3\text{ʒ} - 2N}{5\varphi} = \frac{12 - 2}{40}$
		$30\text{ʒʒ} = 480$	

Abbreviated.

$15\text{ʒʒ} = 240$	$20\text{ʒ} = 80$	$18\text{ʒ} - 12N = 72 - 12$
	$30\varphi = 240$	

CHAP. VIII.

Addition of Broken Cossicks.

Broken Cossicks added. Simple and Homogeneal.

IN adding Cossical Fractions; first if they be Simple, and the Numbers and Cossicks be of one Denomination, then add the Numerators, and subscribe the Common Denominator with the Cossical Character: But if the Numbers be not of one Denomination,

mination, reduce them as Common Fractions, and then add their Numerators as above.

Examples.

As to add $\frac{3}{5}$ʒ to $\frac{1}{5}$ʒ, make the Total $\frac{4}{5}$ʒ, as at *A*.

But $\frac{3}{4}$φ added to $\frac{1}{3}$φ must first be reduced to $\frac{9 \quad 4}{12}$φ, and then added, make together $\frac{13}{12}$φ, or $1\frac{1}{12}$φ, as at *B*.

$$A.\quad \frac{\overline{\quad 4 \quad}}{\frac{3}{5}ʒ + \frac{1}{5}ʒ} = \frac{4}{5}ʒ \qquad (\text{denominator } 5)$$

$$B.\quad \overset{13}{\overset{9 \;+\; 4}{\frac{3}{4}φ \times \frac{1}{3}φ}} = \frac{13}{12}φ \text{ or } \left(1\frac{1}{12}φ\right. \qquad (\text{denominator } 12)$$

2. If the Cossical Quantities of Simple Fractions to be added, be unlike or Heterogeneal, then connex them by the Sign of Addition, or reduce them.

Simple and Heterogeneal.

Example.

As to add $\frac{3}{5}$ʒ to $\frac{1}{2}$φ, I set them as at *C.* or *D.*

$$C.\quad \frac{3}{5}ʒ + \frac{1}{2}φ \qquad\qquad D.\quad \overset{6ʒ \;+\; 5φ}{\frac{3}{5}ʒ \times \frac{1}{2}φ} = \frac{5φ + 6ʒ}{10} \qquad (\text{denominator } 10)$$

3. If the Fractions be Compound, and of like Denominations in Numbers and Cossicks, then add the Numerators as Cossicks are to be added, and subscribe the Common Denominator.

Compound and Homogeneal.

Examples.

As $\frac{3\,N}{10\,ℨ}$ added to $\frac{6\,N}{10\,ℨ}$ shall make the Total $\frac{9\,N}{10\,ℨ}$, as at *E*.

And $\frac{3ℨ + 2N}{20ʒ}$ added to $\frac{2ℨ + 1N}{20ʒ}$ shall make together $\frac{5ℨ + 3N}{20ʒ}$, as at *F*.

And $\frac{3ℨ + 2N}{20ʒ}$ added to $\frac{2ℨ - 5N}{20ʒ}$ shall make the Total $\frac{5ℨ - 3N}{20ʒ}$, as at *G*.

$$E.\quad \overset{9N}{\frac{3N}{10ℨ} + \frac{6N}{10ℨ}} = \frac{9N}{10ℨ} \qquad (10ℨ)$$

$$F.\quad \overset{5ℨ + 3N}{\frac{3ℨ + 2N}{20ʒ} + \frac{2ℨ + 1N}{20ʒ}} = \frac{5ℨ + 3N}{20ʒ} \qquad (20ʒ)$$

$$G.\quad \overset{5ℨ - 3N}{\frac{3ℨ + 2N}{20ʒ} + \frac{2ℨ - 5N}{20ʒ}} = \frac{5ℨ - 3N}{20ʒ} \qquad (20ʒ)$$

4. If the Numbers or Compound Cossicks be not alike, first reduce them, and then add their Numerators, as last above-mentioned.

Compound and Heterogeneal.

Examples.

As to add $\frac{8φ + 9ʒ}{10ʒʒ}$ to $\frac{6φ - 3ʒ}{10φ}$, they are first reduced to the Denomination of 100 Bſß, and then added and abbreviated, as at *H.* and *I.*

$$H.\quad \overset{60\,Bſß + 50ʒφ + 90\,ſß}{\overset{80ʒφ + 90\,ſß \quad + \quad 60\,Bſß - 30ʒφ}{\frac{8φ + 9ʒ}{10ʒʒ} \times \frac{6φ - 3ʒ}{10φ}}} = \frac{60\,Bſß + 50ʒφ + 90\,ſß}{100\,Bſß} \qquad (100\,Bſß)$$

Abbreviated in Numbers and Quantities.

$$I.\quad \frac{60\,Bſß + 50ʒφ + 90\,ſß}{100\,Bſß} = \frac{6\,Bſß + 5ʒφ + 9\,ſß}{10\,Bſß} = \frac{6ʒ + 5ℨ + 9N}{10ʒ}$$

Also

Also $\frac{2ȝ + 3z}{3φ + 4N}$ is added to $\frac{4z - 2N}{2ȝ + 2z}$ in like manner at *K*. The Total whereof may be abbreviated to $\frac{8ȝȝ + 2φ + 3ȝ + 8z - 4N}{3ß + 3ȝȝ + 4ȝ + 4z}$.

$$\begin{array}{ccccc} & & 16ȝȝ + 4φ + 6ȝ + 16z - 8N & & \\ \hline & 4ȝȝ + 10φ + 6ȝ & + & 12ȝȝ + 16z - 6φ - 8N & \\ \hline K. & \frac{2ȝ + 3z}{3φ + 4N} & \times & \frac{4z - 2N}{2ȝ + 2z} & \\ \hline & & 6ß + 6ȝȝ + 8ȝ + 8z & & \end{array}$$

Proof of Cossical Addition of Fractions. Addition of Cossical Fractions is proved both by Substraction, as in the next *Chapter*, and by taking some fit Number for a Root, and accordingly turning the Cossical Fractions into Abstract, and after the Addition to parallel the Totals.

As in the last Example, supposing 2 be a Root, the Fractions given to be added will be in their least Terms, $\frac{1}{2}$ and $\frac{1}{2}$, which added make 1 Integer; and so much is the added Cossick.

$$\frac{2ȝ + 3z}{3φ + 4N} = \frac{8 + 6}{24 + 4} = \frac{14}{28} = \frac{1}{2}$$

$$\frac{4z - 2N}{2ȝ + 2z} = \frac{8 - 2}{8 + 4} = \frac{6}{12} = \frac{1}{2}$$

And as $\frac{\overset{2}{\overline{1}}}{2} + \frac{1}{\underset{2}{\underline{2}}} = \frac{1}{1}$, or 1, so is $\frac{336}{336}$ (1

Numerator.		Denominator.	
$16ȝȝ =$	256	$6ß =$	192
$+ 4φ =$	32	$+ 6ȝȝ =$	96
$+ 6ȝ =$	24	$+ 8ȝ =$	32
$+ 16z =$	32	$+ 8z =$	16
	344		336
$- 8N$	8		
	336		

CHAP. IX.

Substraction of Broken Cossicks.

Broken Cossicks substracted. Simple and Homogeneal. In substracting Cossical Fractions; first if they be Simple, and the Numbers and Cossicks be of like Denominations, then take the Lesser Numerator from the Greater, and to the Remain subscribe the Common Denominator with the Cossical Character. But when the Subtrahend is the Greater change the Sign to the difference.

Examples. As to take $\frac{2}{5}φ$ from $\frac{3}{5}φ$, the Remain shall be $\frac{1}{5}φ$, as at *A*.

But to take $\frac{3}{5}φ$ from $\frac{2}{5}φ$, the Remain shall be $-\frac{1}{5}φ$, as at *B*.

$$A.\quad \frac{3}{5}φ - \frac{2}{5}φ = \frac{1}{5}φ \qquad\qquad B.\quad \frac{2}{5}φ - \frac{3}{5}φ = -\frac{1}{5}φ$$

Simple and Heterogeneal. 2. If the Numbers be not of one Denomination, or the Simple Cossicks Heterogeneal, then reduce the Numbers as Common Fractions, and then substract the Numerators of the Homogeneal, as above: But let the unlike Cossicks be connexed with the Sign of Substraction.

Example. As to take $\frac{1}{2}ȝ$ from $\frac{2}{3}ȝ$, the Remain after Reduction of the Fractions will be $\frac{1}{6}ȝ$ as at *C*.

But if $\frac{1}{2}ȝ$ were to be taken from $\frac{2}{3}φ$, I set them as at *D*, or *E*.

C. $\begin{array}{c} 1 \\ \hline \frac{4}{\frac{2}{3}ʒ} \;\text{X}\; \frac{3}{\frac{1}{2}ʒ} \\ \hline 6 \end{array} = \frac{1}{6}ʒ$ D. $\frac{2}{3}φ - \frac{1}{2}ʒ$ E. $\begin{array}{c} \frac{4φ}{\frac{2}{3}φ} \;\text{X}\; \frac{3ʒ}{\frac{1}{2}ʒ} \\ \hline 6 \end{array} = \frac{4φ - 3ʒ}{6}$

3. If the Fractions be Compound, and of like Denominations both in Numbers and Cossicks, then substract the Lesser Numerator out of the Greater, as Cossicks are to be substracted, and subscribe the Common Denominator: But when the Subtrahend is the Greater, change the Sign to the Difference, as before. *Compound and Homogeneal.*

As to take $\frac{1ȥ}{3ʒ}$ from $\frac{2ȥ}{3ʒ}$, the Remain will be $\frac{1ȥ}{3ʒ}$, as at *F*. *Examples.*

But if $\frac{2ȥ}{3ʒ}$ be taken from $\frac{1ȥ}{3ʒ}$, the Remain will be $-\frac{1ȥ}{3ʒ}$; as at *G*.

So if $\frac{3ȥ+2N}{9ʒ}$ be taken from $\frac{6ȥ+7N}{9ʒ}$, the Remain will be $\frac{3ȥ+5N}{9ʒ}$, as at *H*.

And if $\frac{3ʒ+5ȥ}{4φ}$ be abated out of $\frac{5ʒ-6ȥ}{4φ}$, the Remain will be $\frac{2ʒ-11ȥ}{4φ}$, as at *I*.

Also $\frac{4ȥ-3N}{10ʒ-2ȥ}$ deducted out of $\frac{7ȥ-2N}{10ʒ-2ȥ}$, the Remain will be $\frac{3ȥ+1N}{10ʒ-2ȥ}$, as at *K*.

F. $\begin{array}{c} 1ȥ \\ \hline \frac{2ȥ}{3ʒ} - \frac{1ȥ}{3ʒ} \\ \hline 3ʒ \end{array} = \frac{1ȥ}{3ʒ}$ G. $\begin{array}{c} -1ȥ \\ \hline \frac{1ȥ}{3ʒ} - \frac{2ȥ}{3ʒ} \\ \hline 3ʒ \end{array} = -\frac{1ȥ}{3ʒ}$

H. $\begin{array}{c} 3ȥ+5N \\ \hline \frac{6ȥ+7N}{9ʒ} - \frac{3ȥ+2N}{9ʒ} \\ \hline 9ʒ \end{array} = \frac{3ȥ+5N}{9ʒ}$ I. $\begin{array}{c} 2ʒ-11ȥ \\ \hline \frac{5ʒ-2ȥ}{4φ} - \frac{3ʒ+5ȥ}{4φ} \\ \hline 4φ \end{array} = \frac{2ʒ-11ȥ}{4φ}$

K. $\begin{array}{c} 3ȥ+1N \\ \hline \frac{7ȥ-2N}{10ʒ-2ȥ} - \frac{4ȥ-3N}{10ʒ-2ȥ} \\ \hline 10ʒ-2ȥ \end{array} = \frac{3ȥ+1N}{10ʒ-2ȥ}$

4. If the Numbers or Compound Cossicks be of unlike Denominations; first reduce them, and then substract their Numerators, as last above-mentioned. *Compound and Heterogeneal.*

As to abate $\frac{2ȥ}{3ʒ}$ from $\frac{9N}{10ȥ}$, they are first reduced to the Denomination of $30φ$, *Examples.* and the Remain then is $\frac{7ʒ}{30φ}$, and abbreviated $\frac{7N}{30ȥ}$, as at *L*.

And so to take $\frac{2}{3}ʒ + \frac{9}{10}ȥ$ from $\frac{3}{4}ʒ + \frac{2}{3}ȥ$, they are first to be reduced severally, then jointly; and then substracted the Remain is $\frac{5ʒ - 14ȥ}{60N}$, as at *M*.

Also $\frac{2ʒ+3ȥ}{3φ+4N}$ substracted, as at *N*, from $\frac{4ȥ-2N}{2ʒ+2ȥ}$, will leave remaining, $\frac{8ʒʒ-16φ-6ʒ+16ȥ-8N}{6ß+6ʒʒ+8ʒ+8ȥ}$ which will be abbreviated to $\frac{4ʒʒ-8φ-3ʒ+8ȥ-4N}{3ß+3ʒʒ+4ʒ+4ȥ}$.

L. $\begin{array}{c} 7ʒ \\ \hline \frac{27ʒ}{\frac{9N}{10ȥ}} \;\text{X}\; \frac{20ʒ}{\frac{2ȥ}{3ʒ}} \\ \hline 30φ \end{array} = \frac{7ʒ}{30φ}$ or $\frac{7N}{30φ}$

M.

$$\frac{9\text{ʒ}+8\text{ℨ}}{\frac{3}{4}\text{ʒ}+\frac{2}{3}\text{ℨ}}\ (12)\quad-\quad\frac{20\text{ʒ}+27\text{ℨ}}{\frac{2}{3}\text{ʒ}+\frac{9}{10}\text{ℨ}}\ (30)\qquad \frac{5\text{ʒ}-14\text{ℨ}}{\dfrac{45\text{ʒ}+40\text{ℨ}}{9\text{ʒ}+8\text{ℨ}}\ (12\ldots 5)\ -\ \dfrac{40\text{ʒ}+54\text{ℨ}}{20\text{ʒ}+27\text{ℨ}}\ (30\ldots 2)}=\frac{5\text{ʒ}-14\text{ℨ}}{60\text{N}}\qquad \underbrace{5\quad 2}_{60}$$

$$\frac{8\text{ʒʒ}-16\varphi-6\text{ʒ}+16\text{ℨ}-8\text{N}}{12\text{ʒʒ}+16\text{ℨ}-6\varphi-8\text{N}\qquad 4\text{ʒʒ}+10\varphi+6\text{ʒ}}$$

N. $$\frac{4\text{ℨ}-2\text{N}}{2\text{ʒ}+2\text{ℨ}}\ \times\ \frac{2\text{ʒ}+3\text{ℨ}}{3\varphi+4\text{N}}$$
$$6\text{ß}+6\text{ʒʒ}+8\text{ʒ}+8\text{ℨ}$$

Proof of Cossical Substraction of Fractions. Substraction of Cossical Fractions is proved, as Addition before, by converting the Cossicks into Abstract Fractions, by some apt Root taken at pleasure, and after Substraction made therewith, the Remains will be alike valuable.

As in the last Example, taking 2 for the Root, the Fractions given in their least Terms will be $\frac{1}{2}$ and $\frac{1}{2}$, which substracted the one from the other, leave o for the Remain. And so is the Cossical Remain 8ʒʒ — 16φ — 6ʒ + 16ℨ — 8N, because the negative Quantities equally counterballance the affirmative.

$$\frac{4\text{ℨ}-2\text{N}}{2\text{ʒ}+2\text{ℨ}}=\frac{8-2}{8+4}=\frac{6}{12}=\frac{1}{2}$$

$$\frac{2\text{ʒ}+3\text{ℨ}}{3\varphi+4\text{N}}=\frac{8+6}{24+4}=\frac{14}{28}=\frac{1}{2}$$

Numerator.

8ʒʒ =	128	— 16φ =	128
+ 16ℨ =	32	— 6ʒ =	24
		— 8N =	8
	160		160

And as $\dfrac{\overset{0}{\overline{\frac{1}{2}-\frac{1}{2}}}}{2}=\frac{0}{2}$, or o; so 160 — 160 = o.

Besides, if the Remain 8ʒʒ — 16φ — 6ʒ + 16ℨ — 8N be Cossically added to 4ʒʒ + 10φ + 6ʒ, the Number substracted, the total will be 12ʒʒ + 16ℨ — 6φ — 8N, as above; whereby Addition and Substraction of Cossical Fractions are seen to be alternate Proofs of each other.

CHAP. X.

Multiplication of Broken Cossicks.

Broken Cossicks multiplyed. Simple. IN multiplying Cossical Fractions; first if they be Simple, increase Numerator by Numerator, and Denominator by Denominator, as Common Fractions are multiplyed, and annex to the Product the Cossical Character due to the Total of both their *Indices*.

Example. As $\frac{1}{2}$ʒ multiplyed by $\frac{5}{6}\varphi$, shall make the Product $\frac{5}{12}$ß, as at *A*.

And $\frac{1}{3}\varphi$ with $\frac{2}{3}$ʒ, and $\frac{4}{5}$ℨ multiplyed together shall make the Product $\frac{8}{45}$ʒφ, as at *B*.

A. $\dfrac{\overset{5}{\overline{\frac{1}{2}\text{ʒ}\times\frac{5}{6}\varphi}}}{12}=\frac{5}{12}\text{ß}$ *Indices.* $\dfrac{\begin{matrix}2\\3\end{matrix}}{5}$ *B.* $\dfrac{\overset{8}{\overline{\frac{1}{3}\varphi\times\frac{2}{3}\text{ʒ}\times\frac{4}{5}\text{ℨ}}}}{45}=\frac{8}{45}\text{ʒ}\varphi$ *Indices.* $\dfrac{\begin{matrix}3\\2\\1\end{matrix}}{6}$

Compound. 2. If the Fractions be Compound, then multiply after the manner of Compound Cossicks,

Cossicks, Numerator by Numerator, and Denominator by Denominator ; and both Cossicks and Signs + and — of the produced Numbers are known, as if they were Whole Cossicks.

Examples.

As to multiply $\frac{1N}{3\text{ℨ}}$ by $\frac{1\text{ℨ}}{2\text{ʒ}}$, the Product will be $\frac{1\text{ℨ}}{6\varphi}$, and abbreviated $\frac{1N}{6\text{ʒ}}$, as at *C*. And to multiply $\frac{2\text{ʒ}+3\text{ℨ}-2N}{4\varphi}$ by $\frac{4\text{ℨ}+2N}{3\varphi-5\text{ʒ}}$, the Numerators and Denominators being multiplyed as at *D*, the Product of the new Fraction is $\frac{8\varphi+16\text{ʒ}-2\text{ℨ}-8N}{12\text{ʒ}\varphi-20\text{ß}}$ and abbreviated $\frac{4\varphi+8\text{ʒ}-1\text{ℨ}-2N}{6\text{ʒ}\varphi-10\text{ß}}$.

$$C.\quad \frac{\overset{1}{\overline{\;1N\;}}}{3\text{ℨ}}\times\frac{1\text{ℨ}}{\underset{6}{\underline{2\text{ʒ}}}}=\frac{1\text{ℨ}}{6\varphi}$$

$$D.\quad \frac{\overset{8\varphi+16\text{ʒ}-2\text{ℨ}-4N}{\overline{2\text{ʒ}+3\text{ℨ}-2N}}}{4\varphi}\times\frac{4\text{ℨ}+2N}{\underset{12\text{ʒ}\varphi-20\text{ß}}{\underline{3\varphi-5\text{ʒ}}}}=\frac{8\varphi+16\text{ʒ}-2\text{ℨ}-4N}{12\text{ʒ}\varphi-20\text{ß}}$$

$$\begin{array}{llll} 2\text{ʒ}+3\text{ℨ}-2N & & & 3\varphi-5\text{ʒ} \\ 4\text{ℨ}+2N & & & 4\varphi \\ \hline 8\varphi+12\text{ʒ}-8\text{ℨ} & & & 12\text{ʒ}\varphi-20\text{ß}\ \text{Denominator.} \\ \quad 4\text{ʒ}+6\text{ℨ}-4N & & & \\ \hline 8\varphi+16\text{ʒ}-2\text{ℨ}-4N\ \text{Numerator.} & & & \end{array}$$

Proof of Cossical Multiplication of Fractions.

Multiplication of Cossical Fractions, is proved as well by Division noted in the next *Chapter*, as by converting the Cossical Fractions into Abstract ; and after Multiplication of them, comparing the equality in their Products.

As in the last Example, the given Cossicks converted into Abstract Fractions supposing the Root 2, are $\frac{12}{32}$ and $\frac{10}{4}$, which multiplyed make the Product without abbreviation $\frac{120}{128}$ and with it $\frac{15}{16}$, which agree with the new Cossical Fraction in equal value.

$$\frac{2\text{ʒ}+3\text{ℨ}-2N}{4\varphi}=\frac{8+6-2}{32}=\frac{12}{32}=\frac{3}{8}$$

$$\frac{4\text{ℨ}+2N}{3\varphi-5\text{ʒ}}=\frac{8+2}{24-20}=\frac{10}{4}=\frac{5}{2}$$

$$\begin{array}{rr} \text{Numerator.} & \\ 8\varphi= & 64 \\ +16\text{ʒ}= & 64 \\ \hline & 128 \\ -2\text{ℨ}= & 4 \\ -4N= & 4 \\ \hline & 8 \\ \hline & 120 \end{array}\qquad \begin{array}{rr} \text{Denominotor.} & \\ 12\text{ʒ}\varphi= & 768 \\ -20\text{ß}= & 640 \\ \hline & 128 \end{array}$$

$$\text{And as } \frac{\overset{120}{\overline{12}}}{32}\times\frac{10}{\underset{128}{\underline{4}}}\ \text{or}\ \frac{\overset{15}{\overline{3}}}{8}\times\frac{5}{\underset{16}{\underline{2}}},\ \text{so is}\ \frac{120}{128}\Big|\frac{60}{64}\Big|\frac{30}{32}\Big|\frac{15}{16}$$

CHAP. XI.

Division of Broken Cossicks.

Broken Cossicks divided. *Simple.*

IN dividing Cossical Fractions ; first if they be Simple, multiply cross wise, as in Common Fractions, the Numerator of the Dividend by the Denominator of the Divisor for the Numerator of the Quotient, and the Denominator of the Dividend by the Numerator of the Divisor for the Denominator of the Quotient : And thereto annex the Cossical Character due to the Remain of the *Index* of the Divisor deducted from the *Index* of the Dividend.

As

Examples. As to divide $\frac{1}{2}\text{ß}$ by $\frac{8}{9}\text{ʒ}$, the Quotient shall be $\frac{9}{16}\varphi$, set as at *A.* or *B.*

And $\frac{3}{4}\varphi$ divided by $\frac{3}{4}\text{ʒ}$, shall make the Quotient $1\,\text{ℨ}$, as at *C.* or *D.*

$$A.\ \frac{1}{2}\text{ß}\ \overset{9}{\underset{16}{\times}}\ \frac{8}{9}\text{ʒ} = \frac{9}{16}\varphi \qquad B.\ \frac{8}{9}\text{ʒ}\Big)\frac{1}{2}\text{ß}\Big(\frac{9}{16}\varphi \qquad C.\ \frac{\overset{1}{3}}{\underset{1}{4}}\varphi\ \overset{1}{\underset{1}{\times}}\ \frac{\overset{1}{3}}{\underset{1}{4}}\text{ʒ} = \frac{1}{1}\text{ℨ} \qquad D.\ \frac{\overset{1}{3}}{\underset{1}{4}}\text{ʒ}\Big)\frac{\overset{1}{3}}{\underset{1}{4}}\varphi\Big(1\,\text{ℨ}$$

Compound. 2. If the Fractions be Compound, then after the manner of Compound Cossicks, multiply the Numerator of the Dividend into the Denominator of the Divisor, and contrariwise, the Denominator of the Dividend into the Numerator of the Divisor; and the Cossicks and Signs + and —, of the produced Quotienary Numbers, shall be as if they were Whole Cossicks.

Examples. As to divide $\frac{4\,N}{5\,\text{ℨ}}$ by $\frac{5\,N}{6\,\text{ℨ}}$ giveth in the Quotient $\frac{24\,\text{ℨ}}{25\,\text{ℨ}}$ or $\frac{24\,N}{25\,N}$, as at *E.* or *F.*

And $\frac{12\,\text{ʒʒ} - 16\,\text{ℨ}}{2\,\text{ʒ} - 5\,N}$ divided by $\frac{9\,\text{ʒ} - 3\,N}{4\,\text{ʒ} + 5\,N}$, maketh the Quotient to be $\frac{48\,\text{ʒ}\varphi + 60\,\text{ʒʒ} - 64\,\varphi - 80\,\text{ℨ}}{18\,\text{ʒʒ} - 51\,\text{ʒ} + 15\,N}$, as at *G.* or *H.*

$$E.\ \frac{4\,N}{5\,\text{ℨ}}\ \overset{24\,\text{ℨ}}{\underset{25\,\text{ℨ}}{\times}}\ \frac{5\,N}{6\,\text{ℨ}} = \frac{24\,\text{ℨ}}{25\,\text{ℨ}} \qquad F.\ \frac{5\,N}{6\,\text{ℨ}}\Big)\ \frac{4\,N}{5\,\text{ℨ}}\ \Big(\frac{24\,\text{ℨ}}{25\,\text{ℨ}} \text{ or } \frac{24\,N}{25\,N}$$

$$G.\ \frac{12\,\text{ʒʒ} - 16\,\text{ℨ}}{2\,\text{ʒ} - 5\,N}\ \overset{48\,\text{ʒ}\varphi + 60\,\text{ʒʒ} - 64\,\varphi - 80\,\text{ℨ}}{\underset{18\,\text{ʒʒ} - 51\,\text{ʒ} + 5\,N}{\times}}\ \frac{9\,\text{ʒ} - 3\,N}{4\,\text{ʒ} + 5\,N} = \frac{48\,\text{ʒ}\varphi + 60\,\text{ʒʒ} - 64\,\varphi - 80\,\text{ℨ}}{18\,\text{ʒʒ} - 51\,\text{ʒ} + 15\,N}$$

$$H.\ \frac{9\,\text{ʒ} - 3\,N}{4\,\text{ʒ} + 5\,N}\Big)\ \frac{12\,\text{ʒʒ} - 16\,\text{ℨ}}{2\,\text{ʒ} - 5\,N}\ \Big(\frac{48\,\text{ʒ}\varphi + 60\,\text{ʒʒ} - 64\,\varphi - 80\,\text{ℨ}}{18\,\text{ʒʒ} - 51\,\text{ʒ} + 15\,N}$$

$$\begin{array}{llll}
 & 12\,\text{ʒʒ} - 16\,\text{ℨ} & & 2\,\text{ʒ} - 5\,N \\
 & 4\,\text{ʒ} + 5\,N & & 9\,\text{ʒ} - 3\,N \\ \hline
 & 48\,\text{ʒ}\varphi - 64\,\varphi & & 18\,\text{ʒʒ} - 45\,\text{ʒ} \\
 & \quad 60\,\text{ʒʒ} - 80\,\text{ℨ} & & \quad - 6\,\text{ʒ} + 15\,N \\ \hline
\text{Numerator} & 48\,\text{ʒ}\varphi + 60\,\text{ʒʒ} - 64\,\varphi - 80\,\text{ℨ} & & 18\,\text{ʒʒ} - 51\,\text{ʒ} + 15\,N \ \text{Denominator}
\end{array}$$

Proof of Cossical Division of Fractions. Division of Cossical Fractions is proved, as other Elementary Operations thereof, by resolving the Cossical Fractions into Abstract, taking some fit Number for a Root at pleasure, and after Division made, as in Common Fractions, the Quotients will be found equal.

As in the last Example, supposing 2 for the Root, the given Fractions will be $\frac{160}{3}$ and $\frac{33}{21}$, and the Quotient thereof $\frac{3360}{99}$, and so is the Cossical Quotient $\frac{48\,\text{ʒ}\varphi + 60\,\text{ʒʒ} - 64\,\varphi - 80\,\text{ℨ}}{18\,\text{ʒʒ} - 51\,\text{ʒ} + 15\,N}$.

$$\frac{12\,\text{ʒʒ} - 16\,\text{ℨ}}{2\,\text{ʒ} - 5\,N} = \frac{192 - 32}{8 - 5} = \frac{160}{3}$$

$$\frac{9\,\text{ʒ} - 3\,N}{4\,\text{ʒ} + 5\,N} = \frac{36 - 3}{16 + 5} = \frac{33}{21}$$

Numerator.		Denominator.	
$48\,\text{ʒ}\varphi =$	3072	$18\,\text{ʒʒ} =$	288
$+ 60\,\text{ʒʒ} =$	960	$+ 15\,N =$	15
	4032		303
$- 64\,\varphi =$	512	$- 51\,\text{ʒ} =$	204
$- 80\,\text{ℨ} =$	160		99
	672		
	3360		

And so $\frac{33}{\underset{7}{21}}\Big)\ \frac{160}{\underset{1}{3}}\ \Big(\frac{3360}{99}$ or $\frac{1120}{33}$ being abbreviated.

Besides

Besides if the Quotient be multiplyed by the Divisor, the Dividend of the Simple Fractions will be returned or reduced thereto by abbreviation. But in Compound Fractions other Cossical Fractions in shew will be produced, yet the same in value; which serveth to evidence the alternate Proof of Multiplication and Division of Cossical Fractions one by the other.

For if $\frac{4\varphi + 9\text{ʒ}}{12\,N}$ be divided by $\frac{3\text{ʒ} + 2\text{ℨ}}{3\text{ℨ}}$, the Quotient will be $\frac{12\text{ʒʒ} + 27\varphi}{36\text{ʒ} + 24\text{ℨ}}$ and by abbreviation $\frac{4\varphi + 9\text{ʒ}}{12\text{ℨ} + 8N}$; wherefore if this Quotient be multiplyed by $\frac{3\text{ʒ} + 2\text{ℨ}}{3\text{ℨ}}$ the Divisor, the Product abbreviated will be $\frac{12\text{ʒʒ} + 35\varphi + 18\text{ʒ}}{36\text{ℨ} + 24N}$, which nevertheless is but equal in value to the Dividend $\frac{4\varphi + 9\text{ʒ}}{12N}$, as by the Root 2 appeareth.

$$\frac{4\varphi + 9\text{ʒ}}{12N} = \frac{32 + 36}{12N} = \frac{68}{12} = 5\tfrac{2}{3}$$

$$\begin{array}{lr} 12\text{ʒʒ} = & 192 \\ 35\varphi = & 280 \\ 18\text{ʒ} = & 72 \\ \hline & 544 \end{array} \qquad \begin{array}{lr} 36\text{ℨ} = & 72 \\ 24N = & 24 \\ \hline & 96 \end{array} \qquad \begin{array}{l} 16 \\ 544 \\ 96 \end{array}\Big(5\tfrac{2}{3}$$

CHAP. XII.

Figuration of Cossicks.

To Figurate any Cossick is Cossically to multiply the same, be it Simple or Compound by it self to produce the Square, and that again by the Root to produce the Cube, *&c.* as other Figural Numbers are produced. *Figurate Cossicks produced.*

As to produce the Cube of 2 ℨ, or the Square of 2 ℨ + 3 N, their Figurate Multiplications bring forth 8 φ for the one, and 4 ʒ + 12 ℨ + 9 N for the other. *Examples.*

	Simple		Compound
Root	2 ℨ	Root	2 ℨ + 3 N
	2 ℨ		2 ℨ + 3 N
Square	4 ʒ		4 ʒ + 6 ℨ
	2 ℨ		6 ℨ + 9 N
Cube	8 φ		4 ʒ + 12 ℨ + 9 N

In extracting the Root of a Cossick several things are to be noted. For first, every Cossick is not Cossically Rooted, no more than every Absolute Number is a Rooted Number. But those Cossicks are Rooted, which have a Root agreeable to the Figure or Character of his Quantity; and therefore no Cossick may be properly called Square, Cubick, or otherwise Rooted, except the Root of the Cossick agreeth with his Character. So 8 φ is a Cubick Cossick, and his Root 2 ℨ; because 8 is a Cubick Number agreeable to his Character φ. But 8 ʒ hath no Cossical Root, because 8 hath no Square Root agreeable to the Character ʒ, neither is it a Cubick Cossick, although the Number have a Cubick Root, because the Cubick Root is disagreeable to the Character ʒ. Likewise 16 φ, is no Rooted Cossick, because 16 hath no exact Cubick Root; but 16 ʒ is Cossically Rooted, and hath 4 ℨ for the Root thereof. *What to be noted in extracting of their Roots.* *1. Which Cossicks are Rooted.*

2. Simple Cossicks Compound in their Characters are not Rooted, unless the Number annexed will yield a Root according to the composition of the Character. So 16 ʒʒ because compound of the Square twice, may either have a ʒℨ, or a ʒʒℨ, the one 4 ʒ, the other 2 ℨ. And 4 ʒʒ hath a Square Root 2 ʒ, but no ʒʒℨ agreeable to the whole Character. And 9 ʒφ hath a ʒℨ, which is 3 φ agreeable to part of his Character, but no ʒφ Root answerable to the compounded Character. *2. Which Simple Cossicks Compound in their Characters are Rooted.*

And therefore if any Cossick Compound in his Character have a Root agreeable to his whole Character, then may he have also as many Roots as their be parts in that composition: For so 4096 ʒʒφ hath not only a ʒʒφ Root, which is 2 ℨ, but also a ʒφ *How many Roots such may have.*

Root which is 4ʒ, and thirdly a ʒʒ Root which is 8φ, and fourthly a φ Root which is 16ʒʒ, and lastly a ʒ Root which is 64ʒφ.

3. What proper to Compound Rooted Cossicks.

3. Compound Cossicks, if they be Cossically Rooted, must first have their greatest Cossick a Rooted Cossick, and the Number annexed to the least Denomination of that Compound Cossick, must be a Rooted Number of the same kind or denomination with the greatest Cossick. As in the Example above of 4ʒ + 12ℨ + 9N, the greatest Cossick 4ʒ is Rooted, and the least Denomination 9N is a ʒ Number.

4. What in the Compounds the part not Rooted is.

4. In a Compound Rooted Cossick, every part that is not a Rooted Cossick is a mean between the greatest Cossick and the least Denomination in that Cossick. And if ℨ be one Denomination, then N shall be another: As appeareth in the said Example, where also is 12ℨ to be seen a mean proportional between 9N and 4ʒ.

Others not with these properties what they are.

Other Cossicks that suit not with these particulars, are either Surdes, or such affected Cossicks, whose Roots are not properly Cossical, but being equal to some other Number, the investigation of their Roots are to be sought hereafter among *Æquations*.

Root of a Simple Cossick extracted.

These things premised, to extract the Root of a Simple Cossick, extract the Root of the Number according to the Character of the Quantity, and let the Character of the Root be set for the Denomination.

Examples.

As 9ʒ hath for his Root 3ℨ; and 64φ hath 4ℨ for his Root.

And if the Simple Cossick be Compound in the Character, then as in the former *Book* in *Figural Numbers*, search out the Root from the Number, and affix thereto the Cossical Character.

As if from 256, a Number of the 8th Quantity from the Root 2, I would search out the ʒʒʒ Root, I find it 2ℨ, as at *A*. If out thereof I would seek the ʒʒ Root, I find it 4ʒ, as at *B*. And if the ʒ Root, I find it 16ʒʒ, as at *C*.

So also 729ʒφ gives the ʒφ Root 3ℨ, as at *D*. And the φ Root 9ʒ, as at *E*. And the ʒ Root 27φ, as at *F*.

```
      333  33  3              333  33              333
      x                       x                    x
A.    256 | 16 | 4 | 2ℨ   B.  256 | 16 | 4ʒ    C.  256 | 16ʒʒ
      . .                     . .                  . .
      1     16   4            1     16             1
      156                     156                  156

      3φ    φ                                      3φ
      3                       3φ                   3
D.    729 | 27 | 3ℨ       E.  729 | 9ʒ         F.  729 | 27φ
      . .   27                .                    . .
      4                                            4
      329                                          329
```

Root of the Compounds extracted.

But if the Cossick be Compound, then prick your Cossick according to the Quantity whose Root you would extract, as was before taught in *Figural Numbers* in the *Second Part* of the *Second Book*; and out of the Left Hand pricked Cossick, take the Cossical Root, and place in the Quotient, with half his Cossical Denomination, for the Square; the third part for the Cube, &c. as was noted before in *Extraction of the Roots of Astronomicals*, and proceed with the rest of the work as before in *Figural Numbers*; Cossical Extraction differing therefrom no otherwise than as Cossical Addition and Substraction, Multiplication and Division, differs from that of Integers.

Example in the Square.

As to extract the Cossical Square Root of 16ʒʒʒ — 16Bβ — 20ʒφ + 20β + 5ʒʒ — 6φ + 1ʒ. The pricked Numbers will be 1ʒ, 5ʒʒ, 20ʒφ, and 16ʒʒʒ, in which last the greatest Cossical Square Root is 4ʒʒ; for 16 being a Square Number hath 4 for the Root, and half the *Index* of ʒʒʒ which is 8, is 4, whose Character is ʒʒ. This Root doubled is 8ʒʒ for the first Divisor, which dividing — 16Bβ gets — 2φ for the Quotient, and the Square of — 2φ is + 4ʒφ to be substracted from the next Right Hand Prick: And then doubling the Root, and proceeding to the end of the work, the whole Root obtained is 4ʒʒ — 2φ — 3ʒ + 1ℨ, as is obvious at *G*.

Example in the Cube.

Also to extract the Cossical Cube Root of 27φφ + 54ʒʒʒ — 72Bβ — 136ʒφ + 96β + 96ʒʒ — 64φ. The pricked Numbers will be 64φ, 136ʒφ, and 27φφ, in which the greatest Cube Root is 3φ; for 3 is the Cube Root of 27, and the *Index* of φφ is 9, whose third part is 3, the Character of which *Index* is φ: This tripled is 9φ, and multiplyed by the Root 3φ is 27ʒφ for the first Divisor; which dividing + 54ʒʒʒ gets + 2ʒ for the Quotient. The other parts of the Gnomon and proceedings in the work are plain at *H*.

G.

G. Square Root

−24ʒφ + 8ſß − 4ʒʒ

16ʒʒʒ − 16Bſß − 20ʒφ + 20ſß + 5ʒʒ − 6φ + 1ʒ (4ʒʒ − 2φ − 3ʒ + 1ℨ

16ʒʒʒ

Gnom. { −16Bſß + 4ʒφ
. . . . −24ʒφ + 12ſß + 9ʒʒ
. . . . + 8ſß − 4ʒʒ − 6φ + 1ʒ

H. Cube Root

−108Bſß − 144ʒφ

27φφ + 54ʒʒʒ − 72Bſß − 136ʒφ + 96ſß + 96ʒʒ − 64φ (3φ + 2ʒ − 4ℨ

27φφ

Gnomons { 54ʒʒʒ + 36Bſß + 8ʒφ
. . . . −108Bſß − 144ʒφ + 96ſß + 96ʒʒ − 64φ

The one part of Cossical Figuration serves for Proof of the other; that is, Production by Extraction, and Extraction by Production, as other Figural Numbers before exemplified clearly shew; and here may further be seen at *I.* and *K.*; where the Productions of the Simple Cossicks, whose Roots were before extracted at *A. B. C.* and *D. E. F.*; and at *L.* and *M.* are the Productions of the Compound Cossicks, whose Roots are extracted at *G.* and *H.* *Proof of Cossical Figuration.*

But besides, if the Cossicks be resolved into Abstract Numbers, and the Figural Roots thereof be taken, the same will not differ with the Cossical Root, unless some mistake occasion it.

Proof of the Simple Cossical Extraction.

I.		Root supposed 2.	*K.*		Root supposed 2.
2ℨ	1	4	3ℨ	1	6
2ℨ	1	4	3ℨ	1	6
4ʒ	2	16	9ʒ	2	36
4ʒ	2	16	3ℨ	1	6
16ʒʒ	4	96	27φ	3	216
16ʒʒ	4	16	27φ	3	216
96		256	189		1296
16		256	54		216
	8			6	432
256ʒʒʒ		1536	729ʒφ		46656
		1280			
		512			
		65536			

65536 = 256ʒʒʒ by the Root 2. 46656 = 729ʒφ by the Root 2.

Proof of the Compound Cossical Extraction.

L.

Root 4ʒʒ − 2φ − 3ʒ + 1ℨ
4ʒʒ − 2φ − 3ʒ + 1ℨ

16ʒʒʒ − 8Bſß − 12ʒφ + 4ſß
− 8Bſß + 4ʒφ + 6ſß − 2ʒʒ
− 12ʒφ + 6ſß + 9ʒʒ − 3φ
4ſß − 2ʒʒ − 3φ + 1ʒ

Square 16ʒʒʒ − 16Bſß − 20ʒφ + 20ſß + 5ʒʒ − 6φ + 1ʒ

Root

M.

Root	3φ + 2ʒ − 4ℨ
	3φ + 2ʒ − 4ℨ
	9ʒφ + 6ſß − 12ʒʒ
	6ſß + 4ʒʒ − 8φ
	− 12ʒʒ − 8φ + 16ʒ
Square	9ʒφ + 12ſß − 20ʒʒ − 16φ + 16ʒ
	3φ + 2ʒ − 4ℨ
	27φφ + 36ʒʒʒ − 60Bſß − 48ʒφ + 48ſß
	18ʒʒʒ + 24Bſß − 40ʒφ − 32ſß + 32ʒʒ
	− 36Bſß − 48ʒφ + 80ſß + 64ʒʒ − 64φ
Cube	27φφ + 54ʒʒʒ − 72Bſß − 136ʒφ + 96ſß + 96ʒʒ − 64φ

So in the tryal by Numbers, fuppofing the Root 2, then,

L.

	4ʒʒ = 64		16ʒʒʒ = 4096
	+ 1ℨ = 2		+ 20ſß = 640
	66		+ 5ʒʒ = 80
			+ 1ʒ = 4
	− 2φ = 16		
	− 3ʒ = 12		4820
	28		− 16Bſß = 2048
True value of the Root	38		− 20ʒφ = 1280
	38		− 6φ = 48
	304		3376
	114		
Square	1444	=====	1444

M.

	3φ = 24		27φφ = 13824
	+ 2ʒ = 8		+ 54ʒʒʒ = 13824
	32		+ 96ſß = 3072
	− 4ℨ = 8		+ 96ʒʒ = 1536
ℨ	24		32256
	24		
	96		− 72Bſß = 9216
	48		− 136ʒφ = 8704
ʒ	576		− 64φ = 512
	24		
	2304		18432
	1152		
φ	13824	=====	13824

Partis Quartæ Libri Tertii

FINIS.

THE

THE FIFTH PART OF THE THIRD BOOK

CHAP. I.

Of SURDES.

NEXT after *Cossicks* follow in order *Surdes*, as the fifth sort of Numbers especially Contract, and the second of those whose Denominations are expressed, because of the variety and uncertainty thereof.

Surdes placed as the fifth of special Contracts, &c.

Surdes are Irrational Numbers, as before in *Book* 1. *Par.* 1. *Chap.* 2. and *Book* 2. *Par.* 2. *Chap.* 3. was noted; that is, Numbers set for Roots that cannot be expressed by any Absolute Number: Or Numbers whose Roots cannot certainly be expressed by Integers, but besides the Integers contain some broken part or parts thereof. As the Square Root of 2, 3, 5, 6, or of any other Number that is not a Square Number. So the Cube Root of 2, 3, 4, 5, or of any other Number that is not Cubick. And in like manner any other Root of any Number that hath no such Root exactly to be measured by Whole Numbers, causeth that Number to be called a *Surde Number*. And perhaps the Reason why so called, it being absurd or irrational to attribute to any thing, or seek out thereof a Root or other thing not there to be had. *Latus inexplicabile* (saith *Alsted*) *dicitur, asymmetrum, incommensurabile, incommunicabile, irrationale, & surdum, quia ejus explicatio à nobis, quasi exaudiri nequeat, ut surdam buccinam, surdos ictus, dicimus qui difficulter & obscurè audiantur.* Encyclop. lib. 14. pag. 830.

Surdes *what they are, and why so called.*

The Denominations of these Roots being different according to the Powers of Numbers before-mentioned in Figural Numbers, maketh it necessary to express them. And for the Reason before rendred in the *First Chapter* of *Cossicks*, by some known Characters to distinguish them the one from the other. Which Characters, as in *Cossicks*, are arbitrary and mutable at the pleasure of the Operator.

Denominators why expressed, and how.

Characters arbitrary.

In the ensuing survey of *Surdes*, the following Characters are used with their significations, thus;

Characters used in this Book.

Characters.	*Significations.*
√	Root. Divers set this for the Square Root.
√: or V or V√	Universal Root.
w or √ʒ	Square Root.
ww or √φ	Cube Root.
www or √ʒʒ	Squared Square Root.
wwww or √ß	Sursolide Root.

And so increasing the Minnoms according to the *Index* of the Figural Number, or adjoyning the Cossical Character of the Power to the Character for the Root, √, you have a Character for any Surde Denomination. Wherefore if I see √ʒφ 8, I read it the Zenzicube Root of 8. And so I understand by this Character wwww 8. And the like is to be done with others; only because the Minnoms increasing in the Higher Powers take up more room, and are not so soon made as the Cossical Characters, it is better to use them altogether for the Powers beyond the Zenzicube.

Cossical Characters best for the Higher Powers.

Surdes have society with Rational Numbers,

Besides the taking Denominations from Figural Numbers, and borrowing the Cossical Characters for them, Surdes admit into their society Rational Numbers; that is, such Numbers whose Roots may be expressed by Integers. As √4, the Square Root of 4; so √√8, the Cube Root of 8; and such others: For the Square Root of 4 is 2; and so is the Cube Root of 8; and so consequently no Surdes, but often set thus for the more apt Operation. And so also Absolute Numbers not Rational are used with them as well as Cossicks. And Cossicks themselves I have seen wrought together with Surdes.

and Absolute Numbers.

Nature of Surdes.

Surdes are Simple or Compound, Integral or Fracted. Of Fractionary Surdes, see *Chap.* 11. following.

Integral and Simple.

The Integral Simple Surdes consist of one Species or Denomination: As √5, which is to be read, the Square Root of 5; so √√4, the Cube Root of 4, and the like of others.

Integral and Compound. These of 2 sorts.

Compound Surdes consist of different Species, or divers Simple Surdes, or some Simple Surde with another Number set for a Surde, and are of two sorts, to wit, Particular or Universal.

Particular.

Examples.

Particular, when the different Denominations compounded by the signs + or —, or both, are to be considered distinct as to their Roots. As √5 + √6, which signifieth, the Square Root of 5, and the Square Root of 6. So √6 — √5 denoteth the Square Root of 6, lacking the Square Root of 5. In both which, and such others, their Roots are considered as two distinct Numbers.

Universal.

Examples.

Universal, when though the Quantities consist of different Species, yet the Root of the whole compounded Number is to be understood thereby. As √: 5 + √36; here it signifieth, that the Square Root of 6 is to be added to 5, and then the Square Root of that summe is to be taken for the Root Universal and summe of that compounded Surde. So √: 6 — √35; here the Square Root of 5 is to be taken from 6, and the Square Root of the Remain is the Root Universal. See further of these *Chap.* 7. *Addition of Compound Surdes.*

Compounds otherwise considered.

These Compound Surdes fall again under a threefold Consideration: 1. In their Signs. 2. In their Characters. 3. In their Numbers.

1\. *In Signs, as* Binomials *or* Bimedials. Residuals *or* Apotomes. Polynomials *or* Multinomials.

1\. As to their Signs, two Surdes, or a Simple Surde with a Rational or other Number conjoyned with the sign +, are called *Binomials*, and sometime *Bimedials*; but conjoyned with the sign —, they are called *Residuals* or *Apotomes*. If three Quantities be conjoyned, and but three, they are sometime called *Trinomials*. But generally where the composition hath more than two parts, the Compound is called a *Polynomial* or a *Multinomial*, that is a many named Number, as was before noted in *Cossicks*.

Examples of Binomials.

Examples of Binomials.

3 + √8	That is, 3 more the Square Root of 8.
√24 + 4	Is, the Square Root of 24 more 4.
√6 + √2	Is, the Square Root of 6, and the Square Root of 2.
√√9 + √√√8	Signifieth, the Cubick Root of 9, and the Squared Square Root of 8.

Residuals.

Examples of Residuals.

25 — √80	That is, 25 lacking the Square Root of 80.
√160 — 9	Is, the Square Root of 160 wanting 9.
√180 — √6	Is, the Square Root of 180 wanting the Square Root of 6.
√√100 — √30	Signifieth, the Cubick Root of 100 abating the Square Root of 30.

Polynomials.

Examples of Polynomials.

3 + √10 + √√9	That is, 3 more the Square Root of 10, and the Cube Root of 9.
100 — √20 — √√5	Is, 100 lacking the Square Root of 20, and the Cube Root of 5.
4 + √√30 — √6	Is, 4 and the Cube Root of 30, wanting the Square Root of 6.
√8 + 100 — √√√7 — √√40	Signifieth, the Square Root of 8, added to 100, lacking the Squared Square Root of 7, and the Cube Root of 40.

2\. *In Characters, as* Homogeneal, Heterogeneal.

2\. Compound Surdes are considered in their Characters, and so they are divided into *Homogeneal* and *Heterogeneal*.

Homogeneal, when their Characters or Denominations are one and the same: *Heterogeneal* when contrary.

Examples

Examples of Homogeneals.

Examples of Homogeneals.

Binomials	w5 + w6	ww 9 + ww10	www19 + www18
Residuals	w6 — w5	ww10 — ww9	www19 — www18
Polynomials	ww5 + ww6 + ww4		www6 + www15 — www20

Examples of Heterogeneals.

Heterogeneals.

Binomials	w5 + ww6	ww5 + www6	www14 + wwww25
Residuals	w5 — ww6	ww9 — www12	wwww14 — www15
Polynomials	www5 + ww6 + w3		www6 + w3 — ww5

3. Their consideration in their Numbers divides them into commensurable and incommensurable. 3.

Commensurable, called also *Symmetral*, is when the given Numbers have a Common Divisor, that will reduce them into less Terms of like nature.

In Numbers as Commensurable or Symmetral.

Examples of Symmetrals.

Examples.

Binomials $\left\{ \begin{matrix} w8 + w32 \\ ww81 + ww24 \end{matrix} \right\}$ Common Divisor $\left\{ \begin{matrix} 2 \\ 3 \end{matrix} \right.$

Residuals $\left\{ \begin{matrix} w8 — w32 \\ ww81 — ww24 \end{matrix} \right\}$ Common Divisor $\left\{ \begin{matrix} 2 \\ 3 \end{matrix} \right.$

Polynomials $\left\{ \begin{matrix} w48 + w75 — w27 \\ ww24 + ww81 — ww192 \end{matrix} \right\}$ Common Divisor 3

Because by these Common Divisors, the Square Surdes will be brought into Square Numbers, and the Cubical Surdes into Cube Numbers, they are therefore called Commensurable. For by the Divisor 2, will 8 and 32 in the upper Binomial and Residual be brought into 4 and 16, which are both Square Numbers. And by the Divisor 3, both 81 and 24 in the lower Binomial and Residual will be brought into 27 and 8, which are both Cube Numbers. And by the same Common Divisor 3, will the upper Polynomial be brought into 16 + 25 — 9, which are all Squares or Rational Numbers, and the lower into 8 + 27 — 64, which are all Cubical Numbers.

Incommensurable, or Asymmetral Surdes, are those which have no such Common Divisors.

Incommensurable or Asymmetral. Examples.

Examples of Asymmetrals.

Binomials	w5 + w6	ww9 + w8	ww12 + www19
Residuals	w6 — w5	w8 — ww9	www12 — ww19
Polynomials	w6 + ww5 + www3		w6 + ww5 — www3

Numbers thus Commensurable or Incommensurable are said to be Commensurable or Incommensurable in Power; to difference this measure of Numbers from plain Commensuration, spoken of in Fractions before. For Numbers may be Commensurable, as 2 and 12, yet Incommensurable in Power: But 3 and 12 are as well Commensurable in Power as otherwise, seeing 12 divided by 3 gives in the Quotient 4, a Square Number.

Commensurable in Power how different from other Commensuration.

Symmetral Surdes are discovered from Asymmetral thus: Divide the greater given Number by the lesser, and if 0 remain, then shall the Quotient be a Number of the same nature with the given Surdes, that is Square, Cube, or other like Quantity accordingly, if the Surdes are Commensurable. But if any thing remain upon the Division, reduce the Fraction into its least Terms, and then reduce all into an Improper Fraction, and this shall represent two Figural Numbers of like Quantity with the given Surdes, if they are Commensurable. And to find the Common Divisor do thus: If 0 remain upon the Division as aforesaid, then by the Quotient of this Division, divide the least of the two given Surdes, and this last Quotient shall be the Common Divisor. But if the Division left a Remain, which is to be brought, as aforesaid, into an Improper Fraction, then by the Numerator thereof divide the greatest given Surde, or by the Denominator the least, and this Quotient shall be the Common Divisor.

How Symmetral Surdes are discovered.

As w8 + w32 was before counted Commensurable, and the Common Divisor 2; because if 8 divide 32, the Quotient will be 4, a Square Number agreeable to the Surde and 0 remain; and by this 4, if 8 the least of the two Surdes be divided, 2 the Common Divisor appears in the Quotient, as at *A*. But if w12 + w147, be given, then

Examples.

then after Division, because 3 remains, I reduce 3 with 12 the Divisor to $\frac{1}{4}$, and then 12 in the Quotient and this $\frac{1}{4}$ into an Improper Fraction, which is $\frac{49}{4}$, and being both Square Numbers of the nature with the given Surdes, shew them to be Symmetral; and by dividing 147 by 49, or 12 by 4, the Common Divisor is found in the Quotient to be 3, as at *B*. Examples of Cubes see at *C*. and *D*.

A. 32 / 8 (4 A Square and 0 left: *Ergo*, $\sqrt{8} + \sqrt{32}$ are Commensurable.

8 / 4 (2 Common Divisor.

B. 2(3 147 / 12 / 12 (12$\frac{1}{4}$ or $\frac{49}{4}$ Squares: *Ergo* $\sqrt{12} + \sqrt{147}$ are Commensurable.

147 / 49 (3 12 / 4 (3 Common Divisor.

C. 128 / 16 (8 A Cube and 0 remaining: *Ergo*, $\sqrt[3]{16} + \sqrt[3]{128}$ are Commensurable.

16 / 8 (2 Common Divisor.

D. (9 81 / 24 (3$\frac{3}{8}$ or $\frac{27}{8}$ Cubes: *Ergo*, $\sqrt[3]{24} + \sqrt[3]{81}$ are Commensurable.

81 / 27 (3 24 / 8 (3 Common Divisor.

Symmetral Surdes may have more Common Divisors than one.

Some Symmetral Surdes may have more Common Divisors than one, which is thus known: Divide one of the given Surdes, according to his Quantity, by any Number of like nature that will part it exactly without leaving a Remain; and by this Quotient divide the other given Surde, and if this second Quotient be a Number of like nature, then those given Surdes have more Common Divisors than one. And so proving with all, less than the given Surdes; so many Quotients as will hold this tryal, so many Common Divisors have those Commensurable Surdes.

Examples.

As in the Square Surdes above at *A*, if 32 be divided by 4, it giveth 8 in the Quotient; and if this 8, divide 8 the other given Surde, the Quotient is 1 another Square Number, therefore shall 8 be another Common Divisor to $\sqrt{8} + \sqrt{32}$ besides 2 found out as above.

So in the Cube Surdes above at *C*, if 128 be divided by 8, it giveth 16 in the Quotient: And if this 16 divide 16, the other given Surde, the Quotient is 1, another Cube Number; therefore shall 16 be another Common Divisor to $\sqrt[3]{16} + \sqrt[3]{128}$, besides 2, found as above.

√ 32 / 3 4 (8 8 / 8 (1 3 ∛ 128 / 0 8 (16 16 / 16 (1 0

Surdes not so strictly used to place the greatest foremost. Use of Signs as in Cossicks.

All further needful to this *Chapter* is, That Authors do not so strictly observe to place the Greatest Surdes foremost, as they do the Cossicks; but sometime the Lesser Surdes are set to the Left Hand of the Greater. But agreeable to Cossicks in the use of the signs with or without Asterisques. And where — is not set + is understood.

CHAP. II.

Reduction of Surdes.

Surdes reduced. To their least Termes.

UNder Reduction of Surdes is comprehended, To lessen their Terms, and alter their different Denominations into one: Both sometime called, *Alteration of Surdes.*

1. To lessen the Terms of a Surde is but abbreviation. And as all Common Fractions

ctions will not be abbreviated; so neither will all Surdes have their Terms lessened. But when the Denomination or Character is a Compound Cossick, and the annexed Number hath a Root that may be expressed by part of that Cossical Character, then reduce the Number and Character thereto accordingly, by extracting the Root of the Number, and clearing the Character of that part of the Composition.

Examples.

As √ʒʒ 25 and √ʒφ 81, the Characters being compounded of ʒ with ʒ in the first, and ʒ with φ in the second, and the Numbers 25 and 81 being both Square Numbers, the Square Roots thereof are to be taken, and Square in the Characters abated from both their Quantities, and both are to be expressed in less Terms by √ʒ 5 and √φ 9, or √√ 5 and √√√ 9.

So also √ʒφ 27 may be reduced to the √ʒ 3, being discharged of φ. And √ʒſß 32 may be abbreviated to the √ʒ 2, and discharged of ſß.

To reduce Surdes *to one Denominator.*

2. To bring the different Denominations of Surdes into one, belongs to Heterogeneal Surdes, or one Surde with an Absolute Number.

Surdes *and Absolute Numbers.*

(1.) If an Absolute Number be given to be reduced into the Denomination of a Surde, then multiply the Absolute Number according to the Denomination of the Surde, and set before it the like Character.

Examples.

As if √√ 8 and 2 be reduced into one Denomination; 2 must be multiplyed Squarely, because the Denomination of the Surde 8 is such. And so will the Numbers stand thus, √√ 8 and √√ 4.

And if √√√ 9 and 4 be reduced into one Denomination, 4 must be multiplyed Cubically: And thus reduced the Surde shall be √√√ 9 and √√√ 64.

Surdes *with uncompound* Indices.

(2.) If two different Surdes be given to be reduced into one Denomination, and their *Indices* be uncompounded; then alternately multiply the Number of the one Surde according to the Denomination of the other, and to both the Products adjoyn both the Characters for the Common Denominator.

Example.

As if √√ 8 and √√√ 10 were to be reduced to one Denominator; then must 10 be multiplyed Squarely and 8 Cubically, and each production shall be Square Cube, set after the manner of Surdes thus, √ʒφ 100 and √ʒφ 512; or thus, √√√√√ 100 and √√√√√ 512.

√ʒ 8 × 8 × 8 = 512 } ʒφ
√φ 10 × 10 = 100 }

√ʒφ 512		√ʒφ 100
√ʒ 8	X	√φ 10
	√ʒφ	

Surdes with Compound Indices.

But (3.) If the *Indices* of the given Quantities be Numbers Compound, then by the greatest Common Divisor divide them, and then by the least Terms of the *Indices* multiply alternately as well the *Indices* of the one by the other for a new *Index*, as the Numbers given by the Powers of these alternate *Indices* for the reduced Surdes.

Example.

As √ʒʒ 10 and √ʒφ 7, thus reduced shall be √ʒʒφ 1000 and √ʒʒφ 49. For the *Index* of ʒʒ is 4, and the *Index* of ʒφ is 6, the Common Divisor of 4 and 6 is 2, which reduceth the one to 2 the *Index* of ʒ, and the other to 3 the *Index* of φ; therefore alternately Squaring 7 it is 49, and Cubing 10 it is 1000, and multiplying 6 by 2, or 4 by 3, the Product 12 is the *Index* of the reduced Powers, which is ʒʒφ.

	√	[12] 1000		√	[12] 49
2)	√	[4] 10		√	[6] 7
		[2]			[3]

Proof of Reduction *of* Surdes.

Besides the Proof of that sort of Reduction which lesseneth the Terms, by exalting the lessened Surdes into the Powers from whence they were abated; and reciprocally that sort of Reduction which increaseth their Terms and Denominations, by extracting the Roots, and abating the Characters of the Surdes accordingly; all sorts of Reduction of Surdes may be proved, by supposing Rational Numbers instead of the Surdes, and working with them as if they were Surdes.

As in the former Example, where √ʒʒ 25 was reduced to √√ 5; therefore if √√ 5 be multiplyed Squarely, it shall be √√√ 25: And in Rational Numbers it is evident that √√ 4 and the √√√ 16 are equal, being in each but 2.

Again in the second sort of Reduction, and first Example, there 2 reduced to a Square Denomination shall be 4, the Rational Number, and √√ 8 and √√ 4, is all one as √√ 8 and 2; for they are both equal: And so is the √√√ 64 or 4 Absolute Numbers in the next Example.

Likewise if W 4 and the ww 27, both Rational Numbers, be reduced to one Denomination, they shall be √3φ 64 and √3φ 729, agreeable to the next Example of Reduction. And the √3φ of 64 being but 2, equal to the W 4, and the √3φ of 729 being 3, as the ww 27 shew the Reduction right.

Moreover, agreeable to the last Example of Reduction, if I take √33 16 and the √3φ 64, that are both Rational Numbers, and have 2 for their Roots, and reduce them to one Denomination, they shall be √33φ 4096 and √33φ 4096, and be equal to the other. For the √33φ of 4096 is 2; whence may be also observed, that if the Roots of the given Surdes be equal, the reduced Surdes will be equal.

CHAP. III.

Addition of Simple Surdes.

Addition of Simple Surdes.

TO understand Addition of Surdes the better, it is meet to Analyse them.

Into
- Simple
  - Homogeneal and Commensurable.
  - Homogeneal and Incommensurable.
  - Heterogeneal.
- Compound
  - Particular.
  - Universal.

1. *Homogeneal and Commensurable.*

1. To add two Simple Surdes that are both Homogeneal and Commensurable, divide them by the Common Divisor, and extract the Roots of the Quotients, then multiply the Total of the Roots Figurately according to their Quantities, and this Product multiplyed by the Common Divisor shall be the Total of the added Surdes.

Examples.

As to add W 12 and W 147; both Numbers divided by 3, the Common Divisor, giveth 4 and 49, which are Square Numbers, and their Roots 2 and 7, the Total whereof 9 multiplyed Squarely produceth 81; this multiplyed by 3 is 243. So is W 243 the Total of W 12 + W 147, as at *A*.

Also to add ww 81 and ww 24, Commensurable also by 3; after Division thereby the Cubes are 27 and 8, and their Roots 3 and 2, which together make 5, the Cube whereof is 125; this multiplyed by 3 produceth 375. So is ww 375 the summe of ww 81 + ww 24, as at *B*.

An Example of Squared Square Surdes is set at *C*.

	A.	*B.*	*C.*
Addends	W 12 + W 147	ww 81 + ww 24	wwW 648 + wwW 5000
	3)	3)	8)
	43 493	27φ 8φ	8133 62533
	2√ + 7√	3√ + 2√	3√ + 5√
	9	5	8
	9ℨ	253	512φ
	813	125φ	409633
	3	3	8
Totals	W 243	ww 375	wwW 32768

Where the Data have many Common Divisors.

If the given Surdes have many Common Divisors, any one of them may be used. As in W 1152 + W 288, there are 5 Common Divisors, *viz.* 2, 8, 18, 32 and 72, the Addition by all which agree to be W 2592, as in the Operations following.

Examples.

W 1152 + W 288	W 1152 + W 288	W 1152 + W 288
2)	8)	18)
5763 1443	1443 363	643 163
24√ + 12√	12√ + 6√	8√ + 4√
36	18	12
36ℨ	18ℨ	12ℨ
12963	3243	1443
2	8	18
W 2592	W 2592	W 2592

W

```
    W 1152 + W 288              W 1152 + W 288
32)--------------           72)--------------
    363      93                 163      43
    6√  +   3√                  4√  +   2√
         9                           6
         9 z                         6 z
       -----                       -----
       81 3                        363
        32                          72
       -----                       -----
       162                          72
       243                         252
       -----                       -----
      W 2592                      W 2592
```

2. Simple Square Surdes Homogeneal and Incommensurable, if the Product of both multiplyed together bring forth a Figural Number of the same kind (as many times happeneth when the Numbers are Commensurable, or Rational Numbers used as Surdes) then are they called reducible, and are added thus: To the Total of the Surdes given prefix their proper Character, afterward either extract the Root of their Product and multiply this Root by the *Index*, and add this Product to the former Total; or else multiply the Product of both Surdes by the double *Index*, and extract the Root of this last Product, and add this Root to the Total first reserved; and this Number with his Character shall be the summe of the added Surdes. *2. Square Surdes Incommensurable. Reducible what, and how added.*

As to add W 3 and W 12; because 3 and 12 multiplyed is 36, a Square Number agreeable to the given Surdes, they are reducible; then 3 and 12 added are 15 to be set apart with their Character thus, W 15; afterward the Root of 36, which is 6, is to be multiplyed by 2, the *Index* of Squares, and the Product 12 is to be added to the 15 before reserved; or else 36 is to be multiplyed by 4 the double *Index*, and the Root of 144 which is 12, added to 15, as before, makes the Total of this Addition W 27, as at *D.* and *E.* *Example.*

```
              D.                                 E.
Addends W 3 + W 12                          W 3 + W 12
        ----------                          ----------
            |   3                              |   3
            |  12                              |  12
      W 15  | ----                             | ----
            | W36 is 6                   W15   |  36
            |        2 Index                   |   4
        ----------                             | ----
        W 15 + W 12                            | W 144
Totals  -----------                   ---------------------------
          W 27                        W 15 + 12 or W 15 + W 144
                                      ---------------------------
                                         W 27
```

If Square Surdes are not thus reducible, they with others of higher Denominations Incommensurable, although Homogeneal, are to be joyned together with the sign of Addition +. *When not Reducible.*

As to add W 6 and W 7, they are set thus, W 6 + W 7. *Examples.*

And to add ww 6 and ww 7, they are set thus, ww 6 + ww 7.

Yet some set the Square, although Incommensurable, after the form of Addition abovementioned; whereby W 6 + W 7 is brought to W 13 + W 168; for that by this form they come to be Commensurable with other Numbers with whom they occasionally may be used. *How set by some.*

Example.

```
     W 6 + W 7
     ---------
         |  6
         |  7
   W 13  | ---
         | 42
         |  4
     ----------
   W 13 + W 168
```

3. Simple Heterogeneal Surdes are first to be reduced, and then if by their Reduction they prove Commensurable add them as such; if otherwise, conjoyn them by the sign +. *3. Heterogeneal.*

As to add W 3 and 9, Absolute Numbers together, being reduced they are W 3 and W 81, and because Incommensurable abide so; unless set after the form of Addition last mentioned, and then they stand thus, W 84 + W 972. *Examples.*

And if W 3 be given to be added to ww 2, after Reduction to √3φ27 + √3φ4, because Incommensurable they are left so without further work.

But

But if the Rational Number W 16 be given to be added to ww 8, another Rational Number, by Reduction they are brought to √3φ 4096 + √3φ 64, and being Commensurable, and added as before, make √3φ 46656, which is also a Rational Number, and hath 6 for Zenzicube Root thereof.

```
                                   √3φ 4096 + √3φ 64          4 + 2 = 6
                              64)―――――――――――――――――
                                     64 3φ       1 3φ
 √3φ 4096      √3φ 64                2 √    +    1 √
 W 16      X   ww 8                        3
 ―――――――――――――――――――――                   243 ß
          √3φ                          ――――――
                                        729 3φ
                                        64
                                       ――――――
                                        2916
                                       4374
                                     ――――――――――――
                                     √3φ 46656 is 6
```

4\. *Different Signs*

4\. If two Simple Surdes of different signs be given to be added together, then conjoyn them by the sign of Substraction —.

As to add W 3 with — W 5, they shall be set thus, W 3 — W 5.

To add any Surde to it self.

By what hath been said of Addition of Simple Surdes, appeareth this Consectary; That to add any Surde to it self is but to multiply the Square Surdes by 4, Cube Surdes by 8, Squared Squares by 16, *&c.* See the Examples following.

Examples.

```
Addends  W 8 + W 8            ww 9 + ww 9              www 10 + www 10
      8)――――――――――――       9)――――――――――――          10)――――――――――――――――
          13    13             13    13                 1 33      1 33
          1 √ + 1 √            1 √ + 1 √                1 √   +   1 √
             2                    2                          2
             2 ʑ                  4 3                        8 φ
         ――――――――――          ――――――――――               ――――――――――
            4 3      W 8          8 φ      ww 9             16 33     www 10
            8          4          9          8              10            16
         ――――――――  ――――――    ――――――――  ――――――           ――――――――    ――――――
Totals     W 32      W 32        ww 72     ww 72           www 160    www 160
```

Addition of Compounds why deferred.

Addition of Compound Surdes is deferred to the *Seventh Chapter*, till Substraction of Simple Surdes be learned; because the knowledge thereof is necessary to the Addition of some Compound Surdes.

Proof of Addition of Simple Surdes.

Substraction of Simple Surdes is the Proof of Addition of Simple Surdes, and there to be seen. But the truth of Addition will be clear, if instead of the Surdes, Rational Numbers be taken, and operation made therewith.

For in the last Example of reduced Surdes, the Zenzicube Root of 4096 is 4, and of 64 is 2, which 4 and 2 make 6, and the Total of the Addition bringeth forth 46656 a Zenzicube Number that hath 6 for the Root, as was before noted.

```
         3φ      φ  ʑ              6 ʑ
         25                        6                               Value
         46656 , 216 | 6         ――――――          Indices.            4
          .   .   .              36 3                                2
                                   6                                ――――
         4  :  :                 ――――――
Gnomon { 4  :  :                  216 φ              3              6 Total
         1  :                     216                3              ――――
Gnomon { 252 :                  ――――――             ――――
         36                      1296
                                 216
                                432
                               ――――――――            ――――
                               46656 3φ              6
```

C H A P. IV.
Substraction of Simple Surdes.

Substraction of Simple Surdes.

IN the same method with Addition doth Substraction proceed.

1\. *Homogeneal and Commensurable.*

1\. Then to substract one Homogeneal and Commensurable Surde from another, after dividing them by the Common Divisor, and extracting the Roots of the Quotients,

tients, take the Leſſer Root from the Greater, then multiply the Remainer Figurately according to their Quantities, and this Product multiplyed by the Common Diviſor ſhall be the remaining Surde deſired with the ſame ſign. But if the Subtrahend be the Greater, then the ſign + ſhall be changed into —.

As to ſubſtract w 12 from w 243 by the Common Diviſor 3, they are reduced into the Squares 81 and 4, whoſe Roots are 9 and 2, the difference 7, this multiplyed Squarely is 49, which increaſed by the Common Diviſor is w 147, the Remainer deſired, as at *A*. *Examples.*

Likewiſe if w 147 were taken from w 243, there would remain w 12, as at *B*.

But if w 243 were to be taken from w 147 or w 12, in this there would want w 147, and in that w 12, and then to be marked with —, as at *C.* and *D.*

Greater Surde. Subtrahend.
w 243 — w 12
3) 81 3 — 4 3
9 √ — 2 √
A. 7
7 ℨ
49 3
3
Remain w 147

Greater Surde. Subtrahend.
w 243 — w 147
3) 81 3 — 49 3
9 √ — 7 √
B. 2
2 ℨ
4 3
3
Remain w 12

Leſſer Surde. Subtrah. Remain.
C. w 147 — w 243 = — w 12

Leſſer Surde. Subtrah. Remain.
D. w 12 — w 243 = — w 147

Commenſurable Surdes of Higher Powers, are likewiſe thus to be ſubſtracted

Examples to take ww 24 from ww 375, the Remain will be ww 81, but will want ww 81, if ww 375 be taken from ww 24

So www 648 taken from www 32768, will leave www 5000; but the Greater ſubſtracted from the Leſſer, the Remain will be ſo much too ſhort.

Greater Surde. Subtrahend.
ww 375 — ww 24
3) 125 φ — 8 φ
5 √ — 2 √
3
9 3
27 φ
3
Remain ww 81

Greater Surde. Subtrahend.
www 32768 — www 648
8) 4096 33 — 81 33
8 √ — 3 √
5
125 φ
625 33
8
Remain www 5000

Leſſer Surde. Subtrah. Remain
ww 24 — ww 375 = — ww 81

Leſſer Surde. Subtrah. Remain.
www 648 — www 32768 = — www 5000

If the given Surdes have many Common Diviſors; any one of them may be uſed in Subſtraction, as before in Addition. *If the Data have many Common Diviſors.*

2. One Simple Square Surde Homogeneal and Incommenſurable to another, may be ſubſtracted therefrom, if the Product of them both multiplyed together produce a Figural Number of the ſame kind: For then to the Total of the Surdes prefix their proper Character; afterward either extract the Root of their Product, and multiply this Root by the *Index*, and deduct this Product from the former Total: Or elſe multiply the Product of both the given Surdes by the double *Index*, and extract the Root of this laſt Product, and ſubſtract this Root from the Total firſt reſerved. And this Number with his Character ſhall be the Remain deſired, with the ſign changed, as before noted, if the Subtrahend be the Greater. *2. Square Surdes Incommenſurable. Reducible how added.*

As to extract w 12 from w 27, being multiplyed they produce 324, the Square Number of 18; therefore 18 multiplyed by the *Index* 2, or 324 by 4 the double *Index*, and the Root of this Product, or the Product 36 ſubſtracted from w 39, the Total of 12 and 27 added, leaves remaining w 3, as at *E.* and *F.* But if w 27 were to be taken from w 12, becauſe the Subtrahend is greateſt, the ſign or places of the Surdes ſhall be changed, as at *G.* *Example.*

Greater Surde √27 — √12 | √27 — √12 Subtrahend

E. √39 | 27, 12 | 54, 27 | √324 is 18, 2 *Ind.*
√39 — 36
Remain √3

F. √39 | 27, 12 | 54, 27 | 324, 4 Double *Index.* | √1296 is 36
√39 — 36 or √39 — √1296
√3

G. √12 — √27 = { √1296 — √1521 or 36 — 39 } = — √3

Not Reducible. If Square Surdes be not thus reducible to Square Numbers by Multiplication, then they with other Surdes Incommenſurable of Higher Powers, although Homogeneal, are to be joyned together with the ſign of Subſtraction —.

Examples. As to ſubſtract √6 from √7, or √7 from √6; they are ſet thus;

√7 — √6   √6 — √7

And to ſubſtract √√6 from √√7, or √√7 from √√6; they are ſet thus;

√√7 — √√6   √√6 — √√7

How ſet by ſome. Nevertheleſs for the Reaſon before rendred in Addition, ſome ſet the Square Surdes, though Incommenſurable, after the form of Subſtraction above-mentioned; whereby √7 — √6 is brought to √13 — √168, and √6 — √7 to √168 — 13.

Examples.

√7 — √6
√13 | 7, 6 | 42, 4
√168
√13 — √168

√6 — √7
√13 | 6, 7 | 42, 4
√168
√168 — 13

3. *Heterogeneal.* 3. Simple Heterogeneal Surdes are firſt to be reduced, and then, if by their Reduction they happen to be Commenſurable, ſubſtract them as ſuch: But if otherwiſe, conjoyn them accordingly with the ſign —.

Examples. As to take √3 from 9, Abſolute Numbers after Reduction, they ſtand, becauſe Incommenſurable, √81 — √3, unleſs after the form of Subſtraction laſt above-mentioned, and then they ſtand thus, √84 — √972.

And if √√2 be ſubſtracted from √3, they are reduced, and becauſe Incommenſurable left thus, √3φ27 — √3φ4.

But if √√8 be taken from √16, they being reduced, and both Rational Numbers and Commenſurable, will be √3φ4096 — √3φ64, that is in concluſion √3φ64, which is alſo a Rational Number, and hath 2 for the Zenzicube Root thereof.

√3φ4096 — √3φ64   4 — 2 = 2
64) 643φ — 13φ
2√ — 1√
1
1 ß
13φ
64
√3φ64

√3φ4096   √3φ64
√16 — √√8
√3φ

And if √16 had been given to have been ſubſtracted from √√8, then had there wanted √3φ64, and the Remain ſhould have been — √3φ64.

4. *Different Signs* 4. If the given Surdes be of different ſigns, the Surdes are to be added, and the ſign of the Remain, as in Coſſicks, ſhall be contrary to the ſign of the Subtrahend, or Number ſubſtracted, that is as the upper Number, if the Surdes be Commenſurable; but if Incommenſurable, then conjoyn them by the ſign of Addition +.

Examples. As to ſubſtract — √13 from √52, they are added becauſe the ſigns are unlike, the one + and the other —, the Total, which is √117, is the Remain, as at *H*.

And √5 — √3 and √5 — √3 being Incommenſurable, have their Remains, as at *I*. and *K*, √5 + √3 and √5 + √3.

√52

$$
\begin{array}{c}
\surd 52 - \surd 13 \\
13)\overline{} \\
\overset{43}{2\surd} + \overset{13}{1\surd} \\
3 \\
H. \quad \underline{3\,z} \\
9\,3 \\
\underline{13} \\
\text{Total} \quad \surd 117 \ \text{Remain.}
\end{array}
\qquad
\begin{array}{c}
I. \\
\surd 5 - \surd 3 \\
\hline
\surd 5 + \surd 3 \\
\hline
\end{array}
\qquad
\begin{array}{c}
K. \\
\surd 5 - \surd 3 \\
\hline
\surd 5 + \surd 3 \\
\hline
\end{array}
$$

From what hath been ſaid of Subſtraction of Simple Surdes, theſe two Conſectaries are apparent. *2 Conſectaries hence.*

1. That to ſubſtract any Surde from it ſelf leaves 0 remaining, as in other Numbers. As to take $\surd$ 10 from $\surd$ 10, the Remain is $\surd$ 0. *A Surde taken from it ſelf leaves 0.*

$$
\begin{array}{c}
\surd 10 - \surd 10 \\
10)\overline{} \\
\overset{13}{1\surd} - \overset{13}{1\surd} \\
0 \\
\underline{0\,z} \\
0\,3 \\
\underline{10} \\
\surd 0
\end{array}
\qquad
\begin{array}{c}
\surd 10 \\
\surd 10 \\
\hline
\surd 0 \\
\hline
\end{array}
$$

2. That to take half any Surde is but to divide Square Surdes by 4, Cube Surdes by 8, Squared Squares by 16, *&c.* as is plain by the Examples following. *To take half a Surde.*

$$
\begin{array}{c}
\text{Greater Surde. Subtrah.} \\
\surd 32 - \surd 8 \\
8)\overline{} \\
\overset{43}{2\surd} - \overset{13}{1\surd} \\
1 \\
\underline{1\,z} \\
1\,3 \\
\underline{8} \\
\text{Remain} \quad \surd 8 \\
\hline
\surd \frac{32}{4}\Big(\surd 8
\end{array}
\quad
\begin{array}{c}
\text{Greater Surde. Subtrah.} \\
\surd\surd 72 - \surd\surd 9 \\
9)\overline{} \\
\overset{8\varphi}{2\surd} - \overset{1\varphi}{1\surd} \\
1 \\
\underline{1\,3} \\
1\,\varphi \\
\underline{9} \\
\text{Remain} \quad \surd\surd 9 \\
\hline
\surd\surd \frac{72}{8}\Big(\surd\surd 9
\end{array}
\quad
\begin{array}{c}
\text{Greater Surde. Subtrah.} \\
\surd\surd 160 - \surd\surd 10 \\
10)\overline{} \\
\overset{1633}{2\surd} - \overset{133}{1\surd} \\
1 \\
\underline{1\,\varphi} \\
1\,33 \\
\underline{10} \\
\text{Remain} \quad \surd\surd 10 \\
\hline
\surd\surd \frac{160}{16}\Big(\surd\surd 10
\end{array}
$$

Subſtraction of Compound Surdes is referred to the *Eighth Chapter*, after their Addition hath been inſpected. *Subſtraction of Compounds why deferred.*

Foraſmuch as ſeveral of the Examples in this *Chapter* from which Subſtraction is made, are the Totals of the Additions in the foregoing *Chapter*, and the Subtrahends here are one of the Surde Numbers added, and the Remains the other, it will apparently manifeſt the Proof of Subſtraction of Surdes by Addition, and Addition by Subſtraction. *Proof of Subſtraction of Simple Surdes.*

But for a full demonſtration of all ſubſtractionary Operations, let Rational Numbers be ſet inſtead of the Surdes, and after Subſtraction made therewith, the truth will appear by the equal value of the Remain.

For in the Rational Numbers above-mentioned $\surd 3\varphi$ 4096 is 4, from which if $\surd 3\varphi$ 64, which is 2, be ſubſtracted, the Remain is $\surd 3\varphi$ 64, which is 2, and anſwers to the Remain of 4 — 2, which is but 2.

$$
\begin{array}{ccc}
3\varphi & \varphi & z \\
64 & | \ 8 \ | & 2
\end{array}
\qquad
\begin{array}{cc}
2\,z & \\
\underline{2} & \\
4\,3 & \textit{Indices.} \\
\underline{2} & \\
8\,\varphi & 3 \\
\underline{8} & \underline{3} \\
64\,3\varphi & 6
\end{array}
\qquad
\begin{array}{c}
\text{Value} \\
4 \\
\underline{2} \\
2 \ \text{Remain}
\end{array}
$$

CHAP. V.

Multiplication of Simple Surdes.

Multiplication *of Simple* Surdes. *Homogeneal.*

TO multiply Simple Surdes observe their Homogenity or Heterogeniety. For,

1. If the Surdes given to be multiplyed be Homogeneal, then multiply Number by Number, Integers as Integers, and Fractions as Fractions, and to the Product prefix the Character common to the given Surdes.

Examples. As to multiply √ 15 by √ 5, the Product is √ 75, set as at *A.* or *B.*
And to multiply √ $12\frac{1}{2}$ by √ $4\frac{1}{2}$, the Product is √ $56\frac{1}{4}$, as at *C.*

A.	Multiplicand	√ 15	*B.*	Md. √ 15 × Mr. √ 5 = Prod. √ 75
	Multiplyer	√ 5		
	Product	√ 75		
C.	Multiplicand	√ $12\frac{1}{2}$		
	Multiplyer	√ $4\frac{1}{2}$		
	Product	√ $56\frac{1}{4}$ or $7\frac{1}{2}$		

$$\frac{225}{\frac{25}{2}\times\frac{9}{2}} = \frac{225}{4}\left(56\tfrac{1}{4}\right.$$

225 | 15 Root; 1, 10, 25 / 4 | 2 ($7\frac{1}{2}$

Other Examples.

Multiplicands	√√ 48	√√√ 12	√√√ 30
Multiplyers	√√ 5	√√√ 6	√√√ 3
Products	√√ 240	√√√ 72	√√√ 90

Heterogeneal.

2. If the Surdes given to be multiplyed be Heterogeneal, or one Surde with an Absolute Number; then first reduce them to one Denomination, and then multiply Number by Number, as before.

Examples. As to multiply √ 10 by 3; first 3 is squared, and then by 9 is 10 multiplyed; so is the Product √ 90, as at *D*.

And to multiply √ 8 by √√ 10, being reduced they are √3φ 512, and √3φ 100, and 512 multiplyed by 100 produce 51200; so is the Product √3φ 51200, as at *E.*

D.

√ 10		√ 9
√ 10	X	3
	√	

√ 10
√ 9
√ 90

E.

√3φ 512		√3φ 100
√ 8	X	√√ 10
	√3φ	

√3φ 512
√3φ 100
√3φ 51200

Consectaries hence.

Out of Multiplication of Simple Surdes issue these ensuing Consectaries.

1. *To double, triple, &c. a Surde, what.* 1. That to multiply any Surde, is to increase him by the Power of a Root Homogeneal: And so to double any Square Surde is to multiply him by 4, which is the Square Power of the Root 2, as before noted in Addition. Likewise to triple any Square Surde is to multiply him by 9, *&c.* And to double any Cube Surde is to multiply him by 8, the Cube of 2. As also to triple him is to multiply him by 27, *&c.*

Examples.

	Doubled.	Tripled.	Quadrupled.
Square Surdes	√ 3	√ 3	√ 3
	4	9	16
	√ 12	√ 27	√ 48
Cube Surdes	√√ 3	√√ 3	√√ 3
	8	27	81
	√√ 24	√√ 81	√√ 243
Squared Squares	√√√ 3	√√√ 3	√√√ 3
	16	81	243
	√√√ 48	√√√ 243	√√√ 729

2. *Product is often Rational.*

2. That Multiplication of Surdes oftentimes produceth Rational Numbers, whose Roots are Absolute Numbers, and may be so expressed.

Examples.

Examples in Surdes	Square.	Cubed	Squared Square.
	√ 48	√√ 9	√√√ 8
	√ 3	√√ 3	√√√ 2
	√ 144 √ 12	√√ 27 √ 3	√√√ 16 √ 2

3. That

3. That to multiply the side of any Power according to the exigency of his own kind; the Character or Note of the side may be cancelled, and the Number left Absolute. For w√ 3 multiplyed by the w√ 3, produceth w√ 9, which is the Absolute Number 3. So the Square of the Square Root of 64, and the Cube of the Cube Root of 64, is 64.

3. When the Character may be cancelled.

Examples.

Examples w√ 3 Also w√ 64 × w√ 64 = 64
w√ 3 And ww√ 64 × ww√ 64 × ww√ 64 = 64

w√ 9 √ 3.

4. That the side of a Power whose *Index* is a Compounded Number, multiplyed into one of the Compounding Powers, produceth a Surde answerable to the Quotient of the Greater *Index* divided by the Lesser, and may be set alone accordingly. As 33 whose *Index* is 4, compounded of 3 whose *Index* is 2; it shall be therefore, that if ww√ 10 were so to be multiplyed, the Product ww√ 100 shall be equal to w√ 10, because w√ answers to the *Index* 2, brought out by the Division of 4 by 2. So the Square of √3φ 64 is √φ 64; and the Cube of √3φ 64 is √3 64; for the √3φ is √ of 2 multiplyed by 3.

4. What produced by the Side of a Power multiplyed, &c. Examples.

ww√ 10 *Ergo* ww√ 10 × ww√ 10 = w√ 10 because 2) 4 (2 [3 33 3]
ww√ 10

ww√ 100 = w√ 10

√3φ 64 A Rational Number, and hath the Root 2.
√3φ 64

Ergo √3φ 64 × √3φ 64 = √φ 64 because 2) 6 (3 [3 3φ φ]

256
384

√3φ 4096 = √φ 64 which is a Rational Number, and the Root 4.
√3φ 64

Ergo √3φ 64 × √3φ 64 × √3φ 64 = √3 64 because 3) 6 (2 [φ 3φ 3]

16384
24576

√3φ 262144 = √3 64, also a Rational Number, and the Root 8.

5. That if a Figural Number be multiplyed by an Homogeneal Figural Number, the Product shall be a Figural Number of the same kind, whose Side or Root shall be equal to the Product of the sides of the Numbers multiplyed. As 4 and 9, both Squares, produce 36, whose Root is 6, equal to 2 × 3, the sides of 4 and 9. So 343 the Cube of 7, if multiplyed into 27, the Cube of 3, shall produce 9261, the Cube of 21, equal to the Product of 7 × 3.

5. Homogeneal Figurals multiplyed what produced. Examples.

Squares { 4 √ 2 Cubes { 343 √ 7
{ 9 3 { 27 3

Square 36 √ 6

2401
686

Cube 9261 √ 21

6. That the sides of Homogeneal Surdes multiplyed procreateth sides of Homogeneal Surdes.

6. Sides of Homogeneals what they produce. Examples.

Ergo w√ 2 × w√ 3 begetteth w√ 6. And ww√ 7 × ww√ 3 begetteth ww√ 21.

Multiplication of Compound Surdes is remitted to the *Ninth Chapter* of this *Fifth Part* of the *Third Book*, that it may follow in order Compound Addition and Substraction.

Multiplication of *Compounds why deferred.*

Division of Simple Surdes is the Proof of Multiplication of Simple Surdes, and there set forth. Yet besides, the truth of this Simple Multiplication will appear by taking Rational Numbers and multiplying them, and extracting the Roots of the Product, which will equalize the Product of the Roots of the Factors multiplyed in Absolute Numbers.

Proof of Multiplication of Simple Surdes.

As if w√ 9 be multiplyed by w√ 16, the Product will be w√ 144, whose Root is 12, and so will be the Product of 3, the Root of 9, multiplyed into 4, the Root of 16, as at *F.*

And so if w√ 9, which is 3, be multiplyed by ww√ 8, which is 2, the Product will be 6, agreeable to the √3φ 46656, as at *G.*

F.				G.		
√q 9	√ 3	√ʒc 729	√ʒc 64	√ʒc 729	√ 3	
√q 16	4	√q 9 X	√c 8	√ʒc 64	2	ʒc c ʒ
√q 144	12	√ʒc		2916		25
3 ʒ				4374		46656(216(6
144(12				√ʒc 46656	√ 6	4
1						41
4						2556
4						

CHAP. VI.

Division of Simple Surdes.

Division *of Simple* Surdes. *Homogeneal.*

AFter Multiplication of Simple Surdes followeth their Division, and with like ease and order, as they are Homogeneal or Heterogeneal.

1. If they be Homogeneal, then divide the Number of the Dividend by the Number of the Divisor, Integers as Integers, and Fractions as Fractions, and to the Quotient annex the Common Character of the Surde.

Examples. As to divide √q 75 by √q 5, the Quotient will be √q 15, as at *A.* or *B.*

And to divide √q $56\frac{1}{4}$ by √q $4\frac{1}{2}$, the Quotient will be √q $12\frac{1}{2}$, as at *C.* or *D.*

A.

Dividend √q 75 (√q 15 Quotient
Divisor √q 5

B.

Divisor. Dividend. Quotient.
√q 5) √q 75 (√q 15

C.

Dividend. Divisor. Quotient.

$$\frac{225}{\sqrt{q}\,56\frac{1}{4}} \overset{450}{\underset{36}{X}} \frac{9}{\sqrt{q}\,4\frac{1}{2}} = \sqrt{q}\,\frac{450}{36}\Big(\sqrt{q}\,12\tfrac{1}{2}$$

D.

Divisor. Dividend. Quotient.

$$\frac{9}{\sqrt{q}\,4\frac{1}{2}}\Big)\ \frac{225}{\sqrt{q}\,56\frac{1}{4}}\ \Big(\sqrt{q}\,\frac{450}{36}\ \Big(\sqrt{q}\,12\tfrac{1}{2}$$

Other Examples.

Dividend √c 240 (√c 48 Quotient
Divisor √c 5

Dividend √qq 72 (√qq 12 Quotient
Divisor √qq 6

Heterogeneal. 2. If the Surdes be Heterogeneal, or one Surde be given to be divided with, or to divide an Absolute Number; then first reduce them to one Denomination, and then divide the Dividend by the Divisor, as above.

Examples. As to divide √q 90 by 3; first 3 is squared, and then by 9 is 90 divided, so will the Quotient be √q 10, as at *E.*

And to divide √ʒc 51200 by √q 8, being reduced they are √ʒc 51200 and √ʒc 512, and then divided, the Quotient will be √ʒc 100, as at *F*, and may be depressed to √c 10.

E.

√q 90 √q 9
√q 90 X 3
√q

√q 90 (√q 10
√q 9

2) √ [2] 8 [1] √ [6] 512 √ [6] 51200 √ [6] 51200 [3]

F.

√ʒc 51200 (√ʒc 100
√ʒc 512

Consectaries hence. The following Consectaries flow from Division of Simple Surdes.

1. *To take the half, third part, &c. of a* Surde.

1. That to divide any Surde is to diminish him by the Power of a Root Homogeneal. So as to take the half of any Square Surde is to divide him by 4, the Power of the Root 2, as was before noted in Substraction. Also to take the third part of any Square Surde is to divide him by 9, *&c.* And to take the half of any Cube Surde is to divide him by 8, the Cube of 2. Likewise to take the third part thereof is to divide him by 27, *&c.*

Examples.

Examples of	The Half.	The Third Part.	The Fourth part.
Square Surdes	√q 12 / 4 (√q 3	√q 27 / 9 (√q 3	√q 48 / 16 (√q 3
Cube Surdes	√c 24 / 8 (√c 3	√c 81 / 27 (√c 3	√c 192 / 64 (√c 3

Squared

Squared Squares www 48/16 (www 3   www 243/81 (www 3   www 729/243 (www 3

2. That Division of Surdes sometimes bringeth forth Rational Numbers in the Quotient: The Roots whereof being Absolute Numbers, may be so expressed.

2. Quotient is often a Rational. Examples.

Examples in Surdes	Square.	Cubed	Squared Square.
	w 27 / w 3 (w 9	ww 24 / ww 3 (ww 8	www 48 / www 3 (www 16
	√ 3	√ 2	√ 2

3. That Division of any Surde by himself, giveth in the Quotient a Surde Unit.

3. Surde dividing himself gives 1. Examples.

Examples in	Square Surdes.	Cube Surdes.	Squared Square Surdes.
	w 5 / w 5 (w 1	ww 9 / ww 9 (ww 1	www 15 / www 15 (www 1

4. That a Power whose *Index* is compounded, divided by one side of the Compounding Powers, shall give the Quotient higher or lower according to the divising Power. For Division made by the Root, or lowest Quantity of the Dividend, the Root of the Quotient shall be equal to the Root of the higher compounding Power of the Divisor; and if by the Higher Power, the contrary.

4. Quotient of a Power, &c. divided by the Side.

As if √3φ 262144 which is 8, be divided by √3φ 64 which is 2, the Quotient will be 4, the √3φ of 4096. And because 64 is the Root, the √φ 64 which is 4, may be taken, (Cube being the higher compounding Power in 3φ). But if √3φ 262144 be divided by √3φ 4096, the Quotient will be √3φ 64 which is 2, and may be taken in stead thereof. *Examples.*

√ 2) √ 8 (√ 4
√3φ 64) √3φ 262144 (√3φ 4096 = √φ 64, that is 4.

√ 4) √ 8 (√ 2
√3φ 4096) √3φ 262144 (√3φ 64 = √ 2

5. That if a Figural Number be divided by a Figural Number Homogeneal, the Quotient shall be a Figural Number of the same kind, whose side is equal to the Quotient of the side of the Dividend applyed to the side of the Divisor, or the Greater divided by the Lesser, and the contrary.

5. Homogeneal Figurals divided, what the Quotient.

As if 36 be divided by 4 (both Squares) the Quotient will be 9, whose Root 3 is equal to 6 divided by 2, the sides of 36 and 4. And if 36 be divided by 9, the Quotient 4, whose Root 2, is equal to 6 divided by 3, the sides of 36 and 9. *Examples.*

So 9261, the Cube of 21, divided by 27 and 243, the Cubes of 3 and 7, gives alternately in the Quotient the Powers whose Roots are equal to the Division of 21 by 3 or 7 accordingly.

Square.		Cube.	
√2) √6 (√3	√3) √6 (√2	√3) √21 (√7	√7) √21 (√3
4) 36 (9	9) 36 (4	27) 9261 (343	343) 9261 (27

6. That the sides of Homogeneal Surdes divided procreateth sides of Homogeneal Surdes.

6. Sides of such divided what begotten. Examples.

Ergo w 2 dividing w 6 begetteth w 3. And ww 3 dividing ww 21 begetteth ww 7.

Division of Compound Surdes is to be found in its proper place, in the *Tenth Chapter* following of this *Book*. *Division of Compounds where.*

Because most of the Divisions of this *Chapter* are the Products of the Multiplications in the foregoing *Chapter* divided by one of the Factors, it will serve sufficiently to prove the truth of Surde Multiplication by Division, and Division by Multiplication. *Proof of Division of Simple Surdes.*

Yet to make all clear, take Rational Numbers for Surdes, and proceed in their Division as if they were Surdes, and the Quotients of such Divisions will be equal to the Division of their Roots in Absolute Numbers.

For if w 144 be divided by w 16, the Quotient will be w 9, whose Root is 3, agreeable to the Quotient of 12 the √ of 144 divided by 4 the √ of 16, as at *G*.

And so if √3φ 46656 which is 6, be divided by w 9 which is 3, the Quotient will be 2, as at *H*.

G. √ 4) √ 12 (√ 3
  w 16) w 144 (w 9

H. √ [6] 46656   √ [6] 729
2) √ [6] 46656   √ [2] 9   46656/729 (64
  [3]   [1]
  √ 3) √ 6 (√ 2
√3φ 729) √3φ 46656 (√3φ 64 = 2

CHAP.

CHAP. VII.

Addition of Compound Surdes.

Compound Surdes *added.*

IN the Addition of Compound Surdes, let them be considered as they are Particular or Universal.

As the Compound Surdes are made of the Simple, or else of Rational Numbers with Surdes; so the work of the Compound dependeth on the work of the Simple, and to be wrought alike. And the signs + and — to be ordered, as in Addition of Compound Cossicks.

Particular.

In particular Compound Surdes, as the parts given to be added be, so shall the Addition be. For like Surdes and Symmetral are to be added with like as Simple, and unlike and Asymmetral with the sign of Addition +.

Because Examples are very demonstrative, the varieties of Examples following with their explanations, are to be born with.

Examples of Binomials.

Examples of Binomials.

Addends { 9 + √40 ; 30 + √10 }
Total 39 + √90

After Addition of the Absolute Numbers 9 and 30, the Surdes √40 and √10, are added as before in Simple Surdes.

√40 + √10
10)
4 3   1 3
2√ + 1√
3
3z
9 3
10
√90

Addends { 7 + √8 ; 5 + √3 }
Total 12 + √8 + √3

The Absolute Numbers 7 and 5 make 12. The Surdes being Incommensurable are conjoyned by +. And after the second form of Addition of Simple Surdes, may be set thus, 12 + √11 + √96.

Addends { √1264 + 8 ; 28 + 316 }
Total √2844 + 36

In this Example the Absolute Numbers make up 36, and the Surdes added as Simple are √2844.

√1264 + √316
79)
16 3   4 3
4√ + 2√
6
6z
36 3
79
√2844

Addends { √√32 + √10 ; √√4 + √19 }
Total √√108 + √29 + √760

The Cube Surdes added as Simple make √√108, but the Square Surdes set after the second form of Addition being Incommensurable might have stood thus, √10 + √19; and the whole Total thus, √√108 + √10 + √19.

Addends { √√48 + √5 ; √√243 + √45 }
Total √√1875 + √80

√√ added.
√√48 + √√243
3)
16 3 3   81 3 3
2√ + 3√
5
625 3 3
3
√√1875

√ added.
√5 + √45
5)
1 3   9 3
1√ + 3√
4
16 3
5
√80

Examples of Residuals.

Examples of Residuals.

Addends { 14 — √3 ; 12 — √12 }
Total 26 — √27

After Addition of the Absolute Numbers which make 26, the Surdes added make √27, which is to be adjoyned to 26 with —, because the Numbers to be added had the same sign.

√3 + √12
3)
1 3   4 3
1√ + 2√
3
9 3
3
√27

Addends { 8 — √5 ; 6 — √3 }
Total 14 — √5 — √3

The Absolute Numbers added make 14. The Surdes are Incommensurable, and so annexed with their proper signs; or else after the second form of Simple Addition set thus, 14 — √8 — √60.

Addends

$$\text{Addends}\begin{cases}\surd 50 - 4\\ \surd 18 - 3\end{cases}$$
$$\text{Total}\quad \surd 128 - 7$$

In this Example the work is like the first Example of Residuals above; for the Surdes added make $\surd 128$, and 4 and 3 the Absolute Numbers make 7 to be set with —, because they were of the same Nature.

$$\begin{array}{l}\surd 50 + \surd 18\\ 2)\overline{}\\ 25ʒ \quad 9ʒ\\ 5\surd + 3\surd\\ \quad 8\\ \hline 64ʒ\\ \quad 2\\ \hline \surd 128\end{array}$$

$$\text{Addends}\begin{cases}\surd 128 - 6\\ 8 - \surd 72\end{cases}$$
$$\text{Total}\quad 2 + \surd 8$$

$$\text{Addends}\begin{cases}\surd 128 - 8\\ 6 - \surd 72\end{cases}$$
$$\text{Total}\quad \surd 8 - 2$$

Because one of the Absolute Numbers is + and the other —, 6 taken from 8 leaves 2, but in the one Example +, in the other —, according to the nature of the greater Number. Then the $\surd 72$ being —, and the other Surde +, the Lesser is taken from the Greater, and the Remain is $+\surd 8$, according to the sign of $\surd 128$.

$$\begin{array}{l}\surd 128 - \surd 72\\ 2)\overline{}\\ 64ʒ \quad 36ʒ\\ 8\surd - 6\surd\\ \quad 2\\ \hline 4ʒ\\ \quad 2\\ \hline \surd 8\end{array}$$

$$\text{Addends}\begin{cases}250 - \surd 99\\ \surd 44 - 76\end{cases}$$
$$\text{Total}\quad 174 - \surd 11$$

The work in this Example is like the last; for —76 taken from +250 leaves +174, and $\surd 44$ which is +, taken from $\surd 99$ which is —, leaves $\surd 11$.

$$\begin{array}{l}\surd 99 - \surd 44\\ 11)\overline{}\\ 9ʒ \quad 4ʒ\\ 3\surd - 2\surd\\ \quad 1\\ \hline 1ʒ\\ \quad 11\\ \hline \surd 11\end{array}$$

$$\text{Addends}\begin{cases}\surd 44 - \surd 27\\ \surd 99 - \surd 3\end{cases}$$
$$\text{Total}\quad \surd 275 - \surd 48$$

Here $\surd 44 + \surd 99$ and $\surd 27 + \surd 3$, are added severally, and their Totals conjoyned by their proper sign.

$$\begin{array}{l}\surd 44 + \surd 99\\ 11)\overline{}\\ 4ʒ \quad 9ʒ\\ 2\surd + 3\surd\\ \quad 5\\ \hline 25ʒ\\ \quad 11\\ \hline + \surd 275\end{array} \qquad \begin{array}{l}\surd 27 + \surd 3\\ 3)\overline{}\\ 9ʒ \quad 1ʒ\\ 3\surd + 1\surd\\ \quad 4\\ \hline 16ʒ\\ \quad 3\\ \hline - \surd 48\end{array}$$

$$\text{Addends}\begin{cases}\surd\surd 72 - \surd 96\\ \surd\surd 9 - \surd 6\end{cases}$$
$$\text{Total}\quad \surd\surd 243 - \surd 150$$

The Cube Surdes added by themselves, and the Squares by themselves, make the Total $\surd\surd 243 - \surd 150$.

$$\begin{array}{l}\surd\surd 72 + \surd\surd 9\\ 9)\overline{}\\ 8\varphi \quad 1\varphi\\ 2\surd + 1\surd\\ \quad 3\\ \hline 27\varphi\\ \quad 9\\ \hline + \surd\surd 243\end{array} \qquad \begin{array}{l}\surd 96 + \surd 6\\ 6)\overline{}\\ 16ʒ \quad 1ʒ\\ 4\surd + 1\surd\\ \quad 5\\ \hline 25ʒ\\ \quad 6\\ \hline - \surd 150\end{array}$$

$$\text{Addends}\begin{cases}\surd\surd 32 - \surd 5\\ \surd\surd 32 - \surd 24\end{cases}$$
$$\text{Total}\quad \surd\surd 512 - \surd 24 - \surd 5$$
$$\text{or}\quad \surd\surd 512 - \surd 29 - \surd 480$$

This Example is like the second of the Residual Examples above in the Square Surdes, and so being Asymmetral may be set differently. The $\surd\surd$ being added to himself, is as 32×16.

Examples of Binomials *with* Residuals.

Examples of Mixt.

$$\text{Addends}\begin{cases}8 - \surd 27\\ 11 + \surd 3\end{cases}$$
$$\text{Total}\quad 19 - \surd 12$$

$$\text{Addends}\begin{cases}8 + \surd 27\\ 11 - \surd 3\end{cases}$$
$$\text{Total}\quad 19 + \surd 12$$

In both these Examples adding 8 and 11, they make 19 absolute Numbers; and $\surd 3$ taken from $\surd 27$, because their Signs are contrary, there rests $\surd 12$, which in the one Total is +, in the other —, according to the nature of the Greater Surde.

$$\begin{array}{l}\surd 27 - \surd 3\\ 3)\overline{}\\ 9ʒ \quad 1ʒ\\ 3\surd - 1\surd\\ \quad 2\\ \hline 4ʒ\\ \quad 3\\ \hline \surd 12\end{array}$$

Addends $\left\{\begin{array}{l}\sqrt{12} - 8 \\ \sqrt{3} + 6\end{array}\right.$

Total $\sqrt{27} - 2$

Addends $\left\{\begin{array}{l}\sqrt{12} + 8 \\ \sqrt{3} - 6\end{array}\right.$

Total $\sqrt{27} + 2$

Both these Examples adding the Surdes make their Total $\sqrt{27}$, and taking 6, one of the Absolute Numbers, from 8, the other leave 2, which in the one is —, and in the other +, according to the sign of the greater Number.

$$\begin{array}{l} \qquad \sqrt{12} + \sqrt{3} \\ 3)\overline{\qquad\qquad\qquad} \\ \quad 4\,3 \qquad 1\,3 \\ \quad 2\surd + 1\surd \\ \qquad \underline{\ 3\ } \\ \qquad 9\,3 \\ \qquad \underline{\ 3\ } \\ \quad \sqrt{27} \end{array}$$

Addends $\left\{\begin{array}{l}\sqrt{50} + \sqrt{7} \\ \sqrt{52} - \sqrt{7}\end{array}\right.$

Total $\sqrt{242}$

Here $\sqrt{7}$ being found with + and —, both are cancelled, and the Total of the other Surdes only set down.

$$\begin{array}{l} \qquad \sqrt{50} + \sqrt{72} \\ 2)\overline{\qquad\qquad\qquad} \\ \quad 25\,3 \qquad 36\,3 \\ \quad 5\surd + 6\surd \\ \qquad \underline{\ 11\ } \\ \qquad 121\,3 \\ \qquad \underline{\ 2\ } \\ \quad \sqrt{242} \end{array}$$

Addends $\left\{\begin{array}{l}\sqrt{63} + \sqrt{20} \\ \sqrt{7} - \sqrt{5}\end{array}\right.$

Total $\sqrt{112} + \sqrt{5}$

In this Addition $\sqrt{5}$ is taken from $\sqrt{20}$ because the Signs are contrary, and there rests $\sqrt{5}$, which is +, because the Lesser Surde was substracted.

$$\begin{array}{l} \qquad \sqrt{63} + \sqrt{7} \\ 7)\overline{\qquad\qquad\qquad} \\ \quad 9\,3 \qquad 1\,3 \\ \quad 3\surd + 1\surd \\ \qquad \underline{\ 4\ } \\ \qquad 16\,3 \\ \qquad \underline{\ 7\ } \\ \quad \sqrt{112} \end{array} \qquad \begin{array}{l} \qquad \sqrt{20} - \sqrt{5} \\ 5)\overline{\qquad\qquad\qquad} \\ \quad 4\,3 \qquad 1\,3 \\ \quad 2\surd - 1\surd \\ \qquad \underline{\ 1\ } \\ \qquad 1\,3 \\ \qquad \underline{\ 5\ } \\ \quad \sqrt{5} \end{array}$$

Addends $\left\{\begin{array}{l}\sqrt{80} - \sqrt[3]{160} \\ \sqrt{5} + \sqrt[3]{20}\end{array}\right.$

Total $\sqrt{125} - \sqrt[3]{20}$

After Addition of the Square Surdes, the lesser Cube Surde is taken from the greater, their Signs being contrary.

$$\begin{array}{l} \qquad \sqrt{80} + \sqrt{5} \\ 5)\overline{\qquad\qquad\qquad} \\ \quad 16\,3 \qquad 1\,3 \\ \quad 4\surd + 1\surd \\ \qquad \underline{\ 5\ } \\ \qquad 25\,3 \\ \qquad \underline{\ 5\ } \\ \quad \sqrt{125} \end{array} \qquad \begin{array}{l} \qquad \sqrt[3]{160} - \sqrt[3]{20} \\ 20)\overline{\qquad\qquad\qquad} \\ \quad 8\,\varphi \qquad 1\,\varphi \\ \quad 2\surd - 1\surd \\ \qquad \underline{\ 1\ } \\ \qquad 1\,\varphi \\ \qquad \underline{\ 20\ } \\ \quad \sqrt[3]{20} \end{array}$$

Addends $\left\{\begin{array}{l}\sqrt{63} + \sqrt{20} + \sqrt[3]{10} \\ \sqrt{7} - \sqrt{5}\end{array}\right.$

Total $\sqrt{112} + \sqrt{5} + \sqrt[3]{10}$

This Example is like the last above save one, only the odd Surde $\sqrt[3]{10}$, is adjoyned to the Total of the others added.

Addends $\left\{\begin{array}{l}\sqrt[3]{40} + \sqrt{24} \\ \sqrt[3]{320} - \sqrt{56}\end{array}\right.$

Total $\sqrt[3]{1080} - \sqrt{80} - \sqrt{5376}$

Addends $\left\{\begin{array}{l}\sqrt[3]{40} - \sqrt{24} \\ \sqrt[3]{320} + \sqrt{56}\end{array}\right.$

Total $\sqrt[3]{1080} + \sqrt{80} - \sqrt{5376}$

These two last Examples make the Cube Surdes in both $\sqrt[3]{1080}$, in the Square Surdes the Total is — $\sqrt{80}$ in one, and + $\sqrt{80}$, because in this the Greater Surde to be added was +, but in the other —. The Totals may be thus set,

$$\sqrt[3]{1080} + \sqrt{24} - \sqrt{56}$$
$$\sqrt[3]{1080} - \sqrt{24} + \sqrt{56}$$

$$\begin{array}{l} \qquad \sqrt[3]{40} + \sqrt[3]{320} \\ 40)\overline{\qquad\qquad\qquad} \\ \quad 1\,\varphi \qquad 8\,\varphi \\ \quad 1\surd + 2\surd \\ \qquad \underline{\ 3\ } \\ \qquad 27\,\varphi \\ \qquad \underline{\ 40\ } \\ \quad \sqrt[3]{1080} \end{array} \qquad \begin{array}{l|l} \multicolumn{2}{l}{\sqrt{24} \mp \sqrt{56}} \\ \hline & 24 \\ & \underline{56} \\ \sqrt{80} & 144 \\ & \underline{120} \\ & 1344 \\ & 4 \\ \hline \multicolumn{2}{l}{\sqrt{80} \mp 5376} \end{array}$$

Universal. In Addition of Universal Surdes respect is to be had to the Mark prefixed; for so properly is the Addition to be: So as if there be given to be added thus differently marked;

$\surd 3\ 12$

√ℨ12 to √ℨ12 or W12 + W12 — Then are the Surdes looked on as Compounds, only Particular and not Universal, and the Total of both their Roots added as before is W48, which is almost 7 absolute Numbers. *How differently marked and taken.*

√:12 to √ℨ12 or √:12 + W12 — Then is to be understood that the Square Root of the Dexter 12 is to be added to the Sinister 12, and the Square Root of that summe to be taken for the Universal Root.

√:ℨ12 to √ℨ12 or √:W12 + W12 — This form is understood by some as the last above; but others more strictly, after the √ℨ of the Dexter 12 is added to the √ℨ of the Sinister 12, take the √ℨ of that Total for the Universal Root.

To make all plain let Rational Numbers be taken; as suppose W81 which is 9, and W361 which is 19, these added together make 28, and so they are considered as Compound only, and their Roots particular and distinct; yet as both Roots are added the Total may seem to be the Universal Root of both Surdes. But usually considered as Universal, then must 19 be added to 81, which make 100, and the Square Root thereof taken which is but 10. And this *Record* in his *Whetstone of Wit*, *p.* 11. counts most aptly the Universal Root. And according to him, and other good Authors, let Universal Roots be so understood here Otherwise more strictly, if the Root of the Dexter Surde be added to the Root of his Sinister Surde alike denominate, and the Root thereof taken for the Universal Root; then after 19 is added to 9, the Root of 28, the Total taken for the Universal Root will be 5, and somewhat more. *Explained by Rationals.*

Addends √ℨ81 + √ℨ361
9√ 19√
Total 28 Particular.

Addends √:81 + √ℨ361
19 19√
Total W100 (10√ Universal.

Addends √:W81 + W361
Total strictly 5 and + Universal.

√ℨ81 + √ℨ361

361
81

W442
361
2888 √
29241 (171
2

√:W442 + W342
Total W784 = 28
√9 + √19
(3
√28(5

ℨ (3 √
784 (28 (5

4
32
64

Hence it is apparent, that the Totals of Simple or Particular Compound Surdes, as before added, if considered as Rooted after a sort may be taken for the Universal Root of the Surdes given to be added. But if from the Total the Character of the Universal Surdes or √: be removed or cancelled, and the Sinister Number left absolute, the Surdes are commonly considered as distinct, and their Roots particular.

To the right understanding of the Addition of Universal Roots, it is meet to proceed in the following steps. *Steps to the Addition of Universals.*

1. If the Numbers or Surdes given be Incommensurable or Heterogeneal, then conjoyn them with +, and before them prefix √:, the Mark that signifies the Universal Root. *1. If the Data be Incommensurable or Heterogeneal Example.*

As to add the Universal Root of 39, absolute Numbers, the wW9 and the W8 together, the Total shall be √:39 + wW9 + W8; or thus, √:wW9 + W8 + 39, of which this latter form is the better, to set the absolute Numbers to the Right Hand of the Surdes; lest standing next the Character √: they be taken for a Surde: For there sometime the Denomination is omitted, and the Number valued as his next Dexter Surde.

2. If the one be an Universal Surde, and the other a Particular, then add the Particular Surde to the Sinister part of the Universal, as Particulars are added; and to the Dexter part of the Universal, add double the Root of the Particular (or the Square multiplyed by 4, which is all as one) if the Root of that Dexter part of the Universal added to the Sinister part, make up the next Square Number thereto. *2. One Universal, and the other Particular, two varieties.*

As to add W16 to √:36 + W169, the Square Roots of 16 and 36 added, which are 4 and 6, make 10, the Square whereof is 100; for the Sinister work to the Dexter W169, whose Root is 13, must be added 8, the double of 4, which is 21; or 16 multiplyed by 4 makes W64, which added as Surdes to W169, makes the Dexter part W441, whose Root is 21. Another Example is at *B.* *Examples of the first variety.*

Roots

	Roots.	Rationals.		*B.*
Addends	7	√: 36 + W 169	4	√: 9 + W 49
	4	√3 16	2	√3 4
Total	11	√: 100 + W 441	6	√: 25 + W 121

Other variety. But if the Root of the Dexter part of the Universal added to the Sinister part, make not up the next Square Number, then if one be omitted, quadruple the Root of the Particular which before you doubled, if 2 be omitted then sexcuple the Root, &c.

Examples. As to add W 25 to √: 4 + W 144; because 12 the Root of 144, added to 4, make 16, which is not the next Square to 4, but 9 is omitted; therefore 5 the Root of 25 must be quadrupled which is 20, and the Square therefore added to 144.

So to add W 9 to √: 4 + W 441; because 21, the Root of 441, added to 4 make 25, which is not the next Square to 4, but 9 and 16, two Squares are omitted, therefore 3, the Root of 9, must be sexcupled, which is 18, and the Square thereof added to 441.

Addends	4	√: 4 + W 144	5	√: 4 + W 441
	5	√3 25	3	√3 9
Total	9	√: 49 + W 1024	8	√: 25 + W 1521

3\. *Data both Universals, and o in the Sinister part of one.* 3. If both Numbers given to be added be Universals, and in the Sinister part of one be o, then subscribe the Sinister part as it is, without any alteration, and double the Root Universal of that Number wherein the Cypher is not, multiply the other Root thereby, and the Square thereof add with both the Dexter Squares.

But if the Number that hath the Cypher be a Rational, the Square Root thereof may be taken and placed under the Sinister part, and added as last before.

Example. As to add √: 4 + W 25 to √: 0 + W 256; because 256 is a Rational, and hath 16 for its Root, this may be added with √: 4 + W 25, as before.

But otherwise 4 is subscribed in the sinister part only, and 3 the Root Universal doubled is 6, which multiplying 4, the other Root makes 24, whose Square is to be added with both the other Dexter Squares. The Totals of both Additions are here set down.

Addends	3	√: 4 + W 25	3	√: 4 + W 25
	4	√: 0 + W 256	4	√3 16
Total	7	√: 4 + W 2025	7	√: 36 + W 169

4\. *Data both Universals, without o.* 4. If both Universals be without Cyphers, then as before, add the Sinister Surdes as Particular: And the Dexter must be increased to such a Number, that the Root thereof added to the Sinister may be equal to the Roots of the Numbers given to be added. But herein great circumspection is to be used; for though the Roots be Universal, no Universal Rule can be given that I know of to work by, but sometime the double of the Dexters, sometime the √33 of the Dexters severally taken, added, and then figurate accordingly and set as Squares, sometime the method used in the Pathway, but sometime neither will serve.

Examples. As in the two Examples following, the Dexters in both have their 33 Roots, yet the Dexters at *D.* require 169, a greater Number than their Roots 33 will give, which will be but 81.

		C.		*D.*
Addends	8	√: 48 + W 256	7	√: 48 + W 1
	4	√: 12 + W 16	4	√: 12 + W 16
Total	12	√: 108 + W 1296	11	√: 108 + W 169

Other Examples.

Addends	7	√: 48 + W 1	4	√: 12 + W 16
	2	√: 3 + W 1	2	√: 3 + W 1
Total	9	√: 75 + W 36	6	√: 27 + W 81

5\. *Residual Universals. Examples.* 5. If the Universals be Residuals, then add them as above, and keep the sign Residual to the Total.

As to add √: 48 — W 144 which is 6, to √: 12 — W 9 which is 3, the Total will be √: 108 — W 729, whose Root is 9. Another Example is set at *F.*

E.

	E.			F.
Addends 6	√:48 — W 144		6	√:48 — W 144
3	√:12 — W 9		5	√:27 — W 4
Total 9	√:108 — W 729		11	√:147 — W 676

6. If the Universals be mixt then after Addition of the Sinisters, one of the Dexters is to be taken from the other, as in Addition of Particulars, yet the Remain must be left valuable to make the Root of the Sinisters sufficient for the summe of the Addition. See the Examples following. *6. Mixt Universals.* *Examples.*

Addends 8	√:48 + W 256	6	√:28 + W 64
5	√:27 — W 4	2	√:7 — W 9
Total 13	√:147 + W 484	8	√:63 + W 1
Addends 6	√:48 — W 144	7	√:50 — W 1
4	√:12 + W 16	2	√:2 + W 4
Total 10	√:108 — W 64	9	√:72 + W 81

7. If any Universal Square Surde be added to himself, the Sinister may be multiplyed by 4, the next Dexter by 16, and the next by 256, *&c.* *7. Square Universals added to themselves.*

But if any Absolute Number be intermixed amongst the Surdes, they are to be multiplyed by the next Sinister Multiplyers before them.

As to add √:3 + W 30 + W 36 to it self, that is 3 to 3, the Total will be √:12 + W 480 + W 9216, whose Root Universal is 6: For 96, the Root of 9216, added to 480, and the Root of 576, the summe which is 24, brought and added to 12, makes 36, whose Root is equal to the other Roots of the Addends. *Example.*

	Multiplyers 4 16 256
Addends 3	√:3 + W 30 + W 36
3	√:3 + W 30 + W 36
Total 6	√:12 + W 480 + W 9216

8 If any Square Universal be added to himself, the Multiplications may be shortned thus: Let the Sinister Number be multiplyed by 2, the next Dexter by 4, and the next Dexter by 16, *&c.* and the Absolute Numbers accordingly: For if the outmost Dexter be Absolute and not Figurate, then multiply that by 4; if the middlemost be Absolute, then multiply that by 2; and the outmost Dexter by 4, though Figurate. Also in getting the absolute value of the Total so added, when the Root is brought to be added to or substracted from the Sinister, the summe or difference is accordingly to be multiplyed again by 2. *8. To shorten the Multiplications in such Additions.*

As in the last Example the Total will be √:6 + W 120 + W 576; but after 24, the Root of 576, is added to 120, and the Root of 144, the summe which is 12, brought and added to 6, then 18, the summe there is to be multiplyed by 2, the Root of which Product 36, is 6, the Total as before. *Example.*

	Multiplyers 2 4 16
Addends 3	√:3 + W 30 + 36
3	√:3 + W 30 + 36
Total 6	√:6 + W 120 + W 576

576 | 24 √
+ 120
144 | 12 √
+ 6
18
× 2
36 | 6 √ value

Other Examples follow with their Multiplyers at top, and the summe of their Roots under the Totals. *Other Examples.*

	2 4 16	2 4 16	2 4 16
Addends	√:5 + W 12 + W 16	√:5 + W 20 — W 16	√:5 — W 20 — W 16
	√:5 + W 12 + W 16	√:5 + W 20 — W 16	√:5 — W 20 — W 16
Total	√:10 + W 48 + W 256	√:10 + W 80 — W 256	√:10 — W 80 — W 256
	8 16	8 —16	—8 —16
	18 64 (8 √	18 64 (8 √	2 64 (8 √
	36 (6 √ value	36 (6 √ value	4 (2 √ value

	$\overset{2}{\sqrt{}}:5+\overset{4}{W}20-\overset{4}{1}6$	$\overset{2}{\sqrt{}}:2+\overset{4}{W}20-\overset{4}{1}6$	$\overset{2}{\sqrt{}}:25-\overset{2}{1}2+\overset{4}{W}16$
Addends	$\sqrt{}:5+W20-16$	$\sqrt{}:2+W20-16$	$\sqrt{}:25-12+W16$
Total	$\sqrt{}:10+W80-64$	$\sqrt{}:4+W80-64$	$\sqrt{}:50-24+W64$
	$4 \quad -64$	$4 \quad -64$	$-32 \quad 8$
	$14 \quad 16\,(4\sqrt{}$	$8 \quad 16\,(4\sqrt{}$	$18 \quad 32$
	$W28$ value	$16\,(4\sqrt{}$ value	$36\,(6\sqrt{}$ value

Whence this work last mentioned proceeds. This and the precedent work take original from the *First Consectary* in *Chap.* 5. *Multiplication of Simple Surdes*, and according thereto may also Universal Surdes of Higher Powers than Squares be doubled, tripled, *&c.* but little use being of any higher Universals than Squares, the foregoing Operations are fitted for them.

Proof of Addition of Compound Surdes. The Proof of Addition of Compound Surdes is like the Proof of Simple Addition, either by Substraction, Particular by Particular, and Universal by Universal; or by taking Rational Numbers, and working therewith instead of the Surdes. And seeing several of the Examples are of Rational Numbers, they may serve without further instance.

CHAP. VIII.

Substraction of Compound Surdes.

Compound Surdes substracted. Particular. IN the Substraction of Compound Surdes, as in Addition, let them be considered as they are Particular or Universal.

In Particular Compound Surdes, to Heterogeneal and Incommensurable set the sign of Substraction —; and for Commensurable, as the parts given to be substracted, so shall the Substraction be. Like is to be substracted from like; and the use of the signs + and — is here as in Substraction of Compound Cossicks.

Examples of Binomials.

Examples of Binomials.

Greater Surde	$10+W48$
Subtrahend	$7+W27$
Remain	$3+W3$
Lesser Surde	$10+W27$
Subtrahend	$7+W48$
Remain	$3-W3$

After Substraction of the Absolute Number 7 from 10, and the Remain 3 set down, in both these Examples the $W27$ and $W48$ are substracted as before in Simple Surdes, and the Greater of them being taken from the Lesser, makes the Remain to be —.

$$3)\,W48 - W27$$
$$16\quad 9$$
$$4\sqrt{} - 3\sqrt{}$$
$$1$$
$$1\ 3$$
$$3$$
$$W3$$

Greater Surde	$12+W8+W3$	Greater Surde	$12+W11+W96$
Subtrahend	$5+W3$	Subtrahend	$5+W3$
Remain	$7+W8$	Remain	$7+W8$

The Absolute Numbers being substracted in both these Examples, there remains 7; then in the one finding $W3$ in the Greater Surde and Subtrahend both, they are both cancelled, for taking one from the other, 0 remains; so $W8$ is only set down. But in the latter Example, where the Greater Surde is set as the Total of the second form of Simple Addition, 96 is to be divided by 4, and the Quotient by $W3$, and so $W8$ is gotten for the Remain.

$$W\,96\,(\ 24\,(\ W8$$
$$4 \quad W3$$

Greater Surde	$W2844+36$
Subtrahend	$28+W316$
Remain	$W1264+8$

In this Example taking $W316$ from $W2844$, the Remain is $W1264$. And 28 Absolute Numbers taken from 36, there Remains + 8.

$$316)\,W2844 - W316$$
$$9\quad 1$$
$$3\sqrt{} - 1\sqrt{}$$
$$2$$
$$4$$
$$316$$
$$W1264$$

Greater

Greater Surde	√√108 + √29 + √760	
Subtrahend	√√ 4 + √19	
Remain	√√32 + √10	

Here taking √√4 from √√108 remains √√32, then taking √19 from √29, rests √10, rejecting √760, or dividing as before by 4, and the Quotient by 19, the √10 is obtained, rejecting √29.

√760 / 4 (√190 / √19 (√10

√√108 — √√4
4)
27φ 1φ
3√ — 1√
2
8φ
4
√√32

Greater Surde	√√1875 + √5	
Subtrahend	√√ 48 + √80	
Remain	√√243 — √45	

substracted.
√√1875 — √√48
3)
62533 1633
5√ — 2√
3
8133
3
√√243

substracted.
√5 — √80
5)
13 163
1√ — 4√
—3
—93
5
— √45

Examples of Residuals.

Examples of Residuals.

Greater Surde	26 — √27
Subtrahend	12 — √12
Remain	14 — √3
Lesser Surde	26 — √12
Subtrahend	12 — √27
Remain	14 + √3

After Substraction of 12 from 26, the Absolute Number 14 is left in both these Examples. And √12 taken from √27 leaves √3, but in the one —, in the other +, according as the Subtrahend was the Greater or Lesser Surde.

√27 — √12
+
3)
93 43
3√ — 2√
+
1
13
3
√3

Greater Surde	14 — √5 — √3
Subtrahend	8 — √5
Remain	6 — √3

The Absolute Number 6, is left when 8 is taken from 14. And √5 taken from √5 leaves 0; so both cancelled, √3 is brought down to the Remain: But if the Surde had been set thus, 14 — √8 — √60, after the second form of Simple Substraction, then 60 divided by 4, and the Quotient 15 divided by 5, gives √3 for the Remain, as before.

Greater Surde	√275 — √48
Subtrahend	√99 — √3
Remain	√44 — √27

Here taking √99 from √275, and √3 from √48, the Remain appears thus,

√275 — √99
11)
253 93
5√ — 3√
2
43
11
√44

√48 — √3
3)
163 13
4√ — 1√
3
93
3
√27

Greater Surde	√√243 — √150
Subtrahend	√√72 — √96
Remain	√√9 — √6

Both Cube Surdes and Square Surdes severally substracted as Simple their Remains are thus;

√√243—√√72
9)
27φ 8φ
3√ — 2√
1
1φ
9
√√9

√150—√96
6)
253 163
5√ — 4√
1
13
6
√6

Greater Surde	√√512 — √24 — √5
Subtrahend	√√32 — √24
Remain	√√32 — √5
Greater Surde	√√512 — √24
Subtrahend	√√32 — √24 — √5
	√√32 + √5

In both these Examples the Remain of the Squared Square Surdes is √√32. The Remain of the Square Surdes in the one is +, in the other —, as the odd √5 in the one was in the Subtrahend, in the other in the Number from which Substraction is made. And if the Square Surdes had been set after the second form of Addition, that is √29 — √480, then dividing 480 by 4, and the Quotient 120 by 24, the Remain will be √5, as before.

√√512 — √√32
2)
25633 1633
4√ — 2√
2
1633
2
√√32

Ex-

Examples of Mixt.

Examples of Binomials *with* Residuals.

Greater Surde	$19 + \sqrt{12}$
Subtrahend	$11 - \sqrt{3}$
Remain	$8 + \sqrt{27}$
Greater Surde	$19 - \sqrt{27}$
Subtrahend	$11 + \sqrt{3}$
Remain	$8 - \sqrt{27}$

In both these Examples the Absolute Numbers 11 taken from 19, leaves 8. And $\sqrt{3}$ added to $\sqrt{12}$ because their Signs are contrary, makes the Remain $\sqrt{27}$, in the one $+$, in the other $-$, as the sign of the Number from which Substraction is made.

3) $\sqrt{12} + \sqrt{3}$
$4\square \quad 1\square$
$2\sqrt{} + 1\sqrt{}$
3
$9\square$
3
$\sqrt{27}$

Greater Surde	$\sqrt{72} - 2$
Subtrahend	$6 - \sqrt{8}$
Remain	$\sqrt{128} - 8$

Both Surdes and Absolute Numbers being of contrary Signs, the respective sums of both are taken.

8) $\sqrt{72} + \sqrt{8}$
$9\square \quad 1\square$
$3\sqrt{} + 1\sqrt{}$
4
$16\square$
8
$\sqrt{128}$

Greater Surde	$174 - \sqrt{11}$
Subtrahend	$\sqrt{44} - 76$
Remain	$250 - \sqrt{99}$

The sum of the Absolute Numbers, and the Total of the Surdes, is the Remain of this Substraction, the signs of both being contrary.

11) $\sqrt{44} + \sqrt{11}$
$4\square \quad 1\square$
$2\sqrt{} + 1\sqrt{}$
3
$9\square$
11
$\sqrt{99}$

Greater Surde	$\sqrt{242}$
Subtrahend	$\sqrt{72} - \sqrt{7}$
Remain	$\sqrt{50} + \sqrt{7}$
Greater Surde	$\sqrt{242}$
Subtrahend	$\sqrt{50} + \sqrt{7}$
Remain	$\sqrt{72} - \sqrt{7}$

In both these Examples the work is alike; for $\sqrt{72}$ taken from $\sqrt{242}$ leaves $\sqrt{50}$, and $\sqrt{50}$ taken out of $\sqrt{242}$ leaves $\sqrt{72}$. Then $\sqrt{7}$ in both having none to be taken from doth remain; but the Signs are changed; for where $+$ is substracted $-$ shall remain, and where less, more.

2) $\sqrt{242} - \sqrt{72}$
$121\square \quad 36\square$
$11\sqrt{} - 6\sqrt{}$
5
$25\square$
2
$\sqrt{50}$

Greater Surde	$\sqrt{112} + \sqrt{5}$
Subtrahend	$\sqrt{7} - \sqrt{5}$
Remain	$\sqrt{63} + \sqrt{20}$

Here $\sqrt{7}$ taken from $\sqrt{112}$ leaves $\sqrt{63}$, and $-\sqrt{5}$ added to $+\sqrt{5}$ makes $\sqrt{20}$, whose sign is as the upper Surde.

7) $\sqrt{112} - \sqrt{7}$
$16\square \quad 1\square$
$4\sqrt{} - 1\sqrt{}$
3
$9\square$
7
$\sqrt{63}$

5) $\sqrt{5} + \sqrt{5}$
$1\square \quad 1\square$
$1\sqrt{} + 1\sqrt{}$
2
$4\square$
5
$\sqrt{20}$

Greater Surde	$\sqrt{125} - \sqrt{\sqrt{20}}$
Subtrahend	$\sqrt{5} + \sqrt{\sqrt{20}}$
Remain	$\sqrt{80} - \sqrt{\sqrt{160}}$

The work in this is like that in the last Example above.

5) $\sqrt{125} - \sqrt{5}$
$25\square \quad 1\square$
$5\sqrt{} - 1\sqrt{}$
4
$16\square$
5
$\sqrt{80}$

20) $\sqrt{\sqrt{20}} + \sqrt{\sqrt{20}}$
$1\varphi \quad 1\varphi$
$1\sqrt{} + 1\sqrt{}$
2
8φ
20
$\sqrt{\sqrt{160}}$

Greater Surde	$\sqrt{\sqrt{1080}} - \sqrt{80} - \sqrt{5376}$
Subtrahend	$\sqrt{\sqrt{320}} - \sqrt{56}$
Remain	$\sqrt{\sqrt{40}} + \sqrt{24}$
Greater Surde	$\sqrt{\sqrt{1080}} - \sqrt{80} - \sqrt{5376}$
Subtrahend	$\sqrt{\sqrt{40}} + \sqrt{24}$
Remain	$\sqrt{\sqrt{320}} - \sqrt{56}$

The Operations in both these last Examples are much alike; for $\sqrt{\sqrt{320}}$ substracted from $\sqrt{\sqrt{1080}}$ leaves $\sqrt{\sqrt{40}}$, and therefore $\sqrt{\sqrt{40}}$ substracted must leave $\sqrt{\sqrt{320}}$. In the one $\sqrt{56}$ may be substracted from $\sqrt{80}$, and the Remain set down with the sign of 80; in the other it may be added to $\sqrt{80}$. Or else 5376 divided by 4, and the Quotient divided by 56 in the one, or 24 in the other, gives in the Quotient the Remain accordingly.

40) $\sqrt{\sqrt{1080}} - \sqrt{\sqrt{320}}$
$27\varphi \quad 8\varphi$
$3\sqrt{} - 2\sqrt{}$
1
1φ
40
$\sqrt{\sqrt{40}}$.

40) $\sqrt{\sqrt{1080}} - \sqrt{\sqrt{40}}$
$27\varphi \quad 1\varphi$
$3\sqrt{} - 1\sqrt{}$
2
8φ
40
$\sqrt{\sqrt{320}}$

$\sqrt{5376}$ (1344 ($\sqrt{56}$
4 (24 (

$\sqrt{5376}$ (1344 ($\sqrt{24}$
4 (56 (

In

In *Substraction of Universal Surds*, proceed with respect to the Mark prefixed, as in *Addition of Universals* was before noted. For so accordingly must the *Substraction* be. *Universal.*

As in Rational Numbers it may be clearly demonstrated, that if √ʒ25+√ʒ121, which are 5 and 11, be substracted from √ʒ81+√ʒ361, which are 9 and 19, they being marked only as *Compounds*, shall make the particular Remain as at A, 12 Absolute Numbers. But marked as Universal, the Remain shall be but 4 as at B. And if strictly taken, as before shewed in the foregoing Chapter, it shall not amount to 4; but be W12, which is a single *Surd*, and hath 3 and somewhat more for his Root, as at C. *Explained by Rationals.*

Greater Surd.	√ʒ81+√ʒ361	A	W81+W361=9+19=28
Subtrahend.	√ʒ25+√ʒ121		W25+W121=5+11=16
Remain.	12	Particular.	W16+W 64=4+ 8=12

			W81
Greater Surd.	√:81+√ʒ361	B	W361= 19
Subtrahend.	√:25+√ʒ121		W100=10
Remain.	4	Universal.	W 25
			W121= 11
			W 36=6
			4

Greater Surd.	√:W81+W361	C.	
Subtrahend.	√:W25+W121		
Remain.	W12	Strictly Universal.	√9+19=28
			√5+11=16
			W12

Hereby it appeareth that the Remains of the Substraction of Simple or Particular *Compound Surds* as before substracted, if considered as rooted, are after a sort to be taken for the Universal Root. But if the Character of the *Universal Surds* be removed or cancelled, or the *Remain* but a single Number, then they cease to be *Universal*, but are understood as Simple or Particular *Compound Surds*.

Further to understand the Substraction of Universal Roots, it may be safe to tread in the like Steps as before in their *Addition*. *Steps to the Substraction of Universals.*

1. If the Numbers or *Surds* given be Incommensurable or Heterogeneal; then conjoin them with —, and prefix before them √: to signify the Root Universal. *1. If the Data be Incommensurable or Heterogeneal.*

As to take W8 from √:ww9+W8+39, the Remain shall be √:ww9+39. Example.

2. If a Particular *Surd* be to be substracted from some Universal; then take the Particular *Surd* from the sinister Part of the Universal, as Particulars are substracted: And from the dexter Part of the Universal, take the Square of double the Root of the Particular, when the Root of that dexter Part of the Universal added to the sinister Part make up the next square Number thereto; but if one be omitted, then quadruple the Root; if two be omitted, sexcuple the Root, &c. *2. One Particular, and the other Universal, 2 Varieties.*

As to take W16 from √:36+W169, the Root of 16, which is 4 doubled, is 8; so must 64 the Square thereof be taken from 169, after the manner of *Surds*. *Example of the first Variety.*

But if W25 be taken from √:36+W784, there 5 the Root of 25 must be multiplied by 4; because 28, the Root of 784, added to 36, makes 64; which is not the next square Number to 36, but one is omitted, to wit 49. *Other Variety.*

		Roots.		*Roots.*	
Greater Surds.	√:36+W169	7	√:36+W784	8	Example.
Subtrabends.	√ʒ16	4	√ʒ25	5	
Remains.	√:4+W25	3	√:1+W64	3	

3. If both Numbers given be Universals, and the sinister Part of the *Subtrabend* be a Cipher; then if the Dexter thereof be Rational, take the Root and place in the sinister Place of the Cipher, and proceed as in the last Direction. *3. Data both Universals, and 0 in the Sinister Place of one.*

As to take √:0+W81 from √:49+W225, the Root of 81 being 9, if taken from √:49+W225 as above, leaves the Remain √:16+W81. Example.

Greater Surd.	√:49+W225		8
Subtrahend.	√39	for √:0+W81	3
Remain.	√:16+W81		5

4. Data both Universals without 0.

4. If both Universals be without Ciphers, then take the Sinister of the *Subtrahend* from the Sinister of the other, as particular *Surds* are substracted. And let the Dexter be diminished, that the *Remain* may be just. But herein lies the Skill, as before in *Addition*, wherein the Operator must have under consideration several Directions at once.

Examples.

As to take √:12+W16 from √:48+W256, after 12 is taken from 48, and the Remain 12 subscribed, the Dexter can be but 16, to make 12 half the other *Surd* as it is.

But if √:12+W16 be taken from √:48+W1, the Sinister Part of the Remain will be as before; but the Dexter will be —W9, because W1 was not sufficient to substract W16 from, and therefore the Sign is changed.

Greater Surds.	√:48+W256	8	√:48+W1	7
Subtrahends.	√:12+W16	4	√:12+W16	4
Remains.	√:12+W16	4	√:12—W9	3

5. Residual Universals.

5. If the Universals be Residuals, they are to be substracted as above, and the Residual Sign kept to the Remain, or changed as the Case requires.

Examples.

As to take √:12—W9 from √:48—W144, the Remain shall be √:12—W9. But √:2—W4 taken from √:50—W1, shall leave the Remain √:32+W289. where the Sign is changed, because W1 was too little to substract W4 from.

Greater Surds.	√:48—W144	6	√:50—W1	7
Subtrahends.	√:12—W9	3	√:2 —W4	0
Remains.	√:12—W9	3	√:32+W289	7

6. Mixt Universals.

6. If the Universals be mixt, then after substraction of the Sinisters, the Dexters are to be added; yet so as the Dexter Remain must have respect to the Sinister, and not exceed its due Proportion. See the Examples following.

Examples.

Greater Surds.	√:4+W144	4	√:48+W256	8
Subtrahends.	√:4—W16	0	√:27—W4	5
Remains.	√:0+W256	4	√:3 +W36	3
Greater Surds.	√:48—W144	6	√:50—W1	7
Subtrahends.	√:12+W16	4	√:2 +W4	2
Remains.	√:12—W64	2	√:32—W49	5

7. Sq. Universals halfed.

7. If any Square Universal be to be halved, divide the Sinister by 4, the next Dexter by 16, and the next by 256, &c. And if Absolute Numbers be intermixed, they are to be divided by the next Sinister Divisors before them.

Example.

As to half √:12+W480+W9216, dividing accordingly by 4.16.256. there will be brought forth the √:3+W30+W36, for the half of the former.

8. To shorten the Division in such Substractions.

8. If any Square Universal be divided by 2.4.16, &c. orderly, and the Absolute Numbers, if any, as the Sinisters next before them, the Quotient shall be an Universal, like those in the last Direction mentioned in *Addition*. And if the Root be gotten, the Sum or Difference at last must be divided by 2, and the Root of the Quotient taken for the Remain.

Example.

As if √:12+W480+W9216 be thus divided, the Quotient will be thus, √:6+W120+W576: then 24 the Root of 576 brought and added to 120, make 144, whose Root 12 added to 6 make 18; but this 18 must be divided by 2, and the Root of 9 the Quotient taken.

Whence this last Work proceeds.

This and the precedent Work take their Original from the first Consectary in Chap. 6. *Division of Simple Surds*; and according thereto may higher Universal *Surds* be halfed, &c. But these are fitted only for Squares, others being seldom used, as before noted in *Addition*.

Proof of Substraction of Compound Surds.

The Proof of *Substraction of Compound Surds*, is like the Proof of *Simple Substraction*, either by *Addition*, or by taking Rational Numbers instead of the *Surds*, and working therewith. For making the Total of any *Addition*, the Number from

from which *Substraction* is made, and one of the Numbers added the Subtrahend: then shall the Remain be the other Addend, and so *vice versa*; remembring Particular to try Particular, and Universal, Universal; of which Instances are spared here, forasmuch as many of the Examples in this Chapter are so ordered, as they prove the Additions of the former: And divers of the Examples being of Rational Numbers, may be Instances sufficient without farther explanation.

CHAP. IX.

Multiplication of Compound Surds.

TO multiply *Compound Surds*, let them be considered as they are, Particular or Universal. *Compound Surds multiplied.*

Multiply the Numbers of Particular *Compound Surds*, as Simple *Surds*, like with like, or reduced thereunto; and let the Signs + and — be ordered as in *Multiplication of Compound Cossicks*, for like Signs give +, and unlike —. *Particular.*

Example of Binomials.

Examples of Binomials.

Multiplicand. W26+W3
Multiplier. W5
Product. W130+W15

Multiplicand. 5+W10
Multiplier. 5+W10
25+W250
W250+10
Product. 35+W1000

Multiplicand. 23+W15
Multiplier. 6+W8
138+W540
W4232+W120
Product. 138+W540+W4232+W120

Multiplicand. W120+W12
Multiplier. W12 +W7
W1440+12
W840+W84
Product. 12+W1440+W840+W84

The Multiplier being a Simple *Surd*, tho the Multiplicand a Compound, there is no Difficulty or Difference from Simple Multiplication.

The Absolute Numbers multiplied make 25, and W10 by W10 make 10 Absolute Nnmbers, the 5 squared is 25; which multiplied by 10 is 250, and adding W250 to W250, the Total is W1000, that is 250 multiplied by 4.

After Multiplication of the Absolute Numbers 23 and 6, and the two *Surds* which make W120, the Absolute Numbers are squared; and so multiplied alternately into the other *Surd*, all which collected make the Total Product.

Here W120 multiplied by W12, the Product is W1440; and W12 by W12, gives 12 Absolute Numbers by cancelling W, as was taught in Simple *Surds*; the other Products are plain.

Example of Residuals.

Examples of Residuals.

Multiplicand. W26—W3
Multiplier. W 5
Product. W130—W15

Multiplicand. 10—W5
Multiplier. 10—W5
100—W500
—W500+5
Product. 105—W2000

Multiplicand. 23—W15
Multiplier. 6—W8
138—W540
—W4232+W120
Product. 138—W540—W4232+W120

The Multiplicand being a Compound, but the Multiplier Simple, there is no Difficulty nor Difference from Simple Multiplication.

The Absolute Numbers multiplied, produce 100, and W5 by W5, make 5. Absolute Numbers W being cancelled, the rest of the Work is like the second Example of *Binomials* above.

The Work in this Example is like that in the third Example of *Binomials* above, only altered in the Signs. And W120 which is + may be set next to 138; which is of the same Nature, and so the Product will stand thus;

138+W120—W540—W4232.

Multiplicand.

Multiplicand. W24 —W20
Multiplier. W30 —W24
W720—W600
—24+W480
Product. W720+W480-24-W600

In this Example W24xW30 produceth W720, and W30xW20, makes W600; but W24xW24, giveth 24 Absolute Numbers, cancelling the Character; and W24xW20, makes W480.

Examples of mixt.

Examples of Binomials with Residuals.

Multiplic. 6+W3
Multiplier. 6—W3
36+W108
—W108—3
Product. 33

After multiplication of the Absolute Numbers, whose Product is 36, the *Surds* are multipli'd into the square of 6, which alternately produce +W108 & -W108, and in the addition both cancelled being of contrary Signs, and 3 which is – coming of W3xW3, is taken from 36.

Multiplic. W52+17
Multiplier. 17—W52
W15028+289
—15028-52
Product. 237

Here the Square of 17 multiplied into W52, makes in both places W15028, the one + and the other —, according to the Signs. And 17x17 gives 289; and W52xW52 produceth 52, cancelling the Character.

Multiplic. W124—6
Multiplier. 32+W14
W126976—192
W1736—W504
Product. W126976+W1736-W504-192

The Work in this Example is like the last before, and the total Product is set at large.

Multiplic. W32—3
Multiplier. W 8+2
W256—W72
W128—6
Product. W256+W8 —6

The Multiplication ended in collecting the Total W72 is taken from W128, and the Remain W8 set down.

8) W128—W72
16 9
4√—3√
1
1
8
W8

Polynomial *multiplied squarely, how changeth a Name.*

From hence it is obvious, that if a *Multinomial* be multiplied into it self, with one of his Signs changed, the Product shall be purged of one Name.

Example.

As 3+W5+W2 multiplied by 3+W5—W2, the Product shall be 12+W180.

Multiplic. 3+W5+W2
Multiplier. 3+W5—W2
9+W45+W18
W45+5+W10
—W18—W10—2
9+W180+5—2
Product. 12+W180

45) W45+W45
1 1
1√+1√
2
4
45
W180

W45
4
W180

Universals Homogeneal how multiplied.

Multiply the Numbers of Universal *Surds* that are Homogeneal, as Compound *Surds*, alternately one into the other, with this difference, that the Sinister Numbers be figurate, according to the Denomination of the Dexter into which they are multiplied; And the particular Roots of all the Dexter Multiplees added to the Product of the two Sinister Numbers, and the Root of the Total shall be the Product in Absolute Numbers; which if thereby cannot be expressed, is to be marked accordingly with its proper Character.

Example.

Example, to multiply the Root Universal of W2+W49, by the Universal Root of W3+W36; after multiplication of 3 by 2, both 3 and 2 are squared before they are respectively multiplied into 49 and 36, and then 49 and 36 are multiplied together; the Roots of all which Dexter Multiplees are added to 6, and the Total makes 81, which is a Square, and the Root 9 Absolute Numbers is the Product desired.

Multiplicand.

Multiplicand.	√:2+W49		ʒ √
Multiplier.	√:3+W36		√ 2 × 3 ===== 6
	6+W441		3 . 9 × 49 = 441 | 21
	W144+W1764		
Product.	√:6+W441+W144+W1764		2 . 4 × 36 = 144 | 12
Roots.	6+ 21 + 12 + 42		49 × 36 = 1764 | 42
Total Product.	W81 = 9 Abſolute Numbers.		81

If the given Numbers be *Heterogeneal*, then beſides the Reduction of the one to the other, as in Compounds before, figurate the Siniſter *Surd* according to the Reduction of the Dexter, and then proceed as in other Univerſals. *Univerſals Heterogeneal, how multiplied.*

Example, to multiply the Univerſal Root of W23+ww8, by the Univerſal Root of W2+ww343. Firſt, 23 is multiplied by 2, for they are *Homogeneal*, that is, both Squares. Then W2 is reduced with ww8 to the √ʒφ8, and √ʒφ64: And moreover, becauſe 64 the Dexter Number is now of the Zenzicube Denomination, 2 the Siniſter Number ſhall be made a Zenzicube alſo, which is 64. So ſhall 64 be multiplied by 64, and the Product with his Character ſet down, which is √ʒφ4096. Likewiſe W23 and ww343 reduced, make the Dexter Number √ʒφ117649; which is to be multiplied by 148035889 the Zenzicube of 23, and the product which is the √ʒφ17416274304961 ſet down. Afterward ww8 and ww343, both Homogeneal Cubes, are multiplied, and the Roots ſeverally taken and added to the Siniſter Product 46; the Total is W225, which in Abſolute Numbers is 15. *Example.*

Multiplicand.	√:23+ww8
Multiplier.	√: 2+ww343
	46+√ʒφ4096
	√ʒφ17416274304961+ww2744
Product.	√:46+√ʒφ4096+√ʒφ17416274304961+ww2744
Roots.	46 + 4 + 161 + 14
Total Product.	W 225 = 15 Abſolute Numbers.

If the Univerſal Root be ſtrictly taken, then the Root of theſe laſt Numbers muſt be taken for the Product; which in the former Example would be 3, the Square Root of 9, in this Example W15.

As the Multiplication of Simple *Surds* takes for the Proof thereof, either Simple Diviſion or Rational Numbers; ſo doth Multiplication of Compound *Surds*, Particular and Univerſal, prove it ſelf either by the reſpective Diviſions, *viz.* Particular by Particular, and Univerſal by Univerſal, as in the next Chapter is exemplified; or by working with Rational Numbers, as in the Inſtance following. *Proof of Multiplication of Compound Surds.*

Suppoſe the Roots, Particular or Univerſal of W4+ww125 were to be multiplied by themſelves, the W4 being 2, and ww125 being 5, added, make the Sum of their Particular Roots 7; which multiplied by it ſelf is 49.

But if ww125 which is 5, be added to 4, the Sum is 9, of which the Square Root is 3 for the Root Univerſal, and 3 multiplied into it ſelf is 9, for the Product of the Univerſal Roots; accordingly by the Work thereof are thoſe Numbers produced.

And both if ſtrictly taken muſt be the Roots of theſe Roots, to wit, 7 and 3 the Roots of 49 and 9.

W4+ww125	2+5=7	√:4+ww125	4+5=9 √3
W4+ww125	2+5=7	√:4+ww125	4+5=9 √3
W16+√ʒφ1000000	49	16+√ʒφ64000000	9
√ʒφ1000000+ww15625		√ʒφ64000000+ww15625	
4+10+10+25		16+20+20+25	
49		W81 = 9	

CHAP. X.

Division of Compound Surds.

Compound Surds divided.

TO divide *Compound Surds*, let them also as in *Multiplication*, be considered as they are Particular or Universal.

Particular.

Division of Particular *Compound Surds*, hath sometime the given Divisor Simple, sometime the Dividend Simple, and sometime both Compound: The Varieties of whose Divisions are comprised in the following Cases.

1. Divisor Simple, or an Absolute Number.

Case 1. When the Divisor given is a Simple *Surd*, or Absolute Number, and the Dividend Compound; divide by the Divisor the several Numbers of the Dividend that are of like Denomination; and if any be unlike, reduce the Divisor to the same Denomination, and then divide thereby. And order the Signs + and — as in *Division of Compound Cossicks*; for like Signs give + and unlike —.

Examples of Binomials.

Examples of Binomials.

Dividend. $\sqrt{130}+\sqrt{15}$ ($\sqrt{26}+\sqrt{3}$ *Quotient.*
Divisor. $\sqrt{5}$

Dividend. $\sqrt{56}+\sqrt{24}$ ($\sqrt{9\frac{1}{3}}+\sqrt{4}$ *Quotient.*
Divisor. $\sqrt{6}$

Residuals.

Examples of Residuals.

Dividend. $\sqrt{130}-\sqrt{15}$ ($\sqrt{26}-\sqrt{3}$ *Quotient.*
Divisor. $\sqrt{5}$

Dividend. $\sqrt{56}-\sqrt{24}$ ($\sqrt{9\frac{1}{3}}-\sqrt{4}$ *Quotient.*
Divisor. $\sqrt{6}$

In all these first 4 Examples there appeareth no Difficulty, every Number being divided as Simple *Surds*, and the Quotients + or — according to the likeness or unlikeness of the Divisor's Sign with the Sign of the Numbers of the Dividend.

Mixt.

Examples of Binomials with Residuals.

Dividend. $6+\sqrt{10}-\sqrt[3]{16}$ ($3+\sqrt{2\frac{1}{2}}-\sqrt[3]{2}$ *Quotient.*
Divisor. 2

Dividend. $6-\sqrt{10}+\sqrt[3]{16}$ ($3-\sqrt{2\frac{1}{2}}+\sqrt[3]{2}$ *Quotient.*
Divisor. 2

In both these Examples, after the Absolute Numbers are divided; to divide the Squares, 2 is squared, and to divide the Cubes, 2 is cubed.

2. Dividend Simple, or an Absolute Number.

Case 2. When the Dividend given is a Simple *Surd*, or Absolute Number, and the Divisor a Compound Binomial or Residual, so as the one *Surd* be not equal to the other; then multiply the Divisor, if Binomial, by his Residual; if Residual, by his Binomial. And by the same Number the Divisor is multiplied, multiply the Dividend: this Product divide by the Number remaining of the Multiplication of the Divisor, figurate or not, according to the Denominations of the Dividend so multiplied, ordering the Signs as before.

Examples.

Example, to divide the Absolute Number 49 by $4+\sqrt{9}$. First, the Divisor multiplied by his Residual $4-\sqrt{9}$, produceth 7; which figurated or set as a *Surd*, is $\sqrt{49}$; then the Dividend multiplied by $4-\sqrt{9}$, doth produce $196-\sqrt{21609}$; whereof 196 being Absolute Numbers, divided by 7, the Absolute Number remaining by multiplication of the Divisor, and $\sqrt{21609}$ divided by $\sqrt{49}$, give in the Quotient $28-\sqrt{441}$, as at A.

And if 49 be divided by $4-\sqrt{9}$, the Quotient will be alike in Numbers, only the Sign contrary, *viz.* $28+\sqrt{441}$, as at B.

But if 49 be divided by $\sqrt{9}+4$, the Quotient will be $\overline{\sqrt{441}}+28$, as at C.

And if 49 be divided by $\sqrt{9}-4$, the Quotient will be $\overline{\sqrt{441}}-28$, as at D.

A

	A	B	C	D
	Divisor.	*Divisor.*	*Divisor.*	*Divisor.*
	Binomial. $4+\sqrt{9}$	*Residual.* $4-\sqrt{9}$	*Binomial.* $\sqrt{9}+4$	*Residual.* $\sqrt{9}-4$
	Residual. $4-\sqrt{9}$	*Binomial.* $4+\sqrt{9}$	*Residual.* $\sqrt{9}-4$	*Binomial.* $\sqrt{9}+4$
	$16+\sqrt{144}$	$16-\sqrt{144}$	$9+\sqrt{144}$	$9-\sqrt{144}$
	$-\sqrt{144}-9$	$\sqrt{144}-9$	$-\sqrt{144}-16$	$\sqrt{144}-16$
Product.	$16-9$	$16-9$	$9-16$	$9-16$
Remain.	7	7	-7	-7
Figurated.	$\sqrt{49}$	$\sqrt{49}$	$-\sqrt{49}$	$-\sqrt{49}$
	Dividend.	*Dividend.*	*Dividend.*	*Dividend.*
	49	49	49	49
	$4-\sqrt{9}$	$4+\sqrt{9}$	$\sqrt{9}-4$	$\sqrt{9}+4$
Product.	$196-\sqrt{21609}$	$196+\sqrt{21609}$	$\sqrt{21609}-196$	$\sqrt{21609}+196$

Quotients. *Quotients.*

$$\frac{196\mp\sqrt{21609}}{7 \quad \sqrt{49}} \Big(28\mp\sqrt{441} \begin{cases} A \\ B \end{cases} \qquad \frac{\sqrt{21609}\mp 196}{-\sqrt{49} \quad -7} \Big(\mp\sqrt{441}\pm 28 \begin{cases} C \\ D \end{cases}$$

Another Example, with the Varieties in dividing a Rational Number; the like of which is to be done where the Dividend is a *Surd*. As suppose $\sqrt{81}$ be divided by $3+\sqrt{4}$, or $3-\sqrt{4}$, or $\sqrt{4}+3$, or $\sqrt{4}-3$, the several Quotients appear at E. F. G. H. *Other Examples.*

	E	F	G	H
	Divisor.	*Divisor.*	*Divisor.*	*Divisor.*
	Binomial. $3+\sqrt{4}$	*Residual.* $3-\sqrt{4}$	*Binomial.* $\sqrt{4}+3$	*Residual.* $\sqrt{4}-3$
	Residual. $3-\sqrt{4}$	*Binomial.* $3+\sqrt{4}$	*Residual.* $\sqrt{4}-3$	*Binomial.* $\sqrt{4}+3$
	$9+\sqrt{36}$	$9-\sqrt{36}$	$4+\sqrt{36}$	$4-\sqrt{36}$
	$-\sqrt{36}-4$	$\sqrt{36}-4$	$-\sqrt{36}-9$	$\sqrt{36}-9$
Product.	$9-4$	$9-4$	$4-9$	$4-9$
Remain.	5	5	-5	-5
Figurated.	$\sqrt{25}$	$\sqrt{25}$	$-\sqrt{25}$	$-\sqrt{25}$
	Dividend.	*Dividend.*	*Dividend.*	*Dividend.*
	$\sqrt{81}$	$\sqrt{81}$	$\sqrt{81}$	$\sqrt{81}$
	$3-\sqrt{4}$	$3+\sqrt{4}$	$\sqrt{4}-3$	$\sqrt{4}+3$
Product.	$\sqrt{729}-\sqrt{324}$	$\sqrt{729}+\sqrt{324}$	$\sqrt{324}-\sqrt{729}$	$\sqrt{324}+\sqrt{729}$

Quotients. *Quotients.*

$$\frac{\sqrt{729}\mp\sqrt{324}}{\sqrt{25}} \Big(\sqrt{29\tfrac{4}{25}}\mp\sqrt{12\tfrac{24}{25}} \begin{cases} E \\ F \end{cases} \qquad \frac{\sqrt{324}\mp\sqrt{729}}{-\sqrt{25}} \Big(\mp\sqrt{12\tfrac{24}{25}}\pm\sqrt{29\tfrac{4}{25}} \begin{cases} G \\ H \end{cases}$$

Example,

Examples in Cubes.

Example, to divide $\sqrt[3]{27}$, by $4+\sqrt[3]{8}$, or $4-\sqrt[3]{8}$, or $\sqrt[3]{8}+4$, or $\sqrt[3]{8}-4$, the several Quotients are as at I. K. L. M.

I *Divisor.*	K *Divisor.*	L *Divisor.*	M *Divisor.*
Binom. $4+\sqrt[3]{8}$	*Resid.* $4-\sqrt[3]{8}$	*Binom.* $\sqrt[3]{8}+4$	*Resid.* $\sqrt[3]{8}-4$
Resid. $4-\sqrt[3]{8}$	*Binom.* $4+\sqrt[3]{8}$	*Resid.* $\sqrt[3]{8}-4$	*Binom.* $\sqrt[3]{8}+4$
$16+\sqrt[3]{512}$	$16-\sqrt[3]{512}$	$\sqrt[3]{64}+\sqrt[3]{512}$	$\sqrt[3]{64}-\sqrt[3]{512}$
$-\sqrt[3]{512}-\sqrt[3]{64}$	$\sqrt[3]{512}-\sqrt[3]{64}$	$-\sqrt[3]{512}-16$	$\sqrt[3]{512}-16$
Product. $16-4$	$16-4$	$4-16$	$4-16$
Remain. 12	12	-12	-12
Figurat. $\sqrt[3]{1728}$	$\sqrt[3]{1728}$	$\sqrt[3]{1728}$	$\sqrt[3]{1728}$
Dividend.	*Dividend.*	*Dividend.*	*Dividend.*
$\sqrt[3]{27}$	$\sqrt[3]{27}$	$\sqrt[3]{27}$	$\sqrt[3]{27}$
$4-\sqrt[3]{8}$	$4+\sqrt[3]{8}$	$\sqrt[3]{8}-4$	$\sqrt[3]{8}+4$
Prod. $\sqrt[3]{1728}-\sqrt[3]{216}$	$\sqrt[3]{1728}+\sqrt[3]{216}$	$\sqrt[3]{216}-\sqrt[3]{1728}$	$\sqrt[3]{216}+\sqrt[3]{1728}$

Quotients.

$$\frac{\sqrt[3]{1728}\mp\sqrt[3]{216}}{\sqrt[3]{1728}}\left(\sqrt[3]{1}\mp\sqrt[3]{\tfrac{27}{216}}\begin{cases}\mathrm{I}\\ \mathrm{K}\end{cases}\right.$$

Quotients.

$$\frac{\sqrt[3]{216}\mp\sqrt[3]{1728}}{-\sqrt[3]{1728}}\left(-\sqrt[3]{\tfrac{27}{216}}\pm\sqrt[3]{1}\begin{cases}\mathrm{L}\\ \mathrm{M}\end{cases}\right.$$

3\. *Divisor Compound Binomial, but one part equal to the other.*

Case 3. When the Dividend is Simple or Compound, and the Divisor a Compound Binomial, and one *Surd* is equal to the other; then add them together, and by the Total divide the Dividend, as in the first Case if the Dividend be Compound, or otherwise as a Simple *Surd*. And if such Residual be given for a Divisor whose Parts are equal, because of the contrary Signs, the Value thereof is 0, and so no Division can be made thereby.

Example.

Example, to divide $\sqrt{60}$ by $\sqrt{5}+\sqrt{5}$, both added make $\sqrt{20}$, by which $\sqrt{60}$ divided maketh the Quotient $\sqrt{3}$. But if the Divisor had been proposed $\sqrt{5}-\sqrt{5}$, it being clear the Divisor is 0, no Division can be made thereby but nugatory.

Divisor. $\sqrt{5}$
4
$\sqrt{20}$

Dividend. $\sqrt{60}$
Divisor. $\sqrt{20}$ $\left(\sqrt{3}\right.$ *Quotient.*

4\. *Data both Compound.*

Case 4. When both Dividend and Divisor are Compound, and the Divisor a Binomial or Residual; so as the Parts of the Divisor be not equal, as aforesaid: Then proceed as in the second Case before.

Examples.

Examples of Binomials, Residuals, and mixt Surds.

To divide $\sqrt{68}+\sqrt{54}$ by $\sqrt{6}+\sqrt{3}$ Binomial, as at N.
To divide $\sqrt{68}-\sqrt{54}$ by $\sqrt{6}-\sqrt{3}$ Residual, as at O.
To divide $\sqrt{456}-\sqrt{72}$ by $\sqrt{18}+\sqrt{6}$ Mixt, as at P.
To divide $\sqrt{456}+\sqrt{72}$ by $\sqrt{18}-\sqrt{6}$ Mixt, as at Q.

Divisors.

N $\sqrt{6}+\sqrt{3}$ O
$\sqrt{6}\mp\sqrt{3}$
$6\pm\sqrt{18}$
$\mp\sqrt{18}-3$
$6-3$
3
$\sqrt{9}$

Dividends.

N $\sqrt{68}\pm\sqrt{54}$ O
$\sqrt{6}\mp\sqrt{3}$
$\sqrt{408}\pm\sqrt{324}$
$\mp\sqrt{204}-\sqrt{162}$
$\sqrt{408}\pm\sqrt{324}\mp\sqrt{204}-\sqrt{162}$

Quotients.

Dividends. *Divisor.*
$$\frac{\sqrt{408}\pm\sqrt{324}\mp\sqrt{204}-\sqrt{162}}{\sqrt{9}}\left(\sqrt{45\tfrac{1}{3}}\pm\sqrt{36}\mp\sqrt{22\tfrac{2}{3}}-\sqrt{18}\begin{cases}\mathrm{N}\\ \mathrm{O}\end{cases}\right.$$

Divisors.

Divisors.

$$\begin{array}{lll} P & \sqrt{18}\pm\sqrt{6} & Q \\ & \sqrt{18}\mp\sqrt{6} & \\ \hline & 18\pm\sqrt{108} & \\ & \quad\mp\sqrt{108}-6 & \\ \hline & 18-6 & \\ \hline & 12 & \\ \hline & \sqrt{144} & \end{array} \qquad \begin{array}{lll} P & \sqrt{456}\mp\sqrt{72} & Q \\ & \sqrt{18}\mp\sqrt{6} & \\ \hline & \sqrt{8208}\mp\sqrt{1296} & \\ & \qquad\mp\sqrt{2736}+\sqrt{432} & \\ \hline & \sqrt{8208}\mp\sqrt{1296}\mp\sqrt{2736}+\sqrt{432} & \end{array}$$

Quotients.

$$\begin{array}{ll} \textit{Dividends.} & \sqrt{8208}\mp\sqrt{1296}\mp\sqrt{2736}+\sqrt{432} \\ \textit{Divisor.} & \sqrt{144} \end{array} \Big(\sqrt{57}\mp\sqrt{9}\mp\sqrt{19}+\sqrt{3} \begin{cases} P \\ Q \end{cases}$$

Case 5. When the Divisor is a Multinomial, in multiplying him by his Residual, one Part, or all the Parts thereof except one, may be made Residual at pleasure, always provided the Parts made Residual be not equal in Value to the other as aforesaid. And after addition of the Multiplees, rejecting what may be rejected, by reason of contrary Signs, add the Remains of the Total into one Number, if it may be, and proceed as in the second and fourth Cases before in this Chapter. 5. *Divisor a Polynomial.*

Example, to divide 54 Integers by $\sqrt{4}+\sqrt{9}+\sqrt{16}$ Rational Numbers, the Divisor may be multiplied either by $\sqrt{4}+\sqrt{9}-\sqrt{16}$, or $\sqrt{4}-\sqrt{9}+\sqrt{16}$, or $\sqrt{4}-\sqrt{9}-\sqrt{16}$, or $\sqrt{9}-\sqrt{4}-\sqrt{16}$, or by any other Residual to be made, by exchanging the Places of the Numbers in the Divisor. And accordingly the Divisor multiplied by any of them, and the Remains brought into one Number, by this Number I divide the Product of 54, multiplied by the same the Divisor was multiplied. See two of the Varieties at R and S. Example.

$$\begin{array}{ll} & \textit{Divisor.} \\ R & \sqrt{4}+\sqrt{9}+\sqrt{16} \\ & \sqrt{4}+\sqrt{9}-\sqrt{16} \\ \hline & 4+\sqrt{36}+\sqrt{64} \\ & \quad\sqrt{36}+9+\sqrt{144} \\ & \qquad-\sqrt{64}-\sqrt{144}-16 \\ \hline & 4+\sqrt{144}+9-16 \\ \hline & \sqrt{144}-3 \\ \hline & \sqrt{81} \end{array} \qquad \begin{array}{ll} & \textit{Divisor.} \\ S & \sqrt{4}+\sqrt{9}+\sqrt{16} \\ & \sqrt{4}-\sqrt{9}-\sqrt{16} \\ \hline & 4+\sqrt{36}+\sqrt{64} \\ & \quad-\sqrt{36}-9-\sqrt{144} \\ & \qquad-\sqrt{64}-\sqrt{144}-16 \\ \hline & 4-\sqrt{576}-25 \\ \hline & -\sqrt{576}-21 \\ \hline & -\sqrt{2025} \end{array}$$

$$\begin{array}{l} \textit{Dividend.} \\ 54 \\ \sqrt{4}+\sqrt{9}-\sqrt{16} \\ \hline \sqrt{11664}+\sqrt{26244}-\sqrt{46656} \end{array} \qquad \begin{array}{l} \textit{Dividend.} \\ 54 \\ \sqrt{4}-\sqrt{9}-\sqrt{16} \\ \hline \sqrt{11664}-\sqrt{26244}-\sqrt{46656} \end{array}$$

Quotient.

$$\frac{\sqrt{11664}+\sqrt{26244}-\sqrt{46656}}{\sqrt{81}}\Big(\sqrt{144}+\sqrt{324}-\sqrt{576}. \qquad R$$

Quotient.

$$\frac{\sqrt{11664}-\sqrt{26244}-\sqrt{46656}}{-\sqrt{2025}}\Big(-\sqrt{5\tfrac{1539}{2025}}+\sqrt{12\tfrac{1944}{2025}}+\sqrt{23\tfrac{81}{2025}} \qquad S$$

Case 6. When both Dividend and Divisor are Multinomials, and the Remains of the Total after Multiplication of the Divisor will not be added into one Number as above-mentioned: then divide your given Numbers after the manner of Compound Cossicks, placing the *Surds* orderly; and by every Number gotten for the Quotient, multiply the Divisor, and substract the Product from the Dividend. 6. Data *both Polynomials.*

Example, to divide $9+\sqrt{180}+5-2$ by $3+\sqrt{5}+\sqrt{2}$; at the first Application of the Divisor 3 being gotten for the Quotient, I multiply the Divisor thereby, and the Product is $9+\sqrt{45}+\sqrt{18}$; which substracted from the Dividend, leaves $\sqrt{45}-\sqrt{18}$. Then by Application of the Divisor is gotten for the Quotient $+\sqrt{5}$, by which the Divisor multiplied, produceth $\sqrt{45}+\sqrt{25}+\sqrt{10}$; Example.

this substracted from the Dividend, leaves —W18—W10. And lastly, applying the Divisor, —W2 is gotten for the Quotient, and the Product of the Divisor multiplied thereby is —W18—W10—W4; which substracted leaves 0 remaining, and the Work stands as at W; the other Paragraphs thereof as at T and V.

```
                                      Dividend.
                                     W 45—W18                 T
Divisor.  3+W 5+W 2 )9+W180+5—2( 3                            Quotient.
          3          )9+W 45+W18(
          ____________
          9+W45+W18

                                      Dividend.
                                   W45—W18—W10                V
Divisor.  3+W 5+W 2 )9+W180+5—2( 3+W5                         Quotient.
          3+W 5      )9+W 45+W18(
          ____________
          9+W45+W18       W45+5+W10
            W45+5+W10

                                      Dividend.                W
                                   W45—W18—W10
Divisor.  3+W5 +W2  )9+W180+5—2( 3+W5—W2                      Quotient.
          3+W5 —W2  )9+W45 +W18(
          ____________
          9+W45+W18       W45 +5+W10
            W45+5+W10         —W18—W10—2
              —W18—W10—2
```

When Names are increased and changed in Division.

Thus Division of particular *Surds* give evidence, that every Compound Divisor, by the Work of the 2*d*, 4*th* and 5*th* Cases, increaseth the Names in the Quotient; the one a Binomial, makes the other a Residual, and the Residual begets a Binomial, although the Dividend be single. But if the Dividend be Compound as well as the Divisor, the Quotient shall be a Multinomial. And great heed is to be taken in right placing the Signs, for that according thereto the Quotient is to be valued; all which is plainly to be observed in the foregoing Examples.

Heed to be took in placing the Signs.

Universals Homogeneal, how divided.

Division of Universal *Surds* Homogeneal, imitate the Division of Particular *Surds*, only before the Quotient is to be prefixed the Universal Sign. And if Operation be made according to the sixth Case; then let the Sinister Number of the Divisor upon every Removal be figurate, as the next Dexter Number he is applied to. And if you proceed according to the fourth Case, then in multiplying the Numbers, the Multiplication proper to Universal *Surds* is to be observed; and besides, if the Divisor be Negative, the Order of the Quotient is inverted; for then the Sinister Number of the Quotient shall be substracted from the Dexter, and the Root of the Remainder taken for the Number desired.

Examples.

Example, to divide √:6+W144+W441+W1764 by √:3+W36, upon application of 3 in the Divisor to 6 in the Dividend, 2 is gotten for the Quotient, by which multiplying the Divisor, the Product is 6+W144: Then 3 is squared, and by applying the Product or Square which is 9, to 441, there is W49 gotten for the Quotient; which multiplying the Divisor, produceth W441+W1764, and substracted, leaves 0 remaining. And the Division according to the sixth Case stands as at X. And if Division were made by √:2+W49, the Numbers should be placed thus, √:6+W441+W144+W1764.

But if nine Absolute Numbers (which is the Root Universal of √:6+W144 +W441+W1764) be divided by √:3+W36, according to the 4*th* Case; then multiplying √:3+W36 by his Residual √:3—W36, the Numbers to be used for Divisors will be —27 and —W729. And then √:3—W36 multiplying 9, makes the Dividend 243—W236196, and the Quotient will be —9+W324 as at Y. Then because the Divisor is Negative, 9 shall be taken from the Root of 324, and the square Root of the Remainder, which here will be 3, shall be the Universal Root desired, equivalent, though in other Terms, to the Root of the Quotient at X.

Divisor.	*Dividend.*	*Quotient.*	
√:3+W36)	√:6+W144+W441+W1764	(√:2+W49	X
√:2+W49	6+W144		
6+W144	W441+W1764		
W441+W1764			

Divisor.	*Dividend.*	*Quotient.*	
√:3+W36	9		
√:3—W36	√:3 — W36		
9+W324	243—W236196		
—W324—36			
9—36			
—27	243——W236196 / —27 —W729	(√:—9+W324	Y
—W729			

Nevertheless where the Divisor by this last Sort is Affirmative, the Quotient shall be in the former order.

Example, to divide 30 by √:30+W36, whose Root Universal is 6, or by √:30 —W25, whose Root Universal is 5, the Quotients will accordingly be 5 or 6, and therewith agree the Quotients at Z. z. if their Roots be extracted. *Other Examples.*

Divisor.	*Dividend.*	*Quotient.*	
√:30 +W36	30		
30 —W36	30 —— W36		
900+W32400	27000—W29160000		
—W32400—36			
900—36			Z
864	27000—W29160000 / 864 W746496	(√:31¼—W39 1/16.	
W746496			

Divisor.	*Dividend.*		
√:30 —W25	30		
30 +W25	30 + W25		
900—W22500	27000+W20250000		
W22500—25			
900—25			z
875	27000+W20250000 / 875 W765625	(√:30 6/7+W26 22/49.	
W765625			

If the given Numbers be Heterogeneal, then proceeding by the *6th* Case, besides upon Removal of the Divisor to figurate the Sinister Number thereof, according to the Dexter Number he is applied to; if the Quotient of this Division be of higher Denomination than the next Dexter Number of the Divisor, then let this Quotient be depressed thereto by extracting the Root, and this Root with his proper Character shall be set in the Quotient of the first Division. *Universals Heterogeneal, how divided.*

Example. If √:4+√3ф4096+W16+√3ф4096, the Product of √:2+WW8 by √:2+W4, be divided by the first of the said Factors, then upon the Application of the Divisor 2 to W16 he is to be squared; and the Quotient being a Square, and so not above the Dexter Number of the Divisor, which is of the Cube Denomination, this Quotient shall stand for the Quotient desired. But if Division be made by √:2+W4 the other Factor, then upon application of 2 to √3ф4096, after 2 is exalted to the Zenzicube Power, and 4096 divided thereby, the Quotient 64 being a Zenzicube, (and so above the Denomination of the next Dexter *Examples.*

Dexter Number of the Divisor which is a Square) is to be depressed to a Cube by extracting the Square Root thereof.

Divisor. *Dividend.* *Quotient.*

√:2+ww8) √:4+√ʒ ɸ4096+W16+√ʒ ɸ4096 (√:2+W4

New Divisor. 2×2=W4) 16 (4

Divisor. *Dividend.* *Quotient.*

√:2+W4) √:4+W16+√ʒ ɸ4096+√ʒ ɸ4096 (√:2+ww8

2×ʒ ɸ=64) 4096 (64√ʒ whereof is 8.

If the Universal Root were strictly to be taken in all these Instances, then at X and Y it would be W1. at Z $\frac{W5}{W6}$. at z W1⅕, and in the two last Examples W1.

X $\frac{W3}{W3}$ (W1 Y

Z $\frac{W5}{W6}$ ($\frac{W5}{W6}$

z $\frac{W6}{W5}$ (W1⅕

$\frac{√:2+W4=2}{√:2+ww8=2}$ (W1

Proof of Division of Compound Surds.

As the Proof of Division of Simple *Surds* is by their Simple Multiplication, or by Rational Numbers; so will Division of Compound *Surds* be proved by Compound Multiplication, Particular by Particular, and Universal by Universal respectively; multiplying the Quotient by the Divisor to return the Dividend: And also by working with Rational Numbers.

Among the Examples of Particular Surds in this Chapter at A, 49 was divided by 4+W9, which is 7; so shall the Quotient of the Division be 7; the Quotient being 28—W441, multiplied by the Divisor, shall return 49, and the Square Root of 441, which is 21 taken from 28, shall leave 7.

Among the Examples of Universal *Surds* at X, the Root Universal of √:6+W144+W441+W1764 is 9, and the Root Universal of the Divisor √:3+W36 is 3; so shall the Quotient of that Division be 3, the Quotient at X being √:2+W49 agrees: for if the Square Root of 49 which is 7 be added to 2, it makes 9, whose Square Root is 3. As likewise doth the Quotient at Y, for from the Square Root of 324 which is 18, let 9 be taken, and the Square Root of the 9 remaining is also 3. And if those Quotients be multiplied by the Divisor, the Dividends will respectively be returned, as that at X is to be seen in the foregoing Chapter, the other here follow.

Divisor.	*Dividend.*	*Quotient.*	
4+W9)	49		
4—W9	4—W9		
16—9	196—W21609 (	28—W441 (√ 21	4+3=7) 49 (7
7	7 W49	4+W9	28—21=7
W49		112—W7056	
		W7056—W3969 (√ 63	
		112—63	
		49	

Universal Root.

Divisor.	*Dividend.*	*Quotient.*		
√:3+W36)	√:6+W144+W441+W1764 (	√:2+W49	W81 or 9 (	W9 or 3.
or W9	or W81	or W9	W9 or 3	

Divisor.	*Dividend.*	*Quotient.*
$\sqrt{:3+\sqrt{36}}$)	9	
$3-\sqrt{36}$	$3-\sqrt{36}$	
$9-36$	$243-\sqrt{236196}$	($\sqrt{:-9+\sqrt{324}}$ ($\sqrt{18}$
-27	$-27-\sqrt{729}$	$3+\sqrt{36}$
$-\sqrt{729}$		$-27+\sqrt{2916}$
		$-\sqrt{2916}+\sqrt{11664}$ ($\sqrt{108}$
		$-27+108$
		$\sqrt{81}=9$

$\sqrt{9}=3$) 9 (3

$\sqrt{18-9}=\sqrt{9}=3$

CHAP. XI.

Of Fractionary Surds.

Although little mention of Fractions hath been made before in this Part of *Surds*, yet they oft-times arise upon Division of *Surds*, and sometimes with and without Integers are useful among *Surds*: Wherefore it is necessary to remember, that the Operations proper to common Fractions, mixed with the Operations proper to *Surds*; shall add and substract, multiply and divide them, whether Simple or Compound; so as the particular Rules of their several Elements, being nothing but what hath been before shewed, need no further be spoken to here than to give a few Examples. *Fractionary Surds, how to be ordered.*

Fractionary Surds added and substracted. *Examples of Addition and Substraction.*

Examples. $\left.\begin{array}{l}\sqrt{\frac{8}{10}}\\ \sqrt{\frac{72}{4}}\\ \sqrt{\sqrt{\frac{7}{12}}}\end{array}\right\}$ is added to, and substracted from $\left\{\begin{array}{l}\sqrt{\frac{50}{10}} \text{ at A and B.}\\ 32 \text{ Integers at C and D.}\\ \sqrt{\sqrt{\frac{56}{12}}} \text{ at E and F.}\end{array}\right.$

A . B	C . D	E . F
$\sqrt{\frac{50}{10}}+\sqrt{\frac{8}{10}}$	$32+\sqrt{\frac{72}{4}}$	$\sqrt{\sqrt{\frac{56}{12}}}+\sqrt{\sqrt{\frac{7}{12}}}$
$-$	$-$	$-$
2) $\sqrt{50}$ ($\sqrt{25}$. 5	8) $\sqrt{128}$ ($\sqrt{16}$. 4	7) $\sqrt{\sqrt{56}}$ ($\sqrt{\sqrt{8}}$. 2
$\sqrt{8}$ ($\sqrt{4}$. 2	$\sqrt{72}$ ($\sqrt{9}$. 3	$\sqrt{\sqrt{7}}$ ($\sqrt{\sqrt{1}}$. 1
A. $\sqrt{\frac{98}{10}}$ $\sqrt{49}$. 7	C. $\sqrt{\frac{392}{4}}$. $\sqrt{49}$. 7	E. $\sqrt{\sqrt{\frac{189}{12}}}$. 27 . 3 *Sum.*
B. $\sqrt{\frac{18}{10}}$. $\sqrt{9}$. 3	D. $\sqrt{\frac{8}{4}}$. $\sqrt{1}$. 1	F. $\sqrt{\sqrt{\frac{7}{12}}}$. 1 . 1 *Difference.*

Fractionary Surds *multiplied and divided.* *Examples of Multiplication and Division.*

Examples, $\left.\begin{array}{l}\sqrt{\frac{5}{9}}\ldots\ldots\\ 16 \text{ Integers}\\ \sqrt{\sqrt{\frac{2}{9}}}\ldots\ldots\end{array}\right\}$ is multiplied and divided by $\left\{\begin{array}{l}\sqrt{\frac{5}{7}} \text{ at G and H.}\\ \sqrt{2\frac{1}{4}} \text{ at I and K.}\\ \sqrt{\sqrt{1\frac{1}{3}}} \text{ at L and M.}\end{array}\right.$

G. $\sqrt{\frac{5}{9}}\times\frac{5}{7}=\sqrt{\frac{25}{63}}$ I. $\overset{\sqrt{256}\ (64}{\frac{16}{1}}\times\sqrt{2\tfrac{1}{4}}=\sqrt{\frac{576}{1}}$ L. $\sqrt{\sqrt{\frac{2}{9}}}\times\sqrt{\sqrt{1\tfrac{1}{3}}}=\sqrt{\sqrt{\frac{8}{27}}}$ Product.

H. $\sqrt{\frac{5}{7}}$) $\sqrt{\frac{5}{9}}$ ($\sqrt{\frac{7}{9}}$ K. $\sqrt{2\tfrac{1}{4}}$) $\sqrt{256}\ \frac{16}{1}$ ($\sqrt{\frac{1024}{9}}$ M. $\sqrt{\sqrt{1\tfrac{1}{3}}}$) $\sqrt{\sqrt{\frac{2}{9}}}$ ($\sqrt{\sqrt{\frac{1}{6}}}$ Quotient.

Proof of the Fractionary Operations.

Common with other Fractions are these Operations of *Fractionary Surds* proved, *Addition* by *Substraction*, *Multiplication* by *Division*, and the contrary. Proper to *Surds*, the truth of their Fractionary Operations may be proved, by taking Rational Numbers, and working therewith as in other *Surds*, as may be easily examined.

CHAP. XII.

Figuration of Surds.

Figurate Surds produced.

TO figurate *Surds*, is to multiply them after the manner of *Surds*; for any *Surd* multiplied by himself, produceth the Square and Product again, by the Root bringeth forth the Cube, &c. as other Figural Numbers. But this is proper to *Compound Surds*, for any Simple *Surd* multiplied figuratively, produceth a Rational Number, and so ceaseth to be a *Surd*.

Examples.

As to multiply W3 by W3, produceth W9, which is a Rational Number, and hath 3 for the Root.

To multiply squarely the particular Surds W3+W5, the Square produced is 3+W60+5: but the Universal Root squared, as here appeareth, is √:9+W180+W25.

	Simple.		*Particular.*		*Universal.*
Root.	W3	*Root.*	W3+W5	*Root.*	√:3+W5
	W3		W3+W5		√:3+W5
Square.	W9		3+W15		9+W45
			W15+5		W45+W25
		Square.	3+W60+5	*Square.*	√:9+W180+W25

In extraction of Roots to be noted.

Touching the Extraction of the Root of *Surds* is to be minded,

1. Simple Surds can have no Roots. *Rationals have their Roots as before extracted.*

1. That Simple *Surds* having no Roots to be expressed by Integers or Fractions exactly, can have no Root extracted: but the Rational Numbers set as *Surds*, have their Roots extracted as figural Numbers before spoken to in the second Part of the second Book.

And so W2025 shall be 45, and WW729 shall be 9 Integers; because both 2025 and 729 are Rational Numbers, and not proper *Surds*.

Examples.

4 √ √

Rational. W2025 |45 Root. *Rational.* WW729. |9 Root.

16

Gnomon. { 40, 25

2. Particular Compounds, some in a sort Irradical, what then to be done. *When the Sinister Number is Absolute.*

2. That among particular Compound *Surds*, some are in a sort Irradical, and have their Roots extracted only by altering their Characters. This sort have their Sinister Number, either Absolute or a *Surd*.

If Absolute, then place before the given Number the Character belonging to the Root to be extracted.

Examples.

As to extract the Square Root of 10+W5, or the Cube Root thereof, they are set as at A and B.

A B

Square 10+W5 (W10+W5 Root. Cube 10+W5 (WW10+W5 Root.

When the Sinister is a Surd.

If the Sinister Number be a *Surd*, then multiply the Index of the Root to be extracted, by the Index of the Sinister *Surd*, and the Index amounting shall be the Index of the Root, whose Character is to be prefixed before the given *Surd*.

Examples.

As to extract the Square Root of W10+W5, because 2 and 2 make 4, the Index of squared Squares, the Root shall be WW10+W5; sometime set thus, W.W10+W5, and is as much as to say, the Square Root of the Square Root of 10, and the Square Root of 5.

So to take the Square Root of WW18—2, is W.WW18—2. And the like is to be done for the Cube and Higher Powers.

Square

Square w10+w5 (ww10+w5 Root. Square ww18—2 (√ʒφ18—2 Root.
2×2=4 Index ʒʒ 2×3=6 Index ʒφ.

3. Those Extractions of the Roots of particular Compound *Surds*, that are properly Radical, alter the Numbers in Homogeneals, both Numbers and Characters in Heterogeneals. *3. Particular Compounds Radical, what to be done.*

If the Sinister Number be Absolute, and the Dexter a Square *Surd*, then square the Sinister Number, and substract the Dexter Number from it; take the Square Root of the Difference, which add to the Sinister Number, and also substract it therefrom: half the Sum, and half the Difference, joined with the Sign +, shall be the Binomial Root, and with the Sign — shall be the Residual Root. *When the Sinister is Absolute.*

As to extract the Square Root of 7+w40, the Square of 7 is 49; from which 40 taken, leaves 9, whose Square Root is 3, which added to 7, makes 10; the half is 5, and taken from 7, leaves 4; the half is 2: therefore 2+5 shall be the Binomial Root, and 2—5 the Residual Root, or the contrary. Example.

7 Sinister.	7	7	w2+w5 Binomial Root.
7	3	3	w2—w5 Residual Root.
49 ʒ.	10 Sum.	4 Difference.	of 7+w40
40 Dexter.	5 Half.	2 Half.	or 7—w40
9 Difference.			w5+w2 Binomial Root.
3 Root.			w5—w2 Residual Root.

If the given Numbers be Homogeneal square *Surds*, take the one Square from the other, and the Root of the Difference add to the Root of the greater Square, and also substract from it: half the Sum and half the Difference of these Roots joined with the Sign +, shall be the Binomial Root, and with the Sign — shall be the Residual Root as before. *When the Sinister is a Surd.*

For suppose the Numbers given were w49+w40, or w40+w49: then 40 taken from 49, leaves 9; whose Root 3 added to and substracted from 7, the Root of 49, the Greater, makes the Sum and Difference, and consequently the Halfs, and the Binomial and Residual Roots as before, because the Product of w2+w5, or w5+w2 multiplied squarely, is alike. Example.

49 Greater } Square.	7 Root.	7 Root.	w5±w2 } Roots { Binom.
40 Lesser }	3	3	w2±w5 } { Residual
9 Difference.	10 Sum.	4 Difference.	of w49±w40
3 Root.	5 Half.	2 Half.	or w40±w49

If the given Numbers be Cubes or Higher Powers, or an Absolute Number, and a Cube, or Higher Power, take the Roots of the Numbers, and work as if they were Square Roots till you get the Halfs as before; then advance the Roots of those Halfs into the Powers of the Denominations given, or the Dexter Denomination, if but one. *When the Data are Cubes or Higher Powers.*

As if the Root be desired of 13+ww1728, or ww2197+ww1728, the Cube Root of 2197 is 13, and of 1728 is 12; both Roots squared are 169 and 144, the Difference 25, whose Square Root 5 added to 13, makes 18, the Half thereof is 9, whose Root is 3; the said 5 taken from 13, leaves 8, the half whereof is 4 whose Root is 2: these Roots 2 and 3 advanced to be Cubes, according to the Denominations given, are the desired Numbers. Examples.

```
            √      ʒ
  ww√ 2197 |13×13=169        13
        1  :                  5        13        ww√8+ww√27 Binom.Root.
                             —          5        ww√8—ww√27 Resid. Root.
        ⎧9 :                 18 Sum.   —
Gnomon ⎨27 ·                  9 Half.   8 Differ.       of 13+ww√1728
        ⎩...27                3 Root.   4 Half.         or 13—ww√1728
            √      ʒ         —          2 Root.
  ww√ 1728 |12×12=144        27φ       —             ww√27+ww√8 Binom.Root.
        1  :           25 Difference.   8φ           ww√27—ww√8 Resid. Root.
        ⎧6 :            5 Root.
Gnomon ⎨12 ·
        ⎩...8
```

When the Sinister is Absolute, and the Dexter an Higher Power. If the Sinister Number be Absolute, and the Dexter an Higher Power than a Square, different Roots may be had according to the Powers into which the Roots of the Halfs be exalted; for the same Value in the Roots produce Products of like Value, though different in Terms.

Example. As in the last Example, where the Roots of the Halfs were 3 and 2, let them be both squared, as W√9+W√4, or the one squared and the other cubed, as W√9+ww√8, or ww√27+W√4; any of these shall produce Products alike valuable with the Product of ww√27+ww√8, but all different in Terms.

When the Dexter higher Power may have the Index halfed. Yet if the Higher Power on the Dexter Part of the given Number may have the Index evenly halfed, it is most usual to exalt the Root of one half into the Power answering to half the Index of the Dexter Number, and take the other half for the next Inferior Power thereto.

Example. As suppose the Binomial and Residual Roots be desired of 29+√ʒφ64000000, the Zenzicube Root of 64000000 is 20; which squared is 400, and 29 squared is 841, the Difference is 441, whose square Root is 21; this added to and substracted from 29, makes the Sum 50, the Difference 8, whose Halfs are 25 and 4, their Roots 5 and 2: Of which may be made several Roots, but most usual thus; because the Index of ʒφ which is 6, may be halfed, 3 the Index of Cube shall be taken for one Root; then shall the other half be a Square, which is next inferior to the Cube.

```
              ʒφ       φ    √
29+√ʒφ64000000 |8000|20                   W√4+ww√125 Binomial ⎫Root.
                                          W√4—ww√125 Residual ⎭
                  ʒ                 29          Or,
29 × 29 = 841          29           21    W√25 + ww√8 Binomial ⎫Root.
20 × 20 = 400          21          —      W√25 — ww√8 Residual ⎭
          ——— √        ——           8
          441 |21      50           4 Half.
                       25 Half.     2 √
                        5 √
```

4. Universal Surds, how to extract their Roots. When the Sinister is Absolute. 4. The Roots of Universal *Surds* are extracted in all things like those particular Compound *Surds*, properly Radical, last spoken to, till you come to the Halfs: Then for those that are Homogeneal, the Root of half the Sum shall be the one Root desired, and the half Difference the other, this Dexter, and that Sinister. But if the Root of the half Difference be made the Sinister, then the half Sum shall be the Dexter.

Example. As to extract the Root Universal of 25+W√576, the Square of 576 taken from 625 the Square of 25, leaves 49 whose Root is 7; which added to 25 makes 32, whose half is 16, and the Root thereof 4 for the one Root sought: Then 7 taken from 25 leaves 18, whose half is 9 for the other Root sought. Or if 3 the Root of 9 be the Sinister, then 16 shall be the Dexter.

25

25 + w576	25	25	√:4 + w9 } Root { Binomial.	
25	7	7	√:4 — w9 } Root { Residual.	
125	32 Sum.	18 Difference.		
50	16 Half.	9 Half.	√:3 + w16 } Root { Binomial.	
625	4 Root.	3 Root.	√:3 — w16 } Root { Residual.	
576				
49 Difference.				
7 Root.				

If the Root of Universals be sought, that have the Sinister Number given Absolute, and the Dexter an higher Power than a Square: Then proceed as above till the Roots of the Halfs be obtained, and thereby different Roots may be had according to the Powers into which the Roots of the Halfs are advanced. But most usual it is to exalt the Root of one half into the Power answering to half the Index of the Dexter Denomination given, if the same Index may be equally halfed: And take the Root of the other half, with the Character of the next inferior Power thereto, for the other part of the Root desired. *When the Dexter is an higher Power.*

As if the Roots Binomial and Residual be desired of 41 + √ʒφ4096000000, the Zenzicube Root of 4096000000 is 40, which squared is 1600, taken from 1681 the Square of 41 leaves 81, whose Root is 9; which added to and substracted from 41, makes the Sum 50, the Difference 32, the Halfs whereof are 25 and 16, whose square Roots are 5 and 4. Then because the Index of Zenzicube, which is 6, may be equally halfed into 3, the Index of the Cube, either 5 or 4, may be cubed, and the other that is not shall be set with the Character belonging to the Square. *Example.*

41 + √ʒφ4096000000 | 64000 | 40 (ʒφ, φ, √)			√:4 + ww125 } Root { Binomial.
ʒ			√:4 — ww125 } Root { Residual.
			Or,
41×41 = 1681	41	41	√:5 + ww64 } Root { Binomial.
40×40 = 1600	9	9	√:5 — ww64 } Root { Residual.
81 | 9 √	50	32	
	25 Half.	16 Half.	
	5 √	4 √	

Besides the Proof of Extraction of these Roots by the Production of the *Surds*, and their Production by Extraction, Simple by Simple, Particular by Particular, and Universal by Universal reciprocally: The Truth of all may be tried, by taking Rational Numbers, and working with them instead of the *Surds*; nevertheless for brevity sake Examples thereof are omitted here. *Proof of Figuration of Surds.*

Partis quintæ Libri tertii

FINIS.

The Sixth PART of the Third BOOK.

CHAP. I. *Of Species.*

Species the last Sort of Special Contracts.

SPECIES, as the sixth Sort of Contract Numbers, and Third of those whose Denominations are uncertain, come now to be inspected in the Close of this third Book.

Whence the Name. What Species are. Characters used, and why.

According to the Name, (which with the Latins serveth for the Figure, Form or Shape of any thing) *Species* are Quantities or Magnitudes, denoted by Letters, signifying Numbers, Lines, Lineats, Figures Geometrical, &c. And for avoiding the prolix and often rescription of Words, several Marks or Characters are used therewith for Terms Artificial: Some whereof, most customary, are already specified, *Book* 1. *Part* 1. *Chap*. 3. which being remembred, may save their transcribing here.

The same Letter denotes different Species.

Further let it be remembred here, that A while the Question is in, may signify any Number of Pounds, Yards, Ells, Length, Breadth, &c. But when the Work is ended, and another Question begun, A may denote another Quantity different from the former. And the like is to be understood of B. C. D. or any other *Species*.

Powers how differenced from plain Numbers.

It is also necessary to observe, That sometime to difference Powers (with whom *Species* also converse) from plain Numbers, the Letters for these are commonly Capital, but for those Small. Likewise given Quantities or Numbers, some will have noted with Consonants, and those sought with Vowels; but this is not essentially necessary, so as by any Distinction the *Data* and *Quæsita* be discerned apart.

Form of the Species to be kept while the Question is working.

Moreover, while a Question is working, the Form of the Letters is strictly to be kept: For if A and E be two given Numbers, then AE, or Æ, shall be the Product, Z the Sum, &c. But if *a* and *e* be the given Numbers, then *ae*, or *æ*, shall be the Product, z the Sum, &c.

The same Species differently called.

One and the same Letter in the beginning and end of a Question may be differently called: As in Extraction of Roots and Equations, hereafter spoken to in the 4*th* Book, A in working the Question is called the Supposititious or Quesitious Root; but when the Root is found, or the Question brought to an Equation, A shall be called the Eductitious or Resolved Root.

Figures prefixed make a Number of Quantities, &c.

As every single Letter signifies a certain Quantity or Magnitude, so by prefixing of Figures to their left Hand, there shall be accordingly made a number of Quantities or Magnitudes after the manner of Cossicks. Wherefore if A signify one Yard, B one House, &c. then 10A shall signify 10 Yards, and 19B 19 Houses, &c. But if A signify 10 Yards, B 10 Houses; then shall 10A signify 100 Yards, and 19B 190 Houses; and so of others.

Nature of Species. 1. Whole, and they, Homogeneal, or Heterogeneal.

Touching the Nature of *Species*, they are diversly considered.
First, As they are Whole or Integral, Broken or Fracted.
Integral, as A. B. C. &c. 4A. 5B. 3C. &c. These are also
Homogeneal, as A and A, that is 2A. Or,
Heterogeneal, as A and B, that is A+B.

Broken, & they

Fracted, as $\frac{1}{2}$A. $\frac{A}{B}$. $\frac{3B}{4A}$ or the like. These are also

Commensurable,

Commensurable, as $\frac{BA}{BC}$. $\frac{15BD}{27BD}$ &c. Or,

Or Incommensurable.

Incommensurable, as $\frac{BD}{C}$. $\frac{3AB}{4E}$ &c.

All

Fractions alſo divided as others.

All fracted *Species* may alſo be divided as Common Fractions, into Proper, Improper, and Equal Fractions; and the Proper into Conjunct and Divided, or Fractions of Fractions, as *Book* 1. *Part* 2. *Chap.* 1.

Examples of Fractions.

Examples of
- Equal, as $\frac{A}{A}$ or $\frac{B}{B}$. *&c.* which is always an Unit.
- Improper, as $\frac{BD+C}{D}$, *&c.* when the Numerator is the greater *Species*.
- Proper
  - Conjunct, as $\frac{C}{CA}$ and $\frac{B}{CA}$, *&c.*
  - Divided, as $\frac{C}{CA}$ of $\frac{B}{CA}$, *&c.*

  when the Denominator is the greater.

2. Simple, and they

2. *Species* both Integral and Fracted, are conſidered as they are Simple or Compound.

Simple, are ſuch as have ſome one ſingle *Species* and no more, Whether

Whole, or Broken.

Integral, as 6A. 8B. 9C. *&c.* Or,

Fracted, as $\frac{1}{2}$A. $\frac{2}{3}$B. $\frac{3}{4}$C. *&c.* Theſe, as was noted in Coſſicks before, may be repreſented as Compound, by placing the *Species* to the Numerator, and leaving the Denominator ſolitary, for $\frac{3}{4}$A is all one with $\frac{3A}{4}$.

Compound, and they Whole, as Binomials. Reſiduals. Polynomials.

Compound, conſiſts of ſeveral *Species*. And theſe are alſo

Integral, conjoined with the Signs + or —; or both, As

Binomials, A+E . B+C . B+D. *&c.*

Reſiduals, A—E . B—C . B—D. *&c.*

Polynomials, A+E—B . A—E+B. *&c.* Or,

Broken, as Dual.

Fracted, compound in *Species*, or Signs, or both; as

Dual, compound in *Species* $\frac{A}{B}$. $\frac{2B}{3D}$. *&c.*

Plural.

Plural, compound in Signs $\frac{A+B+E}{D+C}$. $\frac{A-B-E}{D-C}$. *&c.*

Mixt.

Mixt, compound in both $\frac{A+B-E}{D-C}$. $\frac{A-B+E}{D+C}$. *&c.*

3. Plain or

3. *Species*, both Simple and Compound, are conſidered as they are Plain or Figurate.

Plain, as A. B. C. or any other *Species* Integral or Fracted, below the Power of any Number.

Figurate.

Figurate, as Aq. Bq. or Ac. Bc. or any other Figural *Species*.

Theſe are Rational or

Theſe are alſo divided into Rational or Irrational.

Rational, are ſuch as denote ſome figural rooted Number, or Quantity of figural rooted Numbers: The former reſembling the figural rooted Numbers handled in *Book* 2. *Part* 2. The latter Coſſicks in the 4th Part of this third Book: Thoſe without Numbers annexed; theſe with Numbers prefixed to their left Hand.

As Aq. Ac. Aqq. *&c.* Bq. Bc. Bqq. *&c.*
2Aq. 4Ac. 5Aqq. *&c.* 3Bq. 4Bc. 6Bqq. *&c.*
Figural and Coſſical *Species.*

Irrational.

Irrational *Species*, as Surds, treated of before, have no Roots to be expreſſed by Abſolute Numbers.

As $\sqrt{q5}$. $\sqrt{c7}$. *&c.*
$\sqrt{qB}$. $\sqrt{cBA}$. *&c.*
Surd and Irrational *Species.*

Both admit like Diviſions as Coſſicks and Surds reſpectively.

Both Rational and Irrational figurate *Species* are capable of like Diviſions with Coſſicks and Surds into Whole and Broken, and either ſort into Simple and Compound. Compound again into Binomial, Reſidual and Polynomial, Homogeneal and Heterogeneal, Symmetral and Aſymmetral; and their Fractions into Single,

Dual and Plural: And all are ſometime intermixt, and aſſume Society one with another, both Plain and Figural; of all which Examples will but take up room.

Notes of Figurate Species.

Touching Figurate *Species*, let be further noted:

1. Species for all Powers compoſed of the two Prime.

Firſt, That the *Species* for all the Powers, are compoſed of the *Species* for the two Prime Powers, *viz.* the Square and Cube. The *Species* for the Square or Quadrat being q, and for the Cube c: Of theſe are all the others compounded according to the Addition of their Indices. So ſhall the 4*th* Power be noted by qq. the 5*th* by qc. the 6*th* by cc. *&c.* The proper *Species* for Root is l, ſignifying *Latus*, latin for *a Side*; yet the Character √ is uſed as before for Root, and √: for Root Univerſal, and other Characters for Surds occaſionally.

Examples.

Indices.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	*&c.*
Species.	l.	q.	c.	qq.	qc.	cc.	qqc.	qcc.	ccc.	qqcc.	qccc.	cccc.	*&c.*
Powers in Numbers.	2.	4.	8.	16.	32.	64.	128.	256.	512.	1024.	2048.	4096.	*&c.*
Coſſical Characters.	ℨ.	ʒ.	φ.	ʒʒ.	ſs.	ʒφ.	Bſs.	ʒʒʒ.	φφ.	ʒſs.	Cſs.	ʒʒφ.	*&c.*
Example in the Root A.	A.	Aq.	Ac.	Aqq.	Aqc.	Acc.	Aqqc.	Aqcc.	Accc.	Aqqcc.	Aqccc.	Acccc.	*&c.*

For A multiplied into A, or A×A, or AA, is Aq. So A×A×A, or AAA, or AqA, is Ac; and the like is to be underſtood of others.

2. Magnitudes Parodical.

Secondly, All Magnitudes under the Power propoſed, are called *Parodical to the Power*; as A. Aq. Ac. Aqq. are Parodical to Aqc.

3. Coefficient, what.

Thirdly, If the Parodical Degree have a known Magnitude joined therewith, it is called the *Coefficient*, as in Aq—AB, where B is the Coefficient, and A joined to it, is the Parodical Degree under Aq.

Signs uſed as before.

The Signs + and —, with or without Aſterisk, are uſed in all things, as before in *Coſſicks* and *Surds*.

Examples of ſome Species, and what underſtood thereby.

By theſe previous Directions it will not be difficult to underſtand both the Name and Nature of any given *Species*. And ſuppoſing ſome Abſolute Number for the Prime *Species* in any Operation, the Value of the reſt will be ſeen as in a *Speculum*. For if A and E ſignifie two Numbers, and you appoint A the Greater, and E the Leſſer; as let A be 3, and E 2, then ſhall

Species.			*Value.*
Z. or A+E,	ſignify the Sum which is	(by Addition)	3+2= 5
X. or A—E,	Difference	(by Subſtraction)	3−2= 1
Æ. AE. or A×E	Product.	(by Multiplication)	3×2= 6
$\frac{R}{S}$.	Ratio, or Subratio	(by Diviſion)	2) 3 (= $\frac{3}{2}$
Aq.	Greater Square		3×3= 9
Eq.	Leſſer Square		2×2= 4
Ac.	Greater Cube		3×3×3=27
Ec.	Leſſer Cube		2×2×2= 8
Z..	Sum of the Squares		9+4=13
X..	Difference of the Squares		9−4= 5
·Z·	Sum of the Cubes		27+8=35
·X·	Difference of the Cubes		27−8=19
AqE.	Greater Square multiplied by the leſſer Number		9×2=18
EqA.	Leſſer Square multiplied by the greater Number		4×3=12
AqEq.	Greater Square multiplied by the Leſſer		9×4=36
AcEc.	Greater Cube multiplied by the Leſſer		27×8=216
$\frac{Aq}{Eq}$.	Greater Square divided by the Leſſer		4) 9 (=2$\frac{1}{4}$
$\frac{Ac}{Ec}$.	Greater Cube divided by the Leſſer		8) 27 (=3$\frac{3}{8}$

Species.		*Value.*
$ZA+Aq$.	Sum of the two Numbers multiplied by the Greater, and added to the greater Square ———	$5\times3+9=24$
$ZA-Aq$.	Sum of the two Numbers multiplied by the Greater, and made less by the Square of the Greater ———	$5\times3-9=6$
$ZA+Eq$.	Sum of the two Numbers multiplied by the Greater, and added to the lesser Square ———	$5\times3+4=19$
$ZA-Eq$.	Sum of the two Numbers multiplied by the Greater, and made less by the Square of the Lesser ———	$5\times3-4=11$
$ZE+Aq$.	Sum of the two Numbers multiplied by the Lesser, and added to the greater Square ———	$5\times2+9=19$
$ZE-Aq$.	Sum of the two Numbers multiplied by the Lesser, and made less by the greater Square ———	$5\times2-9=1$
$ZE+Eq$.	Sum of the two Numbers multiplied by the Lesser, and added to the lesser Square ———	$5\times2+4=14$
$ZE-Eq$.	Sum of the two Numbers multiplied by the Lesser, and made less by the lesser Square ———	$5\times2-4=6$
$\frac{SqAq}{Rq}$.	Square of the lesser Proportional multiplied by the Square of the greater Number, and the Product divided by the Square of the greater Proportional ———	$\frac{4\times9}{9}=\frac{36}{9}=4$
$\frac{RqEq}{Sq}$.	Square of the greater Proportional multiplied by the Square of the lesser Number, and the Product divided by the Square of the lesser Proportional ———	$\frac{9\times4}{4}=\frac{36}{4}=9$
$Zq+Eq-2ZE$ or $Zq-2ZE+Eq$	Sum of the 2 Numbers multiplied by the Lesser, and the Product doubled, is to be substracted from the Sum of the 2 Numbers squared and added to the lesser Square ———	$25+4-20$ or $25-20+4$ $=9$
$Zq\overset{*}{-}2ZE+Eq$.	Double Sum of the 2 Numbers multiplied by the Lesser, and added to the lesser Square, substracted from the Sum of the 2 Numbers squared ———	$25\overset{*}{-}20+4=1$
$\sqrt{Z}-Aq$.	Square of the greater Number substracted from the Sum of the Squares of the 2 Numbers, and the square Root of the Remain is to be taken ———	$\sqrt{13-9}=\sqrt{4}=2$
$\frac{\sqrt{Z.Rq-RqAq}}{Aq}$.	Sum of the Squares multiplied by the Square of the greater Proportional, made less by the same Square, multiplied into the Square of the greater Number. The Remain is to be divided by the Square of the greater Number, & the square Root of the Quot. to be taken.	$\frac{\sqrt{13\times9-9\times9}}{9}=\frac{36}{9}\left(\sqrt{4}=2\right.$
$\frac{Z}{2}+\frac{\sqrt{Zq-4Æ}}{4}$ or $\frac{Z}{2}+\frac{\sqrt{Zq-4P}}{4}$	Half the Sum of the 2 Numbers added to the square Root of the Sum squared, made less by 4 times, the Rectangle or Product divided by 4 (for P is sometime the Species for Product as well as Periphery).	$\frac{5}{2}+\frac{\sqrt{25-24}}{4}$ that is $2\frac{1}{2}+\sqrt{\frac{1}{4}}=3$

Species.		*Value.*
$\sqrt{:}\frac{Z}{2} - \frac{\mathcal{W}Zq-4Pq}{4}$	The Universal square Root of half the Sum of the Squares, made less by the square Root of the Sum of the Squares squared and lessened by 4 Rectangles squared and divided by 4.	$\sqrt{:}\frac{13}{2} - \frac{\mathcal{W}169-144}{4}$ or $\sqrt{:}\frac{13}{2}-\mathcal{W}\frac{25}{4}$ or $\sqrt{:}\frac{13}{2}-\frac{5}{2}=\frac{8}{2}=\mathcal{W}4=2$
$E=\frac{Zq-X}{2Z}$	The lesser Number is equal to the Quotient of the Sum of both Numbers squared, made less by the Difference of the Squares, and the Remain divided by the doubled Sum ——	$2=\frac{25-5}{10}$ or $2=\frac{20}{10}=2$
$A=\frac{X+Xq}{2X}$.	The greater Number is equal to the Difference of the Squares, and the Difference of the Numbers squared & divided by the same Difference doubled.	$3=\frac{5+1}{2}$ or $3=\frac{6}{2}=3$
$2Aq-2XA=Z-Xq$	The Square of the greater Number doubled, made less by the greater, multiplied by the Difference doubled, is equal to the Sum of the Squares made less by the Difference squared.	$9+9-6=13-1$ or $18-6=12$

Thus with wonderful Variety, and unimaginable Celerity, may *Species* be written and beheld, of both which this is but a Drop; yet may serve as an Introduction, as well to their Knowledg and true Position, as for a Basis to the Resolution of several Propositions deducible therefrom; which is easily discernable by the different *Species*, whereby one and the same Number in value may be expressed. For if the Sum of two Numbers be Z, and the two Numbers A+E, then shall A+E be equal to Z and Z to them. So in like manner shall other *Species* of different Forms be equal in value, of which more in the next Book among *Equations*.

The same Number in value expressed by different Species.

CHAP. II. *Addition of Integral and Rational Species.*

Addition of Integral and Rational Species.

THE *Addition of Integral and Rational Species*, whether Sole or Mixt, with Integers, Simple or Compound, may be comprised under the four Cases following.

1. Homogeneal, and of like Signs.

Case 1. If the *Species* be Homogeneal, and of like Signs, then add the Number of the *Species* as if they were Integers, and annex to the Total the same Signs.

Examples.

As A added to A, shall be 2A; and —A added to —A, shall be —2A. The like is to be observed in the rest of the Examples following.

Addends	A	5A	—A—B	A+B	3C—E
	A	A	−2A	A+B	2C—E
Total.	2A	6A	−3A—B	2A+2B	5C—2E

2. Homogeneal, and of unlike Signs.

Case 2. If the *Species* be Homogeneal and of unlike Signs, then substract the lesser Number from the Greater, and to the Difference subscribe the Sign of the *Species* wherein the Excess lieth.

Examples.

As to add 2A to —3A, the 2 taken from 3, leaves 1; which is here to be Negative, because 3 which exceeded 2 was Negative. See further in the following Examples.

Addends.	A	5A	3B—C	2D—E	A+3C
	—A	−3A	−5B	−D+E	−2A—2C
Total.	oA	2A	−2B—C	D	—A+C Remaining.

Case

Caſe 3. If the *Species* be Heterogeneal, then conjoin the *Species* to be added with their proper Signs. *3. Heterogeneal.*

As A added to B, ſhall be A+B; but −A added to B, ſhall be B−A. This is further clear in the Examples enſuing. *Examples.*

Addends.	A	4B	−A	A+E	A−B	Ac+A
	E	5D	−4B	B	−E	Aq
Total.	A+E	4B+5D	−A−4B	A+B+E	A−B−E	Ac+Aq+A

Caſe 4. If the *Species* be mixt Homogeneal and Heterogeneal, with Signs like and unlike; then as the Caſe is, ſo ſhall the Addition be. *4. Mixt.*

As A+B added to A−B, the *Species* B being of contrary Signs by the ſecond Caſe, are to be ſubſtracted one from the other, and ſo o will remain. But A added to A, makes the Total 2A by the firſt *Caſe*. *Examples.*

Alſo A+B added to A−C, the *Species* B and C being Heterogeneal, are by the third *Caſe* to be conjoined with their proper Signs, and A added to A by the firſt *Caſe*, makes the Total 2A+B−C.

More Examples of this kind follow.

Addends.	−B+C	A+D	3B−A	A−B+C	A+B−C
	C−D	A−5D	5C+B	−A−B	A−B+2C
Total.	2C−B−D	2A−4D	4B+5C−A	C−2B	2A+C

As in other Numbers, ſo in *Species*, *Subſtraction* will prove the Truth of *Addition*, as in the next Chapter appeareth. *Proof of Addition of Int. and Rat. Species.*

Moreover, ſuppoſing the *Species* to be Abſolute Numbers, compare the Addition of the one with the other, and the Truth of the Work will appear thereby. As in the laſt Example where the Total is 2A+C, ſuppoſing A 10, and C 6, then ſhall the Total be 26, and ſo ſhall anſwer the Value of the *Species* to be added when the Quantities − are taken from them with +.

Suppoſing A=10. B=5. C=6. Then ſhall

Species	A+B−C	=10+5−6	=9	Value.
	A−B+2C	=10−5+12	=17	
	2A+C	20+6	26	

CHAP. III. *Subſtraction of Integral and Rational Species.*

INtegral and Rational Species are ſubſtracted under like Caſes with *Addition*. *Subſtraction of Int. and Rat. Species.*

Caſe 1. If the *Species* be Homogeneal and of like Signs, then abate the Quantities to be ſubſtracted from the other; and to the Difference prefix the Common Sign, except the Greater be the *Subtrahend*; then prefix the contrary Sign to the Difference. *1. Homogeneal, and of like Signs.*

As to take 2A from 3A, there reſts 1A. But to take 3A from 2A, there will remain −1A, becauſe the greater Quantity is the Subtrahend. More of like ſort the following Examples ſhew. *Examples.*

From	A	3A	A	E	3A+5B	3C−E
Subſtract	A	A	4A	2E	4A+B	2C−E
Remaineth	oA	2A	−3A	−E	4B−A	C

Caſe 2. If the *Species* be Homogeneal and of unlike Signs, then add the Quantities together, and to the Total prefix the Sign of the *Species* from which Subſtraction is made. And if any odd *Species* have none to fellow him, annex him to the Remain with his own Sign if in the upper Number; but with the contrary Sign if in the *Subtrahend*. *2. Homogeneal, and of unlike Signs.*

As to take 2A from −3A, the Remain ſhall be −5A. But −2A taken from 3A, ſhall leave remaining 5A. The like may be obſerved in the Examples following. *Examples.*

From

From	A	−3A	3B—C	−5B	2D—E
Subſtract	−A	5A	−5B	3B—C	−D+E
Remaineth	2A	−8A	8B—C	−8B+C	3D—2E

3. Heterogeneal.

Caſe 3. If the *Species* be Heterogeneal, then conjoin the *Species* to be ſubſtracted with the contrary Signs.

Examples.

As to take E from A, makes the Remain A—E; but —E taken from A, makes the Remain A+E. See further in the following Examples.

From	4B	—A	A+E	A	A	Aq
Subſtract	5D	—4B	B	B—C	B+C	A
Remaineth	4B—5D	—A+4B	A+E—B	A—B+C	A—B—C	Aq—A

4. Mixt.

Caſe 4. If the *Species* be mixt Homogeneal and Heterogeneal, with Signs like and unlike; then as the Caſe is, ſo ſhall the Subſtraction be.

Examples.

As to take C—D from —B+C, the *Species* C being Homogeneal, and of like Signs, the one taken from the other, by the firſt *Caſe*, leaves the remaining Quantity cleared of both. And —D ſubſtracted from —B being Heterogenal, makes the Remain —B+D by the third *Caſe*.

Alſo A+D ſubſtracted from A—5D, the *Species* A in both Homogeneal, and of like Signs, leaves 0 remaining of that Quantity by the firſt *Caſe*. But +D taken from —5D, leaves —6D by the ſecond *Caſe*. The like is to be obſerved in the Examples enſuing.

From	2A—4D	4B+5C—A	C—2B	2A+C	3Aq—3BA+CD
Subſtract	A—5D	5C+B	−A—B	A—B+2C	CD—Dq—Aq
Remaineth	A+D	3B—A	A—B+C	A+B—C	4Aq—3BA+Dq

Proof of Subſtraction of Int. and Rat. Species.

The Quantities from which Subſtraction is made in four of theſe laſt Examples, being the Total of the Additions in the fourth *Caſe* of the foregoing Chapter: And the Remains here being one of the Addends there, and the Subtrahends here the other; ſufficiently ſhew the Proof of Addition by Subſtraction, and Subſtraction by Addition, without farther Example.

Alſo ſuppoſing the *Species* to be Abſolute Numbers, compare the Subſtraction of the one with the other; and by their exact Agreement in Value of the Remains, will the Truth of the Work be made manifeſt.

As in the laſt Example ſave one, where the Remain is A+B—C, ſuppoſing A10, B5, and C6, then ſhall the Remain be 9, that is 15 lacking 6, and ſo accordingly will remain, when the Value of the *Species* to be ſubſtracted is taken from the other.

Suppoſing $A = 10.$ $B = 5.$ $C = 6.$ Then ſhall

$2A + C = 20 + 6 = 26.$

$A - B + 2C = 10 - 5 + 12 = 17$

$A + B - C \quad 10 + 5 - 6 \quad 9$

CHAP. IV. *Multiplication of Integral and Rational Species.*

Multiplication of Int. and Rat. Species.

FOUR *Caſes* comprehend all needful to the *Multiplication of Integral and Rational Species*, whether Homogeneal or Heterogeneal, Simple or Compound, with or without Integers; in all which, as before in *Coſſicks* and *Surds*, like Signs produce +, and unlike —.

1. Simple and Homogeneal.

Caſe 1. If the *Species* be Simple and Homogeneal, then to the right Hand of one of the given Quantities adjoin q, which ſignifieth a Quadrate or Square; becauſe any Number multiplied by himſelf, produceth the Square thereof.

As

As to multiply A by A. B by B, *&c.* they are ſet thus. Examples.

Multiplicands.	A	B	C	*&c.*	Roots.
Multipliers.	A	B	C		
Products.	Aq	Bq	Cq		Squares.

Caſe 2. If the *Species* be Simple and Heterogeneal; then ſet one of the given *Species* beſides the other, in a Right Line, from the Right Hand to the Left. *2. Simple and Heterogeneal.*

As to multiply A by B, the Product ſhall be AB or BA; and ſometimes ſet with the Sign of Multiplication between them thus, A×B or B×A. Other Examples of this ſort follow. Examples.

Multiplicands.	B	A	D	—B	*&c.*
Multipliers.	C	E	A	A	
Products.	BC	AE	AD	—AB	

Caſe 3. If the *Species* have any Numbers joined with them, or annexed to them by the Signs + or —: then multiply the Numbers as Integers, and the *Species* as *Species.* *3. Species with Numbers.*

Examples.

Multiplicands.	3E	4A	2B+1
Multipliers.	2E	3E	A
Products.	6Eq	12Æ	2BA+A

Caſe 4. If the *Species* or one of them be Compound, or one of the Factors be ſome multiplied *Species*; then multiply every Quantity in the Multiplicand, by every Quantity in the Multiplier, as before in Compound Coſſicks, Integers as Integers, and Species as Species. And place the Multiplees orderly from the Left Hand forwards to the Right, whether the Signs be more or leſs. And when the *Species* are multiplied by themſelves, or any other Power by the Root, or by ſome higher Power, whereby figural Numbers ariſe; let the Character of the Magnitude or Power ariſing by the Addition of their Indices, be added to the Product. *4. Compound Species.*

Examples.

Multiplicands.	AE	A+E	A—E	A+E—I
Multipliers.	AE	B	B	Z
Products.	AqEq	BA+BE	BA—BE	ZA+ZE—ZI

Multiplicands.	A+BE	A—B	A+E	A+E
Multipliers.	B	BA	A+E	A—E
Products.	BA+BqE	BAq-BqA	Aq+AE	Aq+AE
			AE+Eq	—AE—Eq
			Aq+2AE+Eq	Aq— Eq

Multiplicands.	3A—2E	5A+CD
Multipliers.	4B—C	3BA—2CD
	12BA—8BE	15BAq+3BACD
	—3CA+2CE	—10DCA—2CqDq
Products.	12BA—8BE—3CA+2CE	15BAq+3BACD—10DCA-2CqDq

Species, as other Numbers, prove the Truth of their Multiplications by Diviſion, as will appear in the next Chapter. And by ſuppoſing the *Species* to be Abſolute Numbers, the Product of their Multiplication will agree with the Product of their *Species* multiplied, if the Work be right. *Proof of Multiplication of Int. & Rat. Species.*

As in the laſt Example, if A be 3, B 4, C 5, D 6, then ſhall 5A be 15, BA 12, and 3BA 36, CD 30, and 2CD 60. So will the Multiplicand being 5A+CD be 15+30 or 45. And the Multiplier being 3BA—2CD, will be 36—60, that is —24; by which 45 multiplied, makes the Product —1080; and ſo is the Value of the other Product at the ſuppoſed Rate aforeſaid, when the *Species* with + are taken from thoſe with —.

Suppoſing A=3. B=4. C=5. D=6. Then ſhall

$5A + CD = 5\times3+5\times6 = 15+30 = 45$

$3BA - 2CD = 3\times4\times3 - 2\times5\times6 = 36 - 60 = -24$

$15BAq + 3BACD = 15\times4\times9 + 3\times4\times3\times5\times6 = 540 + 1080$

$-10DCA - 2CqDq = -10\times6\times5\times3 - 2\times25\times36 = -900 - 1800$

$15BAq + 3BACD - 10DCA - 2CqDq \qquad 540+1080-900-1800$

15BAq =	540	−10DCA =	900		45
3BACD =	1080	−2CqDq =	1800		−24
	1620				180
			−2700	=	90
					−1080

CHAP. V. Diviſion of Integral and Rational Species.

Diviſion of Int. & Rat. Species. TO divide *Integral and Rational Species*, conſider the ſix *Caſes* following; in all which, as in *Coſſicks*, *Surds*, and other Contract Numbers, like Signs ſhall give + and unlike −. And where Integers are adjoined, divide them as Integers are divided.

1. Simple. *Caſe* 1. If any Simple *Species* be to divide the ſame Simple *Species*, then ſet in the Quotient an Unit; but if Integers be annexed to the given *Species*, divide Integers by Integers, as if there were no *Species*.

Examples. As to divide A by A, or B by B, the Quotients ſhall be 1. But 6A divided by 2A, ſhall give 3 in the Quotient. And 8B by −2B, ſhall give −4B.

Diviſor A) *Dividend* A (1 *Quotient*, or thus $\frac{\textit{Dividend}\ A}{\textit{Diviſor}\ A}$ (1A *Quotient*.

Diviſor B) *Dividend* B (1 *Quotient*, or thus $\frac{\textit{Dividend}\ B}{\textit{Diviſor}\ B}$ (1B *Quotient*.

$\frac{\textit{Dividend}\ 6A}{\textit{Diviſor}\ 2A}$ (3A *Quotient*. $\frac{\textit{Dividend}\ 8B}{\textit{Diviſor}\ -2B}$ (−4B *Quotient*.

2. Simple Species of the Diviſor figurate in the Dividend. *Caſe* 2. If the Simple *Species* of the Diviſor be figurate in the Dividend; then place the Diviſor in the Quotient with ſuch a Note of Abatement in the Power of the *Species*, as the Index of the Diviſor being ſubſtracted from the Index of the Dividend will leave.

Examples. As if Aq be divided by A, the Quotient ſhall be A only; becauſe A being the Root, whoſe Index 1 taken from 2, the Index of Aq, the Square leaves 1 the Index of the Root.

$\frac{\textit{Dividend}\ Aq}{\textit{Diviſor}\ A}$ (A *Quotient*. *Indices* 2−1=1 $\qquad$ $\frac{\textit{Dividend}\ 4Ac}{\textit{Diviſor}\ 2A}$ (2Aq *Quotient*. *Indices* 3−1=2

$\frac{\textit{Dividend}\ AqEq}{\textit{Diviſor}\ AE}$ (AE *Quotient*. *Indices* 2−1=1 $\qquad$ $\frac{\textit{Dividend}\ Aqqc}{\textit{Diviſor}\ Aqq}$ (Ac *Quotient*. *Indices* 7−4=3

3. Simple Species of the Diviſor ſpecified in the Dividend. *Caſe* 3. If the Simple *Species* of the Diviſor be ſpecified in the Dividend, then after the *Species* of like Form are cancelled or ſubſtracted, the one from the other, let the Reſidue be ſet in the Quotient.

Examples. As if BA be divided by A, the *Species* A being in both, only B is placed in the Quotient. So 10AE divided by 5A, makes the Quotient 2E. Other Examples of like ſort are ſet with them as followeth.

$\frac{\textit{Dividends}\ BA}{\textit{Diviſors}\ A}$ (B $\qquad \frac{10AE}{5A}$ (2E $\qquad \frac{BA+A}{A}$ (B+1 *Quotients*.

$\frac{\textit{Dividends}\ BA-BE}{\textit{Diviſors}\ B}$ (A−E $\qquad \frac{BA+BE}{B}$ (A+E *Quotients*.

Caſe

Case 4. If the *Species* of both Dividend and Divisor be Compound, then as before in the third *Case*, substract like *Species*; and for every two of the remaining *Species* in the Dividend, from which Substraction is made, let one be set in the Quotient. *4. Compound Species.*

As to divide BA—BE by A—E, substracting A—E from the Dividend, there is left only B—B, for which B only is put in the Quotient. See further in the following Examples. Examples.

$$\text{Dividends / Divisors}\quad \frac{BA-BE}{A-E}\Big(\,B \qquad \frac{BA+CA}{B+C}\Big(\,A \quad \textit{Quotients.}$$

$$\text{Dividend / Divisor}\quad \frac{BA-BE-CA+CE}{B-C}\Big(\,A-E \qquad \textit{Quotient.}$$

Case 5. If the *Species* be Heterogeneal, then place the Divisor under the Dividend in form of a Fraction. *5. Heterogeneal.*

As to divide B by A, being set thus $\frac{B}{A}$ they are left as not otherwise dividable. The Examples following are of like sort. Example.

$$\text{Dividends / Divisors}\quad \frac{-BC}{E}\Big(-\frac{BC}{E} \qquad \frac{-BC}{-A}\Big(\frac{BC}{A} \qquad \frac{B+C}{A-E}\Big(\frac{B+C}{A-E} \quad \textit{Quotients.}$$

Case 6. If the *Species* given to be divided be mixt, then accordingly let their Division be by mixture of the Cases under which they fall. *6. Mixt.*

As to divide BAc by Aq, by the second *Case*, Aq dividing Ac, shall give A in the Quotient; and because B hath nothing substracted from him, he shall be set in the Quotient by the third *Case*. Examples.

So BA—BE divided by A, maketh the Quotient B—$\frac{BE}{A}$; that is, by the third *Case* is B left for the Quotient, and by the fifth *Case* $\frac{BE}{A}$ because they are Heterogeneals.

$$\text{Dividends / Divisors}\quad \frac{BAc}{Aq}\Big(\,BA \qquad \frac{BA-BE}{A}\Big(\,B-\frac{BE}{A} \quad \textit{Quotients.}$$

The Quotient of every Division in *Species* multiplied by the Divisor, returning the Dividend; and the Product of every Multiplication in *Species* divided by one of the Factors, giving the other in the Quotient, is a sufficient Testimony, that the proof of either is reciprocally by each other. *Proof of Division of Int. and Rat. Species.*

Likewise by supposing the *Species* to be divided Absolute Numbers, and dividing them as such, the Quotient of this Division will be the Value of the *Species* in the Quotient of their Division, if the Work be right.

As in the last Example; Suppose A 6, B 2, E 4, then shall BA be 12, and BE be 8, and the Dividend 12—8 to be divided by 6; so shall the Quotient be $2-\frac{8}{6}$, agreeing with B—$\frac{BE}{A}$.

Supposing A=6. B=2. E=4. Then shall

$$\frac{BA-BE}{A} = \frac{2\times6-2\times4}{6} = \frac{12-8}{6} = 2-\frac{8}{6} = B-\frac{BE}{A}.$$

CHAP. VI. *Reduction of Fracted and Rational Species.*

Integral and Rational *Species* discussed, the next that shew themselves are their Fractions. These in the first Chapter of *Species*, were divided and subdivided into several sorts, and so need no farther Description here; wherefore I proceed to *Reduction*. *Reduction of Fract. and Rat. Species.*

Fracted and Rational *Species* are to be reduced either into their least Terms, or into like Denominators when they will admit thereof.

The first sort is called *Abbreviation*, as in Common Fractions; and if the Numbers be commensurable in Single Fractions, or the *Species* in Dual, or both Num- *To their least Terms.*

bers

bers and *Species* in Plural Fractions, or either of them: then dividing them by the Common Divisor, they will be brought to their least Terms.

Examples. As $\frac{6}{8}$ A will be reduced to $\frac{3}{4}$ A, by the Common Divisor 2; and $\frac{6}{9}$ B, by the Common Divisor 3, will be reduced to $\frac{2}{3}$ B.

And $\frac{BA}{BC}$ will be reduced to $\frac{A}{C}$ by the Common Divisor B. And $\frac{BCD}{BDA}$ by the Common Divisor BD will be reduced to $\frac{C}{A}$.

And $\frac{6BA}{8BC}+\frac{DE}{DC}$ will be reduced in Numbers to $\frac{3BA}{4BC}+\frac{DE}{DC}$ and in *Species* to $\frac{3A}{4C}+\frac{E}{C}$ by the Common Divisors B and D.

			Abbreviated.				
Plain		Simple Fracted *Species*	2)	$\frac{6}{8}$A (	$\frac{3}{4}$A	3) $\frac{6}{9}$B (	$\frac{2}{3}$B.
	Compound	Dual Fracted *Species*	B)	$\frac{BA}{BC}$ (	$\frac{A}{C}$	BD) $\frac{BCD}{BDA}$ (	$\frac{C}{A}$
		Plural Fracted Species	2 B+D)	$\frac{6BA}{8BC}+\frac{DE}{DC}$ (	$\frac{3A}{4C}+\frac{E}{C}$		

To one Denominator. The second sort of Reduction of Fracted and Rational *Species*, to bring different Denominators into one comprehendeth,

1. *Simple.* *First*, To reduce simple Fracted *Species*, in which proceed with the Numbers, as in Common Fractions, without altering the *Species*.

Example. And so $\frac{2}{3}$ A, and $\frac{3}{4}$ B, shall be reduced to $\frac{8A \quad 9B}{12}$

$$\begin{array}{ccc} 8A & & 9B \\ \frac{2}{3}A & \times & \frac{3}{4}B \\ & 12 & \end{array}$$

2. *Dual.* 2*ly*, To reduce Dual Fracted Species into like Denominators, multiply the Numbers as Common Fractions, and the *Species* as *Species*.

Examples. And so $\frac{A}{E}$ and $\frac{B}{D}$ shall be reduced to $\frac{AD \quad BE}{ED}$

And $\frac{2B}{3D}$ and $\frac{3B}{4A}$ shall be reduced to $\frac{8BA \quad 9BD}{12DA}$

$$\begin{array}{ccc} AD & & BE \\ \frac{A}{E} & \times & \frac{B}{D} \\ & FD & \end{array} \qquad \begin{array}{ccc} 8BA & & 9BD \\ \frac{2B}{3D} & \times & \frac{3B}{4A} \\ & 12DA & \end{array}$$

3. *Plural.* 3*ly*, To reduce Plural Fracted *Species* into one Denomination, is alike to the Reduction of Dual, *mutatis mutandis*. And if in either of these the Denominations given are Commensurable, reduce them to their least Terms.

Examples. As to reduce $\frac{A+B}{D}$ and $\frac{C-E}{B}$ into one Denomination, multiplying alternately D into C—E, and B into A+B, the new Numerators are gotten, and D into B is the common Denominator. So are the new Fractions $\frac{BA+Bq}{BD}$ and $\frac{DC-DE}{BD}$.

But if $\frac{B}{CA}$ and $\frac{R}{DA}$ be given to be reduced to one Denomination; then because their Denominators are commensurable by A, they are first reduced to their least Terms C and D, and then multiplied alternately as before.

$$\begin{array}{ccc} BA+Bq & & DC-DE \\ \frac{A+B}{D} & \times & \frac{C-E}{B} \\ & BD & \end{array} \qquad A)\begin{array}{cc} BD & RC \\ \frac{B}{CA} & \frac{R}{DA} \\ \dot{D} & \dot{C} \\ \multicolumn{2}{c}{\underbrace{\qquad}_{DCA}} \end{array} = \frac{BD}{DCA} \quad \frac{RC}{DCA}$$

4ly, To reduce many proper Fractions of *Species* into one Denomination, is like the Method used in vulgar Fractions, by multiplying the Denominators one into another for the common Denominator; and every fraction's Numerator into the other Denominators except his own. *4. Many proper Fractions.*

And so $\frac{B}{D}$ and $\frac{C}{A}$ and $\frac{E}{H}$ reduced to one Denomination, are $\frac{BAH}{DAH}$ & $\frac{CDH}{DAH}$ & $\frac{EDA}{DAH}$ Example.

5ly, To reduce Fractions of Fractions, is to multiply after the manner of *Species*, Numerator by Numerator, and Denominator by Denominator. *5. Fractions of Fractions.*

As $\frac{B}{D}$ of $\frac{E}{A}$ reduced, shall be $\frac{BE}{DA}$ by multiplying B into E, and D into A. Examples.

And $\frac{A}{C}$ of $\frac{3Aq}{2B}$ reduced, shall be $\frac{3Ac}{2BC}$ by multiplying A into 3 Aq, and C into 2B.

6ly, To reduce Integral *Species* and Fracted, into improper Fractions, multiply the Integral *Species* by the Denominator of the Fraction, and to the Product add the Numerator. *6. Mixt into Improper.*

As $B\frac{C}{D}$ and $D\frac{C}{A}$ reduced into improper Fractions, shall be $\frac{BD+C}{D}$ and $\frac{DA+C}{A}$ Examples. by multiplying B into D in the one, and D into A in the other, and adjoining to the Products C, with the Sign of Addition +.

So $B-C\frac{R+D}{S}$ reduced, shall be $\frac{BS-CS+R+D}{S}$ the improper Fraction.

7ly, To reduce improper Fracted *Species* back into Integers, or an Integral and Fracted Species, divide the Numerator by the Denominator after the manner of *Species*. *7. Improper into Mixt.*

As $\frac{BD+C}{D}$ divided by D, makes the Quotient B an Integral *Species*, and the remaining C is set over D as a Fraction thus, $B\frac{C}{D}$. Example.

8ly, To reduce an Integral *Species* into some desired Denomination, multiply the whole *Species* by the given Denominator: And any whole *Species* may be set as a Fraction, by placing 1 under the *Species*. *8. Species turned to a Denomination desired.*

As if B be desired as a Fraction, whose Denominator shall be A, then is B to be multiplied into A, and set thus $\frac{BA}{A}$. Examples.

So $B+C$ into D, shall be $\frac{BD+CD}{D}$.

And B, or $B+C$ set as Fractions, shall be $\frac{B}{1}$ and $\frac{B+C}{1}$. *Integer set as a Fraction.*

Hence it appeareth, that as by this last-mentioned Reduction, whole *Species* may be set as Fractions: So by the first sort of Reduction may some fracted *Species* be turned into Integral, being abbreviated into their least Terms. *Some Fractions by Abbreviation brought to Integers.*

As $\frac{BA}{B}$ may be abbreviated into A the Integer.

And $\frac{4Aq}{2A}$ into 2A. And $\frac{3Aq}{6A}$ into $\frac{A}{2}$.

Reduction of Fracted and Rational Species, as well as other Reductions, may clearly be discerned to prove one part thereof, by the other part reciprocal thereto. And besides by supposing the Fracted *Species* absolute Numbers or common Fractions, every Part of Reduction may be proved with sufficient Demonstration. *Proof of Reduction of Fract. and Rat. Species.*

As in the last Example, if A be supposed 3, then shall 6A be 18, and Aq shall be 9, and 3 Aq 27; which abbreviated or divided by 18, shall be $1\frac{1}{2}$ equal to $\frac{A}{2}$.

Supposing $A = 3$. Then shall

$$\left.\begin{matrix} 3Aq = 3 \times 3 \times 3 = 27 \\ 6A = 6 \times 3 = 18 \end{matrix}\right(\tfrac{3}{2} = \frac{A}{2}.$$

Tttt CHAP.

CHAP. VII. *Addition of Fracted and Rational Species.*

Addition of Fract. and Rat. Species.

THE several Cases and Varieties in Addition of Common Fractions, *Book* 1. *Part* 2. *Chap.* 3. might here be run over again: But three Cases contain sufficient for the *Addition of Fracted and Rational Species*, all the rest differing materially only in Examples.

1. Like Denominators without Numbers.

Case 1. If the Fractions to be added be without Numbers annexed, and of the same Denomination; then add the Numerators as *Species* are added, that is, by conjoining them with the Signs proper thereto, if the Numerators be unlike; or if alike, by adding the Number of them together, their Signs also being alike; but when unlike, take their Difference, and subscribe the common Denominator.

Examples.

As to add $\frac{B}{D}$ to $\frac{A}{D}$ the Total shall be $\frac{B+A}{D}$ by joining B to A, because they are Heterogeneal by the Sign of Addition.

But $\frac{B}{D}$ added to $\frac{B}{D}$ shall make the Sum $\frac{2B}{D}$; because both Numerators are Homogeneal, they are added as Integral *Species*.

So Z the Integer added to the Fraction $\frac{E}{B}$, makes the Total $Z\frac{E}{B}$ or $\frac{ZB+E}{B}$.

And $\frac{3D}{B}$ added to $\frac{-D}{B}$, makes the Sum $\frac{2D}{B}$, because the Signs are contrary; when $-D$ is taken from $+3D$, the Remain is $+2D$.

Other Examples.

Addends.	*Total.*	*Addends.*	*Total.*
$\frac{BA}{DE}$ and $\frac{CD}{DE}$ added, are	$\frac{BA+CD}{DE}$.	$\frac{A+E}{B}+\frac{D-C}{B}=$	$\frac{A+E+D-C}{B}$.

2. Unlike Denominators without Numbers.

Case 2. If the Fractions to be added be without Numbers, and of different Denominations, then first reduce them as fracted *Species* are reduced, and afterwards add their Numerators as before.

Examples.

As $\frac{B}{A}$ added to $\frac{B}{C}$ makes, first by Reduction $\frac{BC}{AC}$ and $\frac{BA}{AC}$, and then joined by the Sign of Addition $\frac{BC+BA}{AC}$. So $\frac{B}{D}$ added to $\frac{D}{A}$ make the Total $\frac{BA+Dq}{DA}$.

The former Example.

$$\begin{array}{ccc} BC & & BA \\ \frac{B}{A} & + & \frac{B}{C} \\ & AC & \end{array} = \frac{BC+BA}{AC}.$$

The latter Example.

$$\begin{array}{ccc} BA & & Dq \\ \frac{B}{D} & + & \frac{D}{A} \\ & DA & \end{array} = \frac{BA+Dq}{DA}.$$

Likewise $\frac{B}{D}$ the proper Fraction added to $\frac{ZB+E}{B}$ the improper Fraction, makes the Total $\frac{ZBD+ED+Bq}{BD}$.

Other Examples.

Addends.	*Total.*	*Addends.*	*Total.*
$\frac{BA}{DE}+\frac{CD}{BE}=$	$\frac{BqA+CDq}{BDE}$.	$\frac{A+B}{E}+\frac{D-C}{B}=$	$\frac{AB+Bq+DE-CE}{EB}$

Thus,

$$\begin{array}{ccc} BqA & & CDq \\ \frac{BA}{DE} & + & \frac{CD}{BE} \\ B & & D \\ & \underbrace{\qquad\qquad}_{BDE} & \end{array} \quad E)$$

Thus,

$$\begin{array}{ccc} AB+Bq & & DE-CE \\ \frac{A+B}{E} & + & \frac{D-C}{B} \\ & EB & \end{array}$$

Addends.

Addends. *Total.*

$$\frac{B+C}{Dq+A}+\frac{DE+1}{A-C}=\frac{BA+CA-BC-Cq+DcE+Dq+ADE+A}{DAq+Aq-DqC-AC.}$$

Case 3. If the Fractions have Numbers annexed to them, then order the Numbers as common Fractions, and the *Species* as *Species*. 3. *With Numbers.*

As to add $\frac{2}{5}A$ to $\frac{1}{5}A$, the Sum shall be $\frac{3}{5}A$.

So 2B and $\frac{1}{2}A$ added, make the Total $2B+\frac{1}{2}A$, or $\frac{4B+A}{2}$. Examples.

Other Examples.

Addends. *Total.*

$$\frac{2}{3}A+\frac{3}{4}B=\frac{8A+9B}{12}. \qquad \frac{2B}{3D}+\frac{3B}{4A}=\frac{8BA+9BD}{12DA}$$

Thus, Thus,

$$\begin{array}{ccc} 8A & + & 9B \\ \frac{2}{3}A & \times & \frac{3}{4}B \\ & 12 & \end{array} \qquad \begin{array}{ccc} 8BA & & 9BD \\ \frac{2B}{3D} & \times & \frac{3B}{4A} \\ & 12DA & \end{array}$$

Here are fitly to be inserted such Propositions as require to add a Part or Parts of a given Number or Magnitude to the same Number or Magnitude, or to any other of his Parts. In both which the Desire is thus obtained; First by the Reduction of Fractions of Fractions get the Part or Parts to be added, and then by Addition add them as the Case requires.

Examples of the first Sort. *Part of a Number added thereto.*

As $\frac{1}{2}$ of $\frac{3A}{5}$ added to $\frac{3A}{5}$, maketh the Total $\frac{9A}{10}$.

And $\frac{2}{5}$ of $\frac{3A+D}{E}$ added to $\frac{3A+D}{E}$, maketh the Total $\frac{21A+7D}{5E}$.

For in the first of these $\frac{1}{2}$ of $\frac{3A}{5}$ is found by Reduction to be $\frac{3A}{10}$, which added to $\frac{3A}{5}$ as Common Fractions are added, make the Total as above.

And in the latter Example $\frac{2}{5}$ of $\frac{3A+D}{E}$ is found by Reduction to be $\frac{6A+2D}{5E}$, which by Addition to $\frac{3A+D}{E}$, makes the Total as before.

$$\begin{array}{c} 3A \\ \hline \frac{1}{2} \text{ of } \frac{3A}{5} \\ \hline 10 \end{array} \qquad \begin{array}{c} 9A \\ \hline 3A \quad 6A \\ 5)\frac{3A}{10}+\frac{3A}{5} \\ \underbrace{1 \quad 2}_{10} \end{array} \qquad \begin{array}{c} 6A+2D \\ \hline \frac{2}{5} \text{ of } \frac{3A+D}{E} \\ \hline 5E \end{array} \qquad \begin{array}{c} 21A+7D \\ \hline 15A+5D \\ \frac{6A+2D}{5E}+\frac{3A+D}{E} \\ \hline 5E \end{array}$$

Examples of the second Sort. *Part of a Number added to other Parts thereof.*

As $\frac{1}{3}$ of 5B added to $\frac{2}{5}$ of 5B, makes the Total $\frac{11B}{3}$.

And $\frac{1}{2}$ and $\frac{1}{3}$ of $\frac{15A-12}{8}$ added, make the Total $\frac{25A-20}{16}$.

For in the first of these $\frac{1}{3}$ of 5B is $\frac{5B}{3}$, and $\frac{2}{5}$ of 5B is $\frac{2B}{1}$; both which added make $\frac{11B}{3}$ as before.

And in the latter Example $\frac{1}{2}$ of the *Data* is $\frac{15A-12}{16}$, and $\frac{1}{3}$ is $\frac{5A-4}{8}$, which added together make the Total as above.

5B

$$\begin{array}{c}\frac{5B}{\frac{1}{3}\text{ of }\frac{5B}{1}} \\ \overline{\quad 3 \quad} \\ \frac{2B}{\frac{1}{3}\text{ of }\frac{5B}{1}} \\ \overline{\quad 1 \quad}\end{array} \qquad \begin{array}{c}\frac{11B}{6B} \\ \frac{5B}{3}+\frac{2B}{1} \\ \overline{\quad 3 \quad}\end{array} \qquad \begin{array}{c}\frac{15A-12}{\frac{1}{2}\text{ of }\frac{15A-12}{8}} \\ \overline{\quad 16 \quad} \\ \frac{5A-4}{5A-4} \\ \frac{1}{2}\text{ of }\frac{15A-12}{8} \\ \overline{\quad 8 \quad}\end{array} \qquad \begin{array}{c}\frac{25A-20}{10A-8} \\ 8)\underbrace{\frac{15A-12}{16} + \frac{5A-4}{8}}_{16}\end{array}$$

Addition of Fracted and Rational Species is to be proved both by Substraction, as in the next Chapter, and by converting the Fracted *Species* into Absolute Numbers or Common Fractions; with the Addition whereof the Addition of these Fracted *Species* will exactly agree.

Proof of Addition of Fract. & Rat. Species.

As in the last Example of the third *Case* above, suppose A2, B3, D4, then shall 2B be 6, and 3D 12. So $\frac{2B}{3D}$ is $\frac{6}{12}$ or $\frac{1}{2}$. And 3B shall be 9, and 4A 8. So $\frac{3B}{4A}$ is $\frac{9}{8}$. And $\frac{1}{2}$ and $\frac{9}{8}$ added together, make $\frac{13}{8}$ or $1\frac{5}{8}$ equal to the Total $\frac{8BA+9BD}{12DA}$

Supposing A=2. B=3. D=4. Then shall

$$\frac{2B=6}{3D=12}+\frac{3B=9}{4A=8} \qquad \left.\begin{array}{r}8BA=8\times3\times2=48\\ 9BD=9\times3\times4=108\end{array}\right\}=156$$

$$2)\underbrace{\frac{4}{\frac{1}{2}}+\frac{9}{\frac{9}{8}}}_{8}=\frac{13}{8}\left(1\frac{5}{8}\right. \qquad 12DA=12\times4\times2=96 \;=\; 96\left(1\frac{60}{96}\text{ or }1\frac{5}{8}\right.$$

CHAP. VIII. *Substraction of Fracted and Rational Species.*

Substraction of Fract. and Rat. Species.

AS in *Addition*, so here, sufficient for the *Substraction of Fracted and Rational Species*, is contained under the three *Cases* following. The other *Cases* mentioned in Substracting of Common Fractions, *Book* 1. *Part* 2. *Chap.* 4. being more occasional than essentially necessary.

1. Like Denominators without Numbers.

Case 1. If the Fractions to be substracted be without Numbers annexed, and of like Denomination, then after the manner of *Species* substract the Numerator of the Subtrahend from the other Numerator set to the left Hand of the Subtrahend; that is, by connexing them with the Sign of Substraction, if the Numerators be unlike and their Signs alike: But if the Numerators be alike, by taking the one from the other, or adding with contrary Signs both Numerators as before in Cossicks, and under such Sum or Difference subscribe the Common Denominator.

Examples.

As to take $\frac{D}{C}$ from $\frac{B}{C}$, the Remain is $\frac{B-D}{C}$ conjoining B to D, because they are Heterogeneal by the Sign of Substraction.

But $\frac{B}{D}$ taken from $\frac{3B}{D}$, shall leave remaining $\frac{2B}{D}$, because both Numerators are Homogeneal, the one is substracted from the other as Integral *Species*.

So $\frac{E}{B}$ the proper Fraction taken from $\frac{ZB+E}{B}$ the improper Fraction, the Remain will be Z.

And $\frac{-D}{B}$ taken from $\frac{2D}{B}$, leaves remaining $\frac{3D}{B}$; for the Signs being contrary, —D shall be added to 2D, which makes 3D as before in Cossicks.

Other

Other Examples.

Subtrahend. *Remain.*

$$\frac{AB}{D} \text{ substracted from } \frac{BC}{D}, \text{ shall leave } \frac{BC-AB}{D}$$

Subtrahend. *Remain.*

$$\frac{A-E}{B} \text{ substracted from } \frac{D-C}{B}, \text{ shall leave } \frac{D-C-A+E}{B}.$$

Case 2. If the Fractions to be substracted be without Numbers, and of different Denominations; then first reduce them as Fracted *Species* are reduced, and then substract the Numerator of the Subtrahend from the other as before. *2. Unlike Denominators without Numbers.*

As from $\frac{A}{B}$ substract $\frac{B}{C}$, the Rednction makes them $\frac{AC}{BC}$ and $\frac{Bq}{BC}$; then by Substraction the Remain is $\frac{AC-Bq}{BC}$. Examples.

And from $\frac{B}{D}$ take $\frac{D}{A}$, the Remain shall be $\frac{BA-Dq}{DA}$.

The former Example.

$$\overset{AC \qquad\quad Bq}{\frac{A}{B} - \frac{B}{C}} = \frac{AC-Bq}{BC}$$
$$BC$$

The latter Example.

$$\overset{BA \qquad\quad Dq}{\frac{B}{D} - \frac{D}{A}} = \frac{BA-Dq}{DA}$$
$$DA$$

Other Examples.

Subtrahend. *Remain.*

$$\frac{BA}{DE} - \frac{CD}{BE} = \frac{BqA-CDq}{BDE}.$$

Subtrahend. *Remain.*

$$\frac{BqA+CDq}{BDE} - \frac{CD}{BE} = \frac{BA}{DE}.$$

Thus,

$$E\Big)\;\underbrace{\overset{BqA}{\frac{BA}{\underset{B}{DE}}} - \overset{CDq}{\frac{CD}{\underset{D}{BE}}}}_{BDE}$$

Thus,

$$BE\Big)\;\underbrace{\frac{BqA+CDq}{\underset{1}{BDE}} - \overset{CDq}{\frac{CD}{\underset{D}{BE}}}}_{BDE} = \frac{BqA}{BDE} = \frac{BA}{DE}$$

In which last Example is to be noted, that in the Reduction the Common Divisor BE takes away all the Denominator of the Subtrahend; and so nothing being left to be brought to the Denominator of the other Number, an Unit is there placed and not a Cipher, lest it should be taken for a *Speeies*. After the Reduction the Numbers stand thus.

$\frac{BqA+CDq-CDq}{BDE}$; where because CDq is both Affirmative and Negative, in the Numerators they are both to be cancelled, and the Remain $\frac{BqA}{BDE}$ abbreviated to $\frac{BA}{DE}$, because Bq in the Numerator is figurate; which divided by B, leaves B only to A for the Numerator.

Case 3. If the Fractions have Numbers annexed to them, then order the Numbers as Common Fractions, and the *Species* as *Species*. *3. With Numbers.*

As from $\frac{2}{7}$ A take $\frac{1}{7}$A, and the Remain shall be $\frac{1}{7}$A. Examples.

But to take $\frac{3}{4}$A from $\frac{13}{12}$A, the Fractions must first be reduced to one Denomination; and then from $\frac{13}{12}$A let $\frac{9}{12}$ A be taken, and the Remain will be $\frac{4}{12}$A or $\frac{1}{3}$ A.

So 2B taken from $\frac{4B+A}{2}$ leaves $\frac{A}{2}$ or $\frac{1}{2}$A.

Other Examples.

Subtrahend.	*Remain.*	*Subtrahend.*	*Remain.*
$\frac{2}{3}A - \frac{3}{4}B =$	$\frac{8A-9B}{12}$.	$\frac{8A+9B}{12} - \frac{3}{4}B =$	$\frac{2}{3}A$.

Thus,

$$\begin{array}{ccc} 8A & - & 9B \\ \frac{2}{3}A & \times & \frac{3}{4}B \\ & 12 & \end{array}$$

Thus,

$$4\Big)\frac{8A+9B}{12} - \frac{\overset{9B}{\overline{}}}{}\frac{3}{4}B = \frac{8A}{12} = \frac{2}{3}A.$$

$$\underbrace{\overset{\cdot\cdot}{1} \qquad \overset{\cdot\cdot}{3}}_{12}$$

In this place may be fitly inserted such Propositions as require to take a Part or Parts of a Number or Magnitude given, from the same Number or Magnitude, or from any other of his Parts. In both which the Quesited is gotten thus: First by Reduction of Fractions of Fractions, get the Part or Parts to be substracted, and then by Substraction, as the Case may require, you will have the Remain.

Part of a Number taken therefrom.

Example of the first Sort.

As to take $\frac{1}{3}$ and $\frac{1}{5}$ of $\frac{B+8A}{C}$ from the same, the Remain will be $\frac{7B+56A}{15C}$. For $\frac{1}{3}$ and $\frac{1}{5}$ being together $\frac{8}{15}$, are $\frac{8B+64A}{15C}$; which substracted from the *Data*, leave the Remain as before.

$$\begin{array}{c} 8 \\ \hline \begin{array}{cc} 5 & 3 \\ \frac{1}{3} & + \frac{1}{5} \end{array} \\ \hline 15 \end{array} \qquad \begin{array}{c} 8B+64A \\ \hline \frac{8}{15} \text{ of } \frac{B+8A}{C} \\ \hline 15C \end{array} \qquad \begin{array}{c} 7B-56A \\ \hline 15B+120A \\ \begin{array}{ccc} \frac{B+8A}{C} & - & \frac{8B+64A}{15C} \end{array} \\ \hline 15C \end{array}$$

Parts of a Number taken from other Parts thereof.

Example of the second Sort.

As to take $\frac{3}{4}$ of $3B+C$ from $\frac{4}{5}$ thereof, the Remain is $\frac{3B+C}{20}$.

For $\frac{3}{4}$ of $3B+C$ is $\frac{9B+3C}{4}$, and $\frac{4}{5}$ is $\frac{12B+4C}{5}$, and the former taken from this latter, leaves the Remain as aforesaid.

$$\begin{array}{c} 9B+3C \\ \hline \frac{3}{4} \text{ of } \frac{3B+C}{1} \\ \hline 4 \end{array} \qquad \begin{array}{c} 12B+4C \\ \hline \frac{4}{5} \text{ of } \frac{3B+C}{1} \\ \hline 5 \end{array} \qquad \begin{array}{c} 3B+C \\ \hline \begin{array}{ccc} 48B+16C & & 45B+15C \\ \hline \frac{12B+4C}{5} & - & \frac{9B+3C}{4} \end{array} \\ \hline 20 \end{array}$$

Proof of Substraction of Fract. and Rat. Species.

Forasmuch as in the last Example here in the third *Case*, and also in the last Example of the second *Case* above, the Numbers from which the Substraction is made, are the Totals of their Additions in the former Chapter; and the Subtrahends in both, are one of the Addends there: It is enough to shew the Reciprocal Proof of Addition by Substraction, and Substraction by Addition in these Fracted *Species*.

Substraction also, as Addition before, may be proved by supposing the *Species* to be Numbers Absolute; because after Substraction made with them, the Remain will be equally valuable with the Fracted *Species* remaining.

As in the last mentioned Example, if A be supposed 2, and B 3, then shall 8A be 16, and B 27. And 8A+9B, that is 16+27, shall be 43; which divided by 12, shall be $3\frac{7}{12}$: From which $\frac{3}{4}$B taken which is $\frac{9}{4}$, the Remain is $\frac{16}{12}$, that is 8A to be divided by 12, or $\frac{2}{3}$A, as the *Species* above shewed.

Supposing A=2. B=3. Then shall

$$\left.\begin{array}{l} 8A=16 \\ 9B=27 \end{array}\right\} \frac{43}{12} - \overset{\textit{Subtrahend.}}{\frac{3}{4}B = \frac{9}{4}} \qquad \overset{\textit{Remain.}}{\frac{2}{3}A = \frac{4}{3} = 1\frac{1}{3}}$$

$$\frac{16}{} \qquad \text{Or thus, } 16 + 27 - 27 = 16$$

$$\frac{27}{}$$

$$4)\underbrace{\frac{43}{12}}_{1} - \underbrace{\frac{9}{4}}_{3} = \frac{16}{12} = 1\tfrac{1}{3}$$

$$12$$

$$\frac{9B}{}$$

$$4)\underbrace{\frac{8A+9B}{12}}_{1} - \underbrace{\frac{3}{4}}_{3}B = 8A$$

$$12 = 12$$

CHAP. IX. *Multiplication of Fracted and Rational Species.*

SPECIES Fracted and Rational, mixed with Integral *Species*, or purely Fracted, are multiplied as Common Fractions, *Book* 1. *Part* 2. *Chap.* 5. by comparing the Heterologal Terms, and multiplying the Homologal. And therefore but two *Cases* are necessary, in both which the Signs are to be ordered as in Cossical Fractions. *Multiplication of Fract. and Rat. Species.*

Case 1. If the Heterologal Terms need no Reduction, then multiply Numerator by Numerator, and Denominator by Denominator, Numbers as Numbers, and *Species* as *Species*. *1. Heterologal Terms not reducible.*

As $\frac{B}{D}$ multiplied into $\frac{C}{A}$, makes the Product $\frac{BC}{DA}$. Examples.

And $\frac{2B}{3D}$ multiplied by $\frac{2C}{3A}$, makes the Product $\frac{4BC}{9DA}$.

So Z the Integral *Species* multiplied by the Fraction $\frac{A}{B}$ makes the Product $\frac{ZA}{B}$; where I set or suppose an Unit for the Denominator to Z, as if it were a Fraction.

Other Examples.

Mds.		*Mrs.*		*Products.*
$\frac{A}{B}$	$\times$	$\frac{ZA}{C}$	$=$	$\frac{ZAq}{BC}$.
$\frac{3B}{4C}$	$\times$	$\frac{1}{5}A$	$=$	$\frac{3BA}{20C}$.

Mds.		*Mrs.*		*Products.*
B	$\times$	$\frac{DA}{C}$	$=$	$\frac{BDA}{C}$.
$\frac{2B+C}{D-E}$	$\times$	$\frac{3A+O}{P}$	$=$	$\frac{6BA+3CA+2BO+CO}{DP-EP}$.

Case 2. If the Heterologal Terms need Reduction, then reduce them according to the Nature of the Fraction, Numbers as Numbers, and *Species* as *Species*; And after Reduction into their least Terms, multiply the Homologal Terms as before. *2. Heterologal Terms reducible.*

As $\frac{B}{A}$ multiplied into $\frac{A}{C}$ produceth $\frac{B}{C}$, because A in the Numerator of the one, and Denominator of the other, being alike, are both set aside. Examples.

And $\frac{2B}{3D}$ multiplied by $\frac{2C}{5B}$ produceth $\frac{4C}{15D}$, the Heterologal B in both Fractions being useless in the Product.

So B the Integral *Species* multiplied into the Fraction $\frac{A}{B}$ makes the Product A, for A is equal to $\frac{BA}{B}$.

Other Examples,

Mds.		*Mrs.*		*Reductions.*				*Products.*
$B\frac{C}{A}$	$\times$	$\frac{C}{D}$	$=$	$\frac{BA+C}{A}$	$\times$	$\frac{C}{D}$	$=$	$\frac{BAC+Cq}{AD}$.
$\frac{BA}{RE}$	$\times$	$\frac{DE}{CA}$	$=$	$\frac{A)\overset{B}{BA}}{E)\underset{R}{RE}}$	$\times$	$\frac{\overset{D}{DE}}{\underset{C}{CA}}$	$=$	$\frac{BD}{RC}$.
$\frac{2B}{3C}$	$\times$	$\frac{1}{4}A$	$=$	$\frac{2)\overset{1}{2B}}{3)\underset{1}{3C}}$	$\times$	$\overset{1}{\underset{1}{\tfrac{1}{4}}}A$	$=$	$\frac{1BA}{2C}$.

Hence

Heterolog. Terms equal. Hence as in Multiplication of Common Fractions, if the Heterologal Terms either way are equal, the other Terms shall stand for the Product. And if they are alike both ways, then shall the Product be an Unit or equal Fraction. For

Examples. $\frac{2B}{3C} \times \frac{3C}{5B} = \frac{2BC}{5CB}$, and omitting the *Species* to $\frac{2}{5}$ the Number 3 in both being set aside. And $\frac{2B}{3C} \times \frac{3C}{2B} = \frac{6BC}{6CB}$, that is 1.

To find a Part or Parts of a Number. Also here properly may be inserted a Proposition to find a Part or Parts of a given Number or Magnitude, which is no more than to multiply the same by the Part or Parts, after the manner of Fractions.

Examples of both Sorts.

Examples. As to know what $\frac{3}{4}$ of 3B is, the Product shews it $\frac{9B}{4}$.

And if $\frac{1}{3}$ and $\frac{1}{4}$ of $\frac{B+8A}{C}$ be demanded, the Multiplication produceth $\frac{7B+56A}{12C}$ for Answer. For $\frac{1}{3}$ and $\frac{1}{4}$ added, make $\frac{7}{12}$, which multiplying the *Data*, makes the Product as last above-mentioned.

Proof of Multiplication of Fract. and Rat. Species. *Multiplication of Fracted and Rational Species* is to be proved, both by Division as in the next Chapter; and by turning the Fracted *Species* into Common Fractions or Absolute Numbers, with the Multiplication whereof will agree the Multiplication of these Fracted *Species*.

For suppose in the Example above-mentioned (where $\frac{2B}{3C} \times \frac{3C}{5B}$ produce $\frac{2BC}{5CB}$) B be 2, and C 3, then shall 2B be 4, and 3C 9. So will $\frac{2B}{3C}$ be $\frac{4}{9}$ and $\frac{3C}{5B}$ will be $\frac{9}{10}$, and the Product by Abbreviation $\frac{2}{5}$, as is the Value of the Product $\frac{2BC}{5CB}$ by that Supposition.

Supposing B = 2. C = 3. Then shall

$$\frac{2B = 4}{3C = 9} \times \frac{3C = 9}{5B = 10} = \frac{2BC = 2\times2\times3}{5CB = 5\times3\times2} = \tfrac{12}{30} \text{ or } \tfrac{2}{5}.$$

CHAP. X. *Division of Fracted and Rational Species.*

Division of Fract. and Rat. Species. Contrary to Multiplication *Species* Fracted and Rational, Pure or Mixt with Integral *Species*, are divided by comparing the Homologal Terms, and multiplying the Heterologal as in Common Fractions, *Book* 1. *Part* 2. *Chap.* 6. And so but two Cases are here necessary, and in both of them the Signs are to be ordered as in Cossical Fractions.

1. Homologal Terms not reducible. *Case* 1. When the Homologal Terms cannot be reduced lower, then multiply the Numerator of the Dividend by the Denominator of the Divisor, to produce the Numerator of the Quotient: And the Denominator of the Dividend by the Numerator of the Divisor, to produce the Denominator of the Quotient; Numbers as Numbers, and *Species* as *Species*.

Examples. As $\frac{B}{D}$ divided by $\frac{C}{A}$ gives in the Quotient $\frac{BA}{DC}$.

And $\frac{2B}{3D}$ divided by $\frac{3C}{4A}$, gives in the Quotient $\frac{8BA}{9DC}$.

So $\frac{Aq}{D}$ divided by the Integral *Species* B, makes the Quotient $\frac{Aq}{BD}$.

Supposing or setting an Unit for Denominator to B.

Other Examples.

Divisors.	*Dividends.*	*Quotients.*	*Divisors.*	*Dividends.*	*Quotients.*
$\frac{Bc}{C}$	$\frac{Ec}{A}$	$\frac{EcC}{BcA}$	$\frac{A}{D}$	$\frac{BC}{1}$	$\frac{BCD}{A}$
$\frac{DA}{C}$	$\frac{B}{1}$	$\frac{BC}{DA}$	$\frac{3B}{4C}$	$\frac{1}{5}A$	$\frac{4AC}{15B}$

Case

Case 2. When the Homologal Terms may be reduced, then, according to the Nature of the Fractions, reduce them, Numbers as Numbers, and *Species* as *Species*: And after Reduction into their least Terms, multiply the Heterologal Terms as before. *2. Homologal Terms reducible.*

As $\frac{A}{C}$ dividing $\frac{B}{C}$ giveth in the Quotient $\frac{B}{A}$, because C the Denominator of both the given Fractions is set aside as useless. Examples.

And $\frac{4B}{15C}$ divided by $\frac{7B}{11D}$ brings in the Quotient $\frac{44D}{105C}$, the Homologal B in both being understood as cancelled.

So BC the Integral *Species* divided by $\frac{B}{A}$, makes the Quotient CA.

Other Examples.

Divisors. Dividends. Reductions. Quotients.

$$\frac{BA}{RT}\Big)\ \frac{CA}{DT} = \frac{A)\overset{B}{\ddot{B}A}}{T)\underset{R}{R\ddot{T}}}\Big)\frac{\overset{C}{\ddot{C}A}}{\underset{D}{\ddot{D}T}}\Big(\frac{CR}{BD}.$$

$$\frac{BA}{CR}\Big)\ \frac{A}{R} = \frac{A)\overset{B}{\ddot{B}A}}{R)\underset{C}{\ddot{C}R}}\Big)\ \frac{\overset{1}{\ddot{A}}}{\underset{1}{\ddot{R}}}\Big(\frac{C}{B}.$$

$$\tfrac{3}{4}A\Big)\ \tfrac{1}{2}\frac{BA}{C} = \frac{A)}{2)}\overset{1}{\underset{2}{\tfrac{3}{4}A}}\Big)\underset{1}{\tfrac{1}{2}}\frac{\overset{B}{\ddot{B}A}}{C}\Big(\frac{2B}{3C}.$$

Hence, as in Division of Common Fractions, if the Homologal Terms be either way equal, the other Terms shall be taken for the Quotient: And if they are alike both ways, the Quotient shall be an Unit or equal Fraction. *Homologal Terms equal.*

For $\frac{2B}{3C}\Big)\frac{2B}{5D}\Big(\frac{3C}{5D}$ the Number 2 being set aside with the *Species* B, because alike in both Divisor and Dividend. And $\frac{2B}{3D}\Big)\frac{1C}{3D}\Big(\frac{1C}{2B}$, because 3D in both Denominators are equal, the other *Species* make up the Quotient. Example.

And $\frac{2B}{3D}\Big)\frac{2B}{3D}\Big(1$ is the Quotient, the Terms and *Species* in both Numerators and Denominators being alike.

Sometimes it is the easiest way to reduce the Numbers given, part before Division and part after; which happens if one or both be figurate. *Best sometime to reduce the Data, part before and part after Division.*

As in dividing $\frac{ZAq}{BC}$ by $\frac{ZA}{C}$ and $\frac{BAC+Cq}{AD}$ by $\frac{C}{D}$ in the first Quotient $\frac{Aq}{BA}$ is reduced to $\frac{A}{B}$, and the other $\frac{BA+C}{A}$ to $B\frac{C}{A}$. Examples.

Divisors. Dividends. Reductions. Quotients Reduced.

$$\frac{ZA}{C}\Big)\ \frac{ZAq}{BC} = \frac{Z)}{C)}\frac{ZA}{\underset{1}{\ddot{C}}}\Big)\frac{ZAq}{\underset{B}{\ddot{B}C}}\Big(\frac{Aq}{BA}\Big(\frac{A}{B}.$$

$$\frac{C}{D}\Big)\frac{BAC+Cq}{AD} = \frac{C)}{D)}\frac{C}{D}\Big)\frac{BAC+Cq}{AD}\Big(\frac{BA+C}{A}\Big(B\frac{C}{A}.$$

Here may be inserted a Proposition, to find the principal Number or Magnitude, if any of his Parts be given; which is done by dividing the same Number or Magnitude by the Part or Parts, after the manner of Fractions. *To find a Number by any of its Parts.*

Examples of both Sorts. Examples.

As to know what is the principal Number whereof B is the [illegible]?

 Answer,

Answer, $\frac{5B}{4}$.

And if $\frac{B+8A}{C}$ be $\frac{2}{3}$ of another Number which is desired, it will be found to be $\frac{3B+24A}{2C}$. For by dividing B by $\frac{4}{5}$, and $\frac{B+8A}{C}$ by $\frac{2}{3}$, their Quotients are as before.

Proof of Division of Fract. & Rat. Species.

Seeing the Quotients of these Divisions multiplied by their Divisors will return the Dividends, and the Dividends in both the Examples above are the Products of their Multiplications in the former Chapter, it is sufficient to shew the Alternate Proof of Multiplication by Division, and Division by Multiplication in these Fracted *Species*.

Division also, as Multiplication before, may be proved by supposing the *Species* to be Numbers Absolute: For after Division made thereby, the Quotient will be equally valuable with the Quotient of the divided Fractions.

As in the Example above, supposing A 2. B 3. C 4. D 5. then will the Divisor $\frac{C}{D}$ be $\frac{4}{5}$; and the Dividend $\frac{BAC+Cq}{AD}$ be $\frac{40}{10}$ or $\frac{4}{1}$, that is $\frac{3\times2\times4+4\times4}{2\times5}$; and the Quotient of that Division 5 Integers equal to $B\frac{C}{A}$ that is $3\frac{4}{2}$.

Supposing A=2. B=3. C=4. D=5. Then shall

$$\begin{matrix} C = 4 \\ \overline{D = 5} \end{matrix}\Bigg)\frac{\overset{24+16}{BAC+Cq} = 40}{AD == 10}\Bigg(B\frac{C}{A} = 3\frac{4}{2} \qquad \begin{matrix}4)\\5)\end{matrix}\frac{\overset{1}{4}}{\underset{1}{5}}\Bigg)\frac{\overset{10}{40}}{\underset{2}{10}}\Bigg(\frac{10}{2}\Big(5$$

C H A P. XI. *Figuration of Rational Species.*

Figurate Rational Species produced.

HOW to add, substract, multiply and divide whole and broken *Species*, as well Plain as Figurate, or Rational, intermixt one with another, is already dispatcht in the foregoing Chapters; wherein because the Cossical or Rational *Species* are subject to like Orders and Directions, and receive like Resolutions with plain *Species*, little need be added here save something of their Figuration.

Common Way.

Touching the Production of Rational *Species*, as others before them, so they are produced by Multiplication: For any *Species* multiplied into it self, produceth the Square; and that multiplied by the Root, produceth the Cube, *&c.* And whether the Root be Simple or Compound, or if one Power be multiplied by another; by Addition of their Indices, the due Index of the Product is had, as was touched in their Multiplication before, and fully seen in Cossicks in the 4*th* Part of this Book, and Figural Numbers in the second Part of the second Book.

Examples

Of Simple Rational Species *produced.*

Examples.

Species	A	Aq	Ac	2Ac	Aqc	Aqc	multiplied.
	A	A	A	3A	Aq	Aqq	
	Aq	Ac	Aqq	6Aqq	Aqqc	Accc	
Indices	1	2	3	3	5	5	added.
	1	1	1	1	2	4	
	2	3	4	4	7	9	

Examples
Of Compound Rational Species *produced.*

√ A + E
A + E
Aq + AE
AE + Eq
q Aq + 2AE + Eq
A + E
Ac + 2AqE + AEq
AqE + 2AEq + Ec
c Ac + 3AqE + 3AEq + Ec
&c.

√ A — E
A — E
Aq − AE
− AE + Eq
q Aq — 2AE + Eq
A — E
Ac — 2AqE + AEq
— AqE + 2AEq — Ec
c Ac — 3AqE + 3AEq — Ec

√ 2A + 3E
2A + 3E
4Aq + 6AE
6AE + 9Eq
q 4Aq + 12AE + 9Eq
2A + 3E
8Ac + 24AqE + 18AEq
12AqE + 36AEq + 27Ec
c 8Ac + 36AqE + 54AEq + 27Ec.
&c.

√ A + B + C
A + B + C
Aq + BA + CA
BA + Bq + BC
CA + BC + Cq.
q Aq + 2BA + Bq + 2CA + 2BC + Cq.

Besides this ordinary way of Production, the Power of any Binomial *Species* may speedily be had, thus: Set down all the Parodical Degrees of both *Species* under the Power to the Root, and then place the highest Powers of each in opposition, and let the rest be coupled contrary, the highest of one sort to the lowest of the other, and to them prefix the Numbers proper to the Power to be produced, mentioned in the Table for Extraction of Roots, *Book* 2. *Part* 2. *Chap.* 3. which said Numbers are sometime called *Unciæ*. *Power of a Binomial otherwise produced.*

Unciæ what.

As if the 7*th* Power of A + E (that is the second Sursolid) be sought. *Example.*

Parodical Degrees.		*Rightly placed*		*with the Unciæ.*
		Aqqc		Aqqc
A	E	Acc	E	7AccE
Aq	Eq	Aqc	Eq	21AqcEq
Ac	Ec	Aqq	Ec	35AqqEc
Aqq	Eqq	Ac	Eqq	35AcEqq
Aqc	Eqc	Aq	Eqc	21AqEqc
Acc	Ecc	A	Ecc	7AEcc
Aqqc	Eqqc		Eqqc	Eqqc

The Parodical Degrees with the *Unciæ* to the 10*th* Power, stand as in the Table following, which may be enlarged at pleasure.

A Table of the Parodical Degrees, with the Unciæ to the 10th Power.

1	2	3	4	5	6	7	8	9	10
A E	Aq 2AE Eq	Ac 3AqE 3AEq Ec	Aqq 4AcE 6AqEq 4AEc Eqq	Aqc 5AqqE 10AcEq 10AqEc 5AEqq Eqc	Acc 6AqcE 15AqqEq 20AcEc 15AqEqq 6AEqc Ecc	Aqqc 7AccE 21AqcEq 35AqqEc 35AcEqq 21AqEqc 7AEcc Eqqc	Aqcc 8AqqcE 28AccEq 56AqcEc 70AqqEqq 56AcEqc 28AqEcc 8AEqqc Eqcc	Accc 9AqccE 36AqqcEq 84AccEc 126AqcEqq 126AqqEqc 84AcEcc 36AqEqqc 9AEqcc Eccc	Aqqcc 10AcccE 45AqccEq 120AqqcEc 210AccEqq 252AqcEqc 210AqqEcc 120AcEqqc 45AqEqcc 10AEccc Eqqcc

By this Table we have a farther Demonstration of the Truth of that Theorem of *Euclid* touching the Square; and the other of *Ramus* touching the Cube, mentioned before, *Book* 2. *Part* 2. *Chap.* 2. *Sect.* 6, 7. *Theorem of Euclid demonstrated.*

As

As in producing the Square and Cube of 845; firſt A ſhall be ſet for 8, and E for 4; then 84 ſhall be A, and 5 ſhall be E; the reſt of the Work, with the *Species* annexed, follow.

Square.

8	4	5		
64			Aq	
6	4		2AE	Gnomon.
	16		Eq	
70	56		Aq	
	84	0	2AE	Gnomon.
		25	Eq	
71	40	25		

Cube.

8	4	5		
512			Ac	
76	8		3AqE	
3	84		3AEq	Gnomon.
	64		Ec	
592	704		Ac	
10	584	0	3AqE	
	63	00	3AEq	Gnomon.
		125	Ec	
603	351	125		

What the Diagonals, Complements, Unciæ and Gnomon. The two extream Powers of every kind in this Table, are ſometime called *Diagonals*, and the intermediate Species *Complements*. The Numbers or *Unciæ* affixed, ſhew the number of Complements to be taken in the Conſtitution of the Power; and all the Complements with the leſſer Power make up the *Gnomon*.

Mr. Oughtred's *Obſervations on the Table.* Mr. *Oughtred* in the 17*th* Chapter of his *Clavis*, makes farther inſpection into the Table, and obſerves:

1. What Species Affirmative, and what Negative, &c. 1. That all the *Species* of the Powers of a Binomial Root are Affirmative, and thoſe Parts ſimply taken without Unity, are in continual Proportion: But all the *Species* of the Powers of a Reſidual Root, are alternately Negative.

Example in Binomials.

As Powers.	*Binomial.*		*Parts.*	*Ratio.*
Q:	A + E	is	Aq + AE + Eq ∴ 9 . 6 . 4 ∵	$\frac{9}{6}(1\frac{1}{2}$ $\frac{6}{4}(1\frac{1}{2}$
C:	3 + 2		Ac + AqE + AEq + Ec ∴ 27 . 18 . 12 . 8 ∵	$\frac{27}{18}(1\frac{1}{2}$ $\frac{18}{12}(1\frac{1}{2}$ $\frac{12}{8}(1\frac{1}{2}$
QQ:			Aqq + AcE + AqEq + AEc + Eqq ∴ 81 . 54 . 36 . 24 . 16 ∵	$\frac{81}{54}(1\frac{1}{2}$ $\frac{54}{36}(1\frac{1}{2}$, &c.

Examples in Reſiduals.

Powers.	*Reſidual.*	*Parts.*
Q:	A — E	is Aq — 2AE + Eq.
C:		is Ac — 3AqE + 3AEq — Ec.
QQ:		Aqq — 4AcE + 6AqEq — 4AEc + Eqq. &c.

2. Difference of Names, what. 2. That the difference of the Names of every Binomial or Reſidual, is the Homogeneal Power of the Difference of the Names of the Root.

Example. *Scilicet*, Ac + 3AEq — 3AqE + Ec, or Ac + 3AEq — 3AqE — Ec, is the Cube of A — E.

3. Difference of Squares, &c. *what.* 3. That the Difference of the Squares of the Names of every Binomial or Reſidual, is the Homogeneal Power of the difference of the Squares of the Names of the Root.

Example. *Scilicet*, Q: Ac + 3AEq — Q:3AqE + Ec, is C: Aq — Eq.

4. Species aggregate in Names, &c. *what.* 4. That if the *Species* aggregate in Names alternately be Affirmative and Negative, the Sum of the Squares of the Names is the Homogeneal Power of the Sum of the Squares of the Names of the Root.

Example. *Scilicet*, Q: Ac — 3AEq + Q:3AqE — Ec, is C: Aq + Eq.

5. Intermediate Species, Powers of the Means. 5. That all the intermediate *Species* of every Order, are alſo Powers of the mean Proportionals between A and E.

Example. *Scilicet*, between Ac and Ec, there are two mean Proportionals AqE and AEq, which are alſo Cubes of M and N, (the four continual Proportionals being A.M.N.E): Wherefore A.√cAqE.√cAEq.E. are continual Proportionals. For AqE = AMN = Mc. And AEq = MNE = Nc.

Invention of Means ariſe hence. And hence ariſeth the Invention of any mean Proportionals between A and E; of which further is to be ſeen in the next Book. As if five mean Proportionals be ſought, then ſhall the Power be the ſixth Quantity or cc; the Index whereof exceeds

exceeds by an Unit the Number of the Means fought, and fhall be A. √ccAqcE. √ccAqqEq. √ccAcEc. √ccAqEqq. √ccAEqc. E. ∸

6. That every mean *Species* in every kind is made of the two Powers of the Names of the Root, the Indices whereof together, are equal to the Index of the fame kind; but the mean Diftances of the fame *Species* from their Extreams, fhall be equal to the Indices of the Alternate Factors, and anfwer to them in the Common Angle. *6. Mean Species made of what.*

Scilicet. AqEc in the 5*th* Power, is made of Aq multiplied into Ec, which anfwer in the common Angle; and is from Aqc the Third, and Eqc the Second. Example.

And hence came the former fpeedy and eafy Way of producing the Power of any Binomial by the Parodical Degrees. The Example given by Mr. *Oughtred* there, is in the 5*th* Power of the Binomial Root A+√qÆ, of which the Parodical Degrees may be thus placed with their Indices. *Hence came the production of Figurals in the Pages before.*

Example.

	4	3	2	1		
	Aqq .	Ac .	Aq .	A		Aqc
5					√qÆqc	5 √qÆAqcc
Aqc					5	10 ÆAc
	√qÆ .	Æ .	√qÆc .	Æq		10 √qÆcAqq
	1	2	3	4		5 ÆqA
						√qÆqc

This explained in Numbers, taking for the Root A+E, 82 of which the Segments being 80+2 or 8. 2. the Rectangle or Æ is 16, the fquare Root or √qÆ is 4. the Numbers that anfwer to the Parodical Degrees, and make up the whole Power of 84, *viz.* 4182119424, are as followeth. *Explained in Numbers.*

A .8	√qÆ .4 .		Aqc	32768
Aq .64	Æ .16 .	5	√qÆAqcc	81920
Ac .512	√qÆc .64 .	10	ÆAc	81920
Aqq .4096	Æq .256 .	10	√qÆcAqq	40960
Aqc .32768	√qÆqc .1024.	5	ÆqA	10240
			√qÆqc	1024
				4182119424

7. That if any *Species* be multiplied in Æ, the Magnitude produced fhall be the mean *Species* collateral in the following alternate Order, and the fame in Number from their Extreams. *7. What produced by a Species, × in Æ.*

As Ac×Æ, is AqqE, which is the firft from Aqc, and the fourth from Eqc. Examples.

So AcE×Æ, is AqqEq, which is the fecond from Acc, and the fourth from Ecc.

8. That if any *Species* be multiplied by A—E or X, the Magnitude produced fhall be the Difference between the two *Species* next each of the following Order. *8. What the Product of A—E× into a Species.*

As AcX =Aqq —AcE. AqEX=AcE—AqEq.

AEqX=AqEq—AEc. EcX =AEc— Eqq. Examples.

Wherefore if all the *Species* of every Order be multiplied by X, the Difference of the two extream Powers of the next upper Order will be produced.

As of Ac+AqE+AEq+Ec multiplied into X, will be made Aqq—Eqq.

9. That in the Orders of unequal Indices (l. c. qc. &c.) the Sum of the two extream Powers; but in the Orders of equal Indices, (q. qq. cc. &c.) the Difference of them, made of A+E multiplied into the fingular *Species* of the leffer Order precedent, is alternately Affirmative and Negative. *9. Sum of unequal difference of the equal Indices × into a Species, what produced.*

As Ac+Ec, made of Aq—AE+Eq, multiplied into A+E. Examples.

Alfo Aqq—Eqq, made of Ac—AqE+AEq—Ec, multiplied into A+E.

10. That if the fame Magnitude be multiplied into two contrary Magnitudes, the Magnitudes made of them fhall be alfo contrary. *10. Magnitude in contrary, what they produce.*

As Aq−2AE+Eq multiplied into A−E, fhall be made Ac−3AqE+3AEq−Ec.

But the fame multiplied into −A+E, fhall be made −Ac+3AqE−3AEq+Ec. Examples.

11. That the *Unciæ* or Numbers prefixt to the *Species*, are numerary Figures: For all under A and E are Roots; all under Aq and Eq are Triangulars; all under Ac and Ec are Pyramidals; all under Aqq and Eqq are Triangle-Triangulars; all under Aqc and Eqc are Triangle-Pyramidals; all under Acc and Ecc are Pyramidal-Pyramidals, *&c.* *11. Unciæ what they are, and how called.*

12. That if in any *Species* the number of the negative Sides be unequal, that *Species* fhall be Negative. *12. Unequal Sides Negative, in the Species.*

Examples.

As Q: A+B—C=Aq+2AB+Bq—2BC+Cq—2AC.
C: A+B—C=Ac+3AqB+3ABq+Bc—3BqC+3BCq—Cc—3CAq+3CqA—6ABC.

Root.	*Square.*		*Cube.*		
A+B—C	Aq	2AB	Ac	Bc	—Cc
	Bq	—2BC	3AqB		3ABq
	Cq	—2CA	—3BqC		3BCq
			3CqA		—3CAq
				—6ABC	

Roots of Rational Species extracted.

Thus much may suffice for the Genesis. Now for the Analysis of Rational *Species* or Extraction of their Roots, wherein nothing of difficulty occurs, the Extraction of Roots in other Numbers being learned, and especially Cossicks, between which and these Rational *Species* there is great Concinity: For Figurate *Species* Simple, as Simple Cossicks, and compound Figurate *Species* Binomial and Residual, as compound Cossicks, have their Roots extracted. Yet *Species* of a Polynomial Root differ in the pointing or pricking of the Number; for whereas in all other Extractions the figural Numbers were pricked according to their Quantities, in *Species* respect is to be had both to the Quantities and to the Number of distinct *Species* in the Root: For the Square that hath three *Species* in the Root, shall leave two Magnitudes unprickt between the right-hand Prick and the next, and but one Magnitude unprickt between the two next left-hand Pricks. But if the Root have four *Species* in it, then between the Dexter and next Sinister Prick, shall three Magnitudes be left unprickt, and 2 between the two next Pricks, and but one between that and the uttermost Sinister Prick. Likewise in pricking Cubes, if the Root consist of three *Species*, there shall be five Magnitudes left unprickt, between the Dexter and next Sinister Prick, and between that and the utmost Sinister Prick but two: And if the Root have four *Species* in it, then are nine Magnitudes left unprickt between the first two right-hand Pricks, five between the two next, and two between those next the left-hand, &c.

Figurate Species of a Polynomial Root pricked different from others.

Examples in the Square.

As to extract the Root of Aq+2AE+Eq, because the Root is a Binomial, only one *Species* is left unprickt as others: And the Root of Aq which is A, is placed in the Quotient; this doubled, that is 2A, is Divisor, which gets E in the Quotient: Then multiplying the Divisor thereby, and adjoining the Square of E, makes the *Gnomon* to be substracted.

Square Aq+2AE+Eq | A+E Root.
Aq
Divisor 2A
E
2AE+Eq *Gnomon.*

But to extract the Square Root of Aq+2BA+Bq+2CA+2BC+Cq, because the Root is a Polynomial of three *Species* Aq. Bq. & Cq. are the pricked *Species*. And after A+B is gotten in the Quotient, as by the Work above, then 2A+2B is Divisor, which gets C in the Quotient; and the Divisor multiplied thereby, and the Square of C adjoined, makes the second *Gnomon*.

Square Aq+2BA+Bq+2CA+2BC+Cq | A+B+C. Root.
Aq
Divisor 2A
B
2BA+Bq *Gnomon.*
Divisor 2A+2B
C
2CA+2BC+Cq *Gnomon.*

Examples in the Cube.

So to extract the Cube Root of Ac+3AqE+3AEq+Ec, the Root being a Binomial, the *Species* unprickt are as others: And the Root of Ac which is A, is placed in the Quotient: This squared and tripled is Divisor, which gets E in the Quotient, whereby the Divisor multiplied, and the triple of A by the Square of E and the Cube of E added together, make up the *Gnomon* to be substracted.

Cube

$$\begin{array}{llll} \text{Cube} & Ac+3AqE+AEq+Ec & | \underline{A+E} & \text{Root.} \\ & Ac & & \\ & \text{Divisor } 3Aq \quad 3A & & \\ & \qquad\quad\; E \quad\;\; Eq & & \\ & \overline{3AqE+3AEq+Ec} & \textit{Gnomon.} & \end{array}$$

But to extract the Cube Root of $Ac+3AqB+3ABq+Bc+3CAq+6ABC+3BqC+3ACq+3BCq+Cc$, because the Root is a Polynomial of three *Species*, according to the Instructions aforesaid, the Number is pricked: And after $A+B$ is gotten in the Quotient, as by the Work of the other Cube last mentioned; then $A+B$ squared and tripled, makes the Divisor $3Aq+6AB+3Bq$, whereby C is gotten for the Quotient; this multiplying the Divisor, and the Square of C by the triple of $A+B$, and added to the Cube of C, makes up the second *Gnomon* to be substracted.

$$\begin{array}{lll} & \textit{Cube.} & \textit{Root.} \\ & Ac+3AqB+3ABq+Bc+3CAq+6ABC+3BqC+3ACq+3BCq+Cc & | \underline{A+B+C} \\ & Ac & \\ \text{Divisor} & 3Aq \quad 3A & \\ & \;\; B \quad\; Bq & \\ \textit{Gnomon} & \overline{3AqB+3ABq+Bc} & \\ & \qquad\qquad \text{Divisor } 3Aq+6AB+3Bq \quad 3A+3B & \\ & \qquad\qquad\qquad\qquad\qquad C \qquad\qquad\quad Cq & \\ & \qquad\qquad \textit{Gnomon } \overline{3AqC+6ABC+3BqC+3ACq+3BCq+Cc.} & \end{array}$$

Besides this real Difference in pointing the Numbers, there is another seeming Difference in extraction of the Roots of Rational *Species* and others in the Divisor of all Powers above the Square: For whereas in all other Extractions, the Power next inferior to the given Number, whose Root is to be extracted, multiplied by the Index of the given Power, is counted the Divisor; As in the Square the Root multiplied by 2, the Index of the Square; in the Cube the Square multiplied by 3, the Index of the Cube; in the squared Square the Cube multiplied by 4, the Index of the squared Square, &c. In *Species* the Divisor is usually reckoned to consist of all the Parodical Degrees under the Power whose Root you are extracting, with the *Unciæ* annexed to those Degrees. *Divisors used in Extraction, how different from others.*

As in the Square $2A$, in the Cube $3Aq$ and $3A$, in the squared Square $4Ac$ and $6Aq$ and $4A$, &c. Nevertheless, inquiry for the next Quotient, Figure or *Species*, is made only by the left-hand *Species* thereof, which is the same with the Divisor in other Extractions: And the other Parts of the Divisor are set down, the better to direct the Operator to place the Parodical Degrees of the new-gotten Quotient, Figure or *Species* thereby, for Multiplication of them one into another to make up the *Gnomon*. Examples.

$$\begin{array}{ccc} \textit{Square.} & \textit{Cube.} & \textit{Squared Square.} \\ \underline{Aq} & \underline{Ac} & \underline{Aqq} \\ \text{Divisor} \mid \left.\begin{array}{r} 2A:E \\ Eq \end{array}\right\} \textit{Gnomon.} & \text{Divisor} \left\{\begin{array}{r} 3Aq:E \\ 3A\;\;:Eq \end{array}\right. \left.\begin{array}{r} \\ \\ Ec \end{array}\right\} \textit{Gnomon.} & \text{Divisor} \left\{\begin{array}{r} 4Ac:E \\ 6Aq:Eq \\ 4A\;\;:Ec \end{array}\right. \left.\begin{array}{r} \\ \\ \\ Eqq \end{array}\right\} \textit{Gnomon.} \end{array}$$

Thus that most curious Analyst Mr. *Oughtred* practises, not only in all his Resolution of Affected Equations, but even in extracting the Roots of plain Figural Numbers, to which as more demonstrative he adjoineth the *Species*, after the manner in the two Examples following, of the Square and Cube of 845; in which latter the first Divisor is 1944, which by the common Way is but 192: And the next Divisor, which ordinarily would be but 21168, is 211932. In the Square there ariseth no Difference in the Divisor, because the Quantity is next the Root. *Practice of Mr. Oughtred.*

Square

Examples.

Square 784 / 714025 — Root. (845

64			Aq
1	6		2A *Divisor.*
6	4		2AE
	16		Eq *Gnomon.*
6	56		
	16	8	2A *Divisor.*
	84	0	2AE
		25	Eq *Gnomon.*
	84	25	

Cube 20 / 91647 / 603351125 — Root. (845

512			Ac
19	2		3Aq *Divisor.*
	24		3A
15	44		
76	8		3AqE
3	84		3AEq *Gnomon.*
	64		Ec
80	704		
2	116	8	3Aq *Divisor.*
	2	52	3A
2	119	32	
10	584	0	3AqE
	63	00	3AEq *Gnomon.*
		125	Ec
10	647	125	

Proof of Figuration of Rational Species.

Production of Rational *Species* and Extraction of their Roots, are mutual Proofs of the Truth of each other's Operations, as in other Figural Numbers. Besides which, if any Scruple arise in either, trial may be made of both, by taking Absolute Numbers, and working therewith. As instead of other Examples, the last of the Analysis of the Square and Cube of 845, with the Examples of their Production before in this Chapter, are sufficient Evidence.

CHAP. XII. *Reduction of Irrational Species.*

Reduction of Irrational Species.

In pursuit of *Species*, I am now come to *Irrationals*, which in their Operations and Resolutions follow *Surds*, as the Rational *Species Cossicks*. And having been described before, *Chap.* 1. of *Species*, I proceed forthwith to their Reduction.

Irrational Species are either to be reduced to their least Terms, or from different Denominations into one.

To their least Terms.

To abbreviate or lessen the Terms of an *Irrational Species*, is when the Denomination is Compound, and the Number annexed hath a Root that may be expressed by part of that Compound Denomination; then extract the Root of the Number, and alter the *Species* accordingly.

Examples.

As √qq25 and √cc81, the Indices 4 and 6 reduced to their least Terms, are 2 and 3, by the common Divisor 2 the Index of the Square: If therefore the Square Root of 25 and 81 be taken, the Surd *Species* shall be reduced to √q5 and √c9.

	2) 4	6	2	3	
√qq25 and √cc81	qq	cc	q	c	√q5 and √c9
	25	81	5	9	

So √cc27 may be reduced to √q3, and discharged of c.
And √qqcc32 discharged of qcc, may be reduced to √q2.

To one Denomination.

To reduce *Irrational Species* of different Denominations into one, comprehends,

With an Absolute Number.

First, To reduce an Absolute Number into the Denomination of a Surd or *Irrational Species*; and this is done by multiplying the Number according to the Power of the *Species*.

Examples.

As 2 and √qB reduced, is √q4 and √qB.
So 2 and √cB reduced, is √c8 and √cB.

Secondly,

Secondly, To reduce two *Irrational Species* of uncompounded Indices into one Denomination; And this is effected by multiplying each Quantity as his Alternate Power or *Species* doth shew. *With uncompound Indices.*

As √qB & √cC reduced, is √ccBc & √ccCq *Example.*

√q		√c		√q
B	X	C		√c
Bc		Cq		√cc

Thirdly, To reduce two *Irrational Species* of compounded Indices into one Denomination. And this is performed by dividing the Indices by the common Divisor, and then by the least Terms thereof multiplying alternately both the given Indices for the new Index, and the Numbers into the Powers of these least Terms. *With compound Indices.*

	Examples in Numbers.		*Species.*		
Surds to be reduced	√qq10 and √cc7		√qAq and √qqBq		*Examples.*
Common Divisor	2) 4qq	6cc	2) 2q	4qq	
	2q	3c	1√	2q	
Least Terms.	3	2	2	1	
Surds reduced	√cccc1000	√cccc49	√qqAqq	√qqBq	

As before in Reduction of Surds one part of Reduction, which lessened the Terms, was observed to prove the other, which exalted the lessened Surds into the Powers from whence they were abated; and reciprocally that part of Reduction which increaseth their Terms and Denominations, by extracting the Roots and abating the Characters: So here. *Proof of Reduction of Irrational Species.*

Also all sorts of Reduction of *Irrational Species* may be proved, by taking in their stead Rational Numbers, and working with them after the manner of Surds, as in the Chapter of *Reduction of Surds* is so fully exemplified that Example here need not.

CHAP. XIII. *Addition of Simple Irrational Species.*

IRrational *Species* agreeing in their Simple Elements with Surds, before explicated in the 5*th* Part of this third Book, the fewer Examples, and a more brief Repetition of the Rules may serve turn here. *Addition of Irrational Species Simple.*

Addition of the Simple may be included in four *Cases*.

Case 1. If the *Species* and Numbers (if any be annexed) be commensurable, then order the Numbers, whether Integers or Fractions, as Simple Surds before, and the *Species* in like manner; that is, figurate the Sum of their Roots according to their Quantities, and multiply that Quantity by the Common Divisor. *1. Commensurable.*

As to add ꟿRPq to ꟿRSq. The Common Divisor is R. The Squares Pq & Sq. The Roots P & S; which squared and multiplied into R, makes the Total ꟿRPq +2PRS+RSq, which is all one in equality with ꟿRPq+ꟿRSq. *Examples.*

		Addends.					
Common Divisor	ꟿR	ꟿRPq ꟿRSq	Squares	Pq Sq	Roots	P. S.	
Total		ꟿRPq+2PRS+RSq					

P+S	Root.
P+S	
Pq+PS	
PS+Sq	
Pq+2PS+Sq	Square.
ꟿR	Com. Divisor.
ꟿRPq+2PRS+RSq	Total.

Another Example with Numbers.

Addends.

ꟿ3) ꟿ27Aq (9Aq . 3A.
ꟿ3Bq (1Bq . 1B.

Total ꟿ27Aq+18BA+3Bq

3A +B
3A +B
9Aq+3BA
3BA+Bq
9Aq+6BA+Bq
ꟿ3
ꟿ27Aq+18BA+3Bq = ꟿ27Aq+ꟿ3Bq.

2. Incommensurable. *Case* 2. If the *Species* or Numbers annexed, or both be incommensurable, then join them together with +.

Examples. As √qB and √qC added, are √qB+√qC.

So √c5B added to √c10C, make √c5B+√c10C.

3. Heterogeneal. *Case* 3. If the Denominations of the Quantities be Heterogeneal, they may be reduced, and then added or conjoined as before.

Example. As to add the √qB & √cC, they are reduced as in the precedent Chapter to the Denomination of cc, and then conjoined with the Sign of Addition thus; √ccBc +√ccCq.

4. Different Signs. *Case* 4. If the Signs be different, conjoin the given Irrationals with the Sign —.

Example. As to add √qB with —√qA, the Total shall be √qB—√qA.

Irrational added to it self. Here also, as in Simple Surds, may be observed, to add any *Irrational Species* to it self, is to multiply the Squares by 4, the Cubes by 8, &c.

Examples. As √qB+√qB, is √q4B. And √cD+√cD, is √c8D.

Proof of Addition of Simp. Irrat. Species. *Addition of Simple Irrational Species*, is to be proved by their Substraction in the next Chapter, and by taking Rational *Species* or Numbers instead of Irrational. As in the last Example, supposing the √cD be 2, then shall D be 8, and 8D 64, whose Cube Root will be 4 equal to 2 added to it self.

CHAP. XIV. *Substraction of Simple Irrational Species.*

Substraction of Irrational Species Simple. As Addition, so the Substraction of these Irrationals may be included in four *Cases*.

1. Commensurable. *Case* 1. If the *Species* and Numbers (if any be annexed) be commensurable, then (whether Integers or Fractions) order the Numbers as in Substraction of Simple Surds before, and the *Species* likewise; that is, figurate the Difference of their Roots according to the given Quantities, and multiply that Quantity by the Common Divisor.

Examples. As to take WRSq from WRPq, the Common Divisor is R, the Squares Pq and Sq, the Roots P and S, the Difference P—S; which squared and multiplied into R, makes the Remain WRPq—2PRS+RSq, which is equal to WRPq—WRSq.

Subtrahend.

WRPq—WRSq Squares (Pq Roots P
Common Divisor WR (Sq S

Remain WRPq—2PRS+RSq

P—S
P—S
Pq-PS
-PS+Sq
Pq-2PS+Sq
WR
WRPq-2PRS+RSq

Another Example with Numbers.

Subtrahend.

W27Aq—W3Bq (9Aq . 3A.
Common Divisor W3 (1Bq . 1B.

Remain W27Aq-18BA+3Bq

3A—B
3A—B
9Aq-3BA
-3BA+Bq
9Aq-6BA+Bq
W3

W27Aq—W3Bq=W27Aq—18BA+3Bq

2. Incommensurable. *Case* 2. If the *Species* or Numbers annexed, or both, be Incommensurable, then join them together with —.

Examples. As to take √qC from √qB, the Remain shall be √qB—√qC.

So √c9B taken from √c10C, shall make the Remain √c10C—√c9B.

3. Heterogeneal. *Case* 3. If the given Denominations of the Quantities be Heterogeneal, they may be reduced, and then substracted or conjoined as before.

Example. As to take √qB from √cC, being reduced as before in *Chap.* 12. *Of Reduction of Irrational Species*, to the Denomination of cc, they are then conjoined with the Sign of Substraction thus, √ccCq—√ccBc.

Case

Case 4. If the Signs be different, the Irrationals given are to be added; And the Sign of the Remain ſhall be contrary to the Sign of the Subtrahend, as before in Simple Surds, if they be commenſurable; but if incommenſurable, conjoin them by the Sign of Addition $+$. 4. *Different Signs.*

As to take $-\surd q3BD$ from $\surd q12A$, becauſe the Signs are unlike, the one $+$ and the other $-$ they are added, and the Remain ſhall be $\surd q12A + \surd q3BD$. Examples.

And $-\surd qC$ taken from $\surd qB$, and $\surd qC$ taken from $-\surd qB$, leave their Remains $\surd qB + \surd qC$, and $-\surd qB + \surd qC$.

Here alſo, as in Subſtraction of Simple Surds, may be obſerved, that to take any Irrational from it ſelf, leaves nothing remaining; but to take half any Irrational *Species*, is to divide the Squares by 4, the Cubes by 8, &c. *Irrational taken from it ſelf, or the Half taken.*

As to take $\surd qB$ from $\surd qB$, the Remain is Nought. Examples.

But to take half the $\surd qB$ is $\frac{\surd B}{4}$. And half the $\surd cB$ is $\frac{\surd\surd B}{8}$

Subſtraction of Simple Irrational *Species* is proved by their Addition: And by taking Rational *Species* or Numbers, inſtead of thoſe Irrational, and working therewith: As in the laſt Example, if half the $\surd cB$ be $\frac{\surd cB}{8}$, then by the Addition thereof to it ſelf, it ſhall be $\frac{\surd c8B}{8}$, and by clearing the Number of 8, it ſhall be $\surd cB$ as before. *Proof of Subſtraction of Sim. Irrat. Species.*

Alſo ſuppoſe B were 64, the $\surd c$ thereof ſhall be 4; which halfed, ſhall be 2 equal to the $\surd c8$.

CHAP. XV. *Multiplication of Simple Irrational Species.*

TO the Multiplication of theſe Irrationals, two Caſes will ſerve.

Caſe 1. If the *Species* be Homogeneal in their figurate Denominations, then multiply Numbers as Numbers, and *Species* as *Species*, Integers as Integers, and Fractions as Fractions. *Multiplication of Irrational Species Simple.* 1. *Homogeneal.*

As $\surd c12B$ multiplied by $\surd c5C$, produceth $\surd c60BC$. Examples.

Factors.	*Product.*	*Factors.*	*Product.*
$\surd qA \times \surd qB$	$= \surd qBA$	$\surd qq12B \times \surd qqD$	$= \surd qq12BD$.

Caſe 2. If the *Species* be Heterogeneal in their figurate Denominations; then after Reduction to one Denomination, multiply them as before. 2. *Heterogeneal.*

As $\surd qA$ to be multiplied into $\surd cB$, is reduced to $\surd ccAc$, and $\surd ccBq$; and then multiplied to $\surd ccAcBq$. Example.

Reduction.

$$\begin{array}{c} A \\ \surd q \\ \hline Bq \end{array} \times \begin{array}{c} B \\ \surd c \\ \hline Ac \end{array} \quad \begin{array}{c} \surd q \\ \surd c \\ \hline \surd cc \end{array}$$

Multiplication.

$$\begin{array}{c} \surd cc\ \begin{matrix} Ac \\ Bq \end{matrix} \\ \hline \surd ccAcBq \end{array}$$

Here, as in Multiplication of Simple Surds, theſe Confectaries take place. *Confectaries.*

1. To multiply any *Irrational Species*, is to increaſe him by the Power of a Root Homogeneal. 1. *How to double, triple, &c.*

Irrational.	*Doubled.*	*Tripled.*	*Quadrupled.*
	$\surd qB$	$\surd qB$	$\surd qB$
Squares.	4	9	16
	$\surd q4B$	$\surd q9B$	$\surd q16B$
	$\surd cD$	$\surd cD$	$\surd cD$
Cubes.	8	27	64
	$\surd c8D$	$\surd c27D$	$\surd c64D$

Examples.

2. To multiply ſome Irrationals, produce Rationals, and the Product, as occaſion is, may be cleared of the Surd Character. 2. *Rationals produced.*

Irrationals.	*Square.*	*Cube.*
	$\surd q18B$	$\surd c250C$
	$\surd q\ 2E$	$\surd c\ 4D$
Rationals.	$\surd q36BE = 6BE$.	$\surd c1000CD = 10CD$.

Examples.

3. To

3. When to cancel the Character. 3. To multiply the Side of any Power according to the Exigency of his Kind, is but to blot out or cancel the Note of the Side, and leave the *Species* absolute.

Examples. As $\surd qB \times \surd qB = B$. And $\surd c12D \times \surd c12D \times \surd c12D = 12D$.

4. What produced by the Side of a Power, &c. 4. To multiply the Side of a Power, whose Index is a Compound Number, according to the Exigency of one of the Kinds compounding; the Side of either Kind may be prefixed to the special Number alone.

Examples. As the Square of the $\surd ccB$, is $\surd cB$; and the Cube of the $\surd ccB$, is $\surd qB$: For $\surd cc$ is $\surd$ of 2×3.

5. Homogeneal Figurals multiplied, what produced. 5. To multiply a figural Number by an Homogeneal figural Number, the Product shall be a figural Number of the same Kind, whose Side or Root shall be equal to the Product of the Sides of the Number multiplied.

Examples.

As	$Aq \times Eq = AqEq$.	And	$Ac \times Ec = AcEc$.
q.	$25 \times 16 = 400$	c	$27 \times 8 = 216$
$\surd$.	$5 \times 4 = 20$	$\surd$	$3 \times 2 = 6$

6. Sides of such multiplied, what produced. 6. To multiply the Sides of Homogeneal *Species* Irrational, begetteth Sides of Irrational *Species* Homogeneal.

Examples. As $\surd qA \times \surd qE = \surd qAE$. And $\surd cB \times \surd cC = \surd cBC$.

Proof of Multiplication of Sim. Irat. Species. Multiplication of Simple Irrational *Species*, is to be proved by their Division in the next Chapter; and by taking Rational *Species* or Numbers, and working therewith instead of Irrational. As in the last Example, if B be 27, and C 8, the Product will be 216; the Cube Root whereof is 6, equal to 3 the Root of 27, multiplied into 2 the Root of 8.

CHAP. XVI. *Division of Simple Irrational Species.*

Division of Irrational Species Simple. As Multiplication, so the Division of these Irrationals may be included in two Cases.

1. Homogeneal. *Case* 1. If the *Species* be Homogeneal in their figurate Denominations, then divide the Dividend by the Divisor, Numbers as Numbers, and *Species* as *Species*, Integers as Integers, and Fractions as Fractions.

Examples. As $\surd qBA$ divided by $\surd qB$, giveth in the Quotient $\surd qA$.

And $\surd c600BC$ divided by $\surd c12B$, giveth in the Quotient $\surd c5C$.

	Dividend.			*Dividend.*	
Divisor $\surd qB$)	$\surd qBA$	($\surd qA$ Quotient.	$\surd c12B$)	$\surd c600BC$	($\surd c5C$.

2. Heterogeneal. *Case* 2. If the *Species* be Heterogeneal in their figurate Denominations, then first reduce them into an Homogeniety, and afterward divide them as before.

Example. As $\surd ccAcBq$ divided by the $\surd qA$, the Divisor is first reduced to $\surd ccAc$; and then Division being made, the Quotient is $\surd ccBq$, which may be depressed to $\surd cB$.

Reduction.

$$
\begin{array}{ccc}
 & \surd ccAc & \surd ccAcBq \\
2) & \surd qA \quad \times & \surd ccAcBq \\
 & \surd & c \\
\hline
 & \surd ccAc & \surd ccAcBq
\end{array}
$$

Division.

$\surd ccAc$) $\surd ccAcBq$ ($\surd ccBq$

Consectaries. Here, as in Division of Simple Surds, these Consectaries are fitly inserted.

1. To take half, &c. 1. To divide any Irrational *Species*, is to diminish him by the Power of a Root Homogeneal.

Examples.

Irrational.	*The Half.*	*The third Part.*	*The fourth Part.*
Squares.	$\frac{\surd q12B}{4}$ ($\surd q3B$	$\frac{\surd q27B}{9}$ ($\surd q3B$	$\frac{\surd q48B}{16}$ ($\surd q3B$
Cubes.	$\frac{\surd c24B}{8}$ ($\surd c3B$	$\frac{\surd c81B}{27}$ ($\surd c3B$	$\frac{\surd c192B}{64}$ ($\surd c3B$

2. Quotient some time Rational. 2. To divide some Irrationals, brings forth Rationals in the Quotient. And the Quotients, as occasion is, may be cleared of the Surd Character.

Square.

Irrational. $\frac{\surd q27BE}{\surd q3B}\big($ $\surd q9E = 3E$ *Rational.* Examples.

Cube.

Irrational. $\frac{\surd c24DC}{\surd c\,3\,C}\big($ $\surd c8D = 2D$ *Rational.*

3. To divide any *Irrational Species* by himself, giveth in the Quotient one *Irrational Species* or Unit. *3. Irrational dividing himself gives 1.*

Examples in *Squares.* *Cubes.*

$\frac{\surd qB}{\surd qB}\big($ 1, or B. $\frac{\surd c9B}{\surd c9B}\big($ 1, or B. Examples.

4. To divide a Power whose Index is compounded, by one Side of the Compounding Powers, shall give the Quotient higher or lower according to the Dividing Power. *4. Quotient of a Power, &c. divided by the Side.*

A's $\surd qB)\ \surd ccB\ (\surd cB.$ And $\surd cB)\ \surd ccB\ (\surd qB.$ Examples.

5. To divide a Figural Number by another Homogeneal figural Number, the Quotient shall be a figural Number of the same Kind, whose Side is equal to the Quotient of the greater Side divided by the Lesser. *5. Homogeneal Figurals divided, what the Quotient.*

As $\surd q\frac{AqEq}{Bq}\big(\frac{AE}{B}.$ Example.

6. To divide the Sides of Homogeneal Irrational *Species*, begetteth Sides of Irrational *Species* Homogeneal. *6. Sides of such divided, what begotten.*

Ergo, $\surd qA)\ \surd qAE\ (\surd qE.$ Example.

Division of Simple Irrational Species, is proved by their Multiplication; and by taking Rational *Species* or Numbers instead of Irrational, and working therewith. As in the last Example, and several others of this Chapter made exemplary in Multiplication, bring here in the Quotient one of the Factors there. *Proof of Division of Simple Irrat. Species.*

Also suppose A 25 and E 9, then $\surd qA$ which is 5 multiplied into $\surd qE$ that is 3, will be 15 for $\surd qAE$: And this divided by $\surd qA$ that is 5, giveth 3 in the Quotient equal to $\surd qE$.

CHAP. XVII. *Addition of Compound Irrational Species.*

ADdition of Irrational *Species* that are Compound, includes the Addition of Particulars and Universals. *Addition of Comp. Irrat. Species. Particulars.*

Particulars, like or unlike, are added as the Simple, and joined together into one Total with the respective Signs.

Examples.	*Binomials.*	*Residuals.*	*Mixt.*
Addends.	$Z + \surd qBC$	$\surd qB - 3D$	$\surd cRq + 6B$
	$R + \surd qDE$	$\surd qB - 2C$	$\surd cRq - 6B$
Totals.	$Z + R + \surd qBC + \surd qDE$	$\surd q4B - 3D - 2C$	$\surd c8Rq$

Examples.

Another Example.

$$\textit{Addends}\begin{cases}\frac{Z}{2} \ast + \surd q\frac{Zq - 4P}{4} \\ \frac{Z}{2} \ast - \surd q\frac{Zq - 4P}{4}\end{cases}$$

$$\textit{Total.}\quad Z$$

Here the plain and fracted Sinister *Species* being Z and Z, that is 2Z to be divided by 2, makes Z only to be left for the Total. All the dexter Part of the Addends being of contrary Signs, *viz.* one + and the other −, and so Equal, are taken away.

Universals are added as other Compound Irrationals, with respect had to their Signs, and the Sign Universal $\surd$: prefixed, as was largely discoursed in the Addition of Universal Surds before. *Universals.*

Examples.

Examples.	*Binomials.*	*Refiduals.*	*Mixt.*
Addends.	$\sqrt{:}\,Zq+4Pq$	$\sqrt{:}\,Zq-4Pq$	$\sqrt{:}\,Zq+4Pq$
	$\sqrt{:}\,Zq+4Pq$	$\sqrt{:}\,Zq-4Pq$	$\sqrt{:}\,Zq-4Pq$
Totals.	$\sqrt{:}\,4Zq+16Pq$	$\sqrt{:}\,4Zq-16Pq$	$\sqrt{:}\,2Zq+\sqrt{q}\,4Zqq-64Pqq$

The two firſt of theſe Examples are wrought according to the 7*th* Direction in Addition of Univerſal Surds, *Chap.* 7. *Book* 3. *Part* 5. but the laſt according to the Second Form of Addition of Square Surds, *Chap.* 3. *Book* 3. *Part* 5.

Other Examples explained in Numbers.

Other Examples in Species *and Numbers correſponding.*

$$\text{Suppoſing}\begin{cases} A\,3.\ E\,2.\ Z\,5.\ P\,6. \\ Aq\,9.\ Eq\,4.\ Z\,13.\ X\,5. \end{cases}$$

$$\begin{array}{lll}
\sqrt{:}\,Z+\sqrt{q}\,Zq-4Pq & \sqrt{:}\,13+\sqrt{q}\,169-144 & \text{or}\ \sqrt{:}\,13+\sqrt{q}\,25 \quad \text{or}\ \sqrt{q}\,18 \\
\sqrt{:}\,Z+\sqrt{q}\,Zq-4Pq & \sqrt{:}\,13+\sqrt{q}\,169-144 & \phantom{\text{or}}\ \sqrt{:}\,13+\sqrt{q}\,25 \quad \phantom{\text{or}}\ \sqrt{q}\,18 \\
\hline
\sqrt{:}\,4Z+\sqrt{q}\,16Zq-64Pq & \sqrt{:}\,52+\sqrt{q}\,2704-2304 & \sqrt{q}\,72
\end{array}$$

$$20 \quad -\frac{2304}{400}\Big(20\ \sqrt{} \qquad \sqrt{q}\,72\ \textit{Value}$$

$$\begin{array}{ll}
\sqrt{:}\,\frac{Z}{2} \overset{*}{+} \sqrt{q}\,\frac{Zq-4Pq}{4} & \sqrt{:}\,\frac{13}{2} \overset{*}{+} \sqrt{q}\,\frac{169-144}{4} \\
\sqrt{:}\,\frac{Z}{2}+\sqrt{q}\,\frac{Zq-4Pq}{4} & \sqrt{:}\,\frac{13}{2}+\sqrt{q}\,\frac{169-144}{4} \\
\hline
\sqrt{:}\,\frac{4Z}{2}+\sqrt{q}\,\frac{16Zq-64Pq}{4} & \sqrt{:}\,\frac{52}{2}+\sqrt{q}\,\frac{2704-2304}{4} \\
\hline
\text{or}\ \sqrt{:}\,2Z+\sqrt{q}\,4Zq-16Pq & \sqrt{:}\,26+\sqrt{q}\,676-576
\end{array}
\quad \text{or} \begin{cases} \sqrt{:}\,\frac{13}{2}+\sqrt{q}\,\frac{25}{4} \\ \sqrt{:}\,\frac{13}{2}+\sqrt{q}\,\frac{25}{4} \end{cases}
\text{or} \begin{cases} \sqrt{q}\,\frac{18}{2}=3 \\ \sqrt{q}\,\frac{18}{2}=3 \end{cases} \ \frac{}{6}$$

$$10 \quad -\frac{576}{100}\Big(10\ \sqrt{} \qquad \sqrt{q}\,36 \quad (6\ \textit{Value.}$$

$$\begin{array}{lll}
\sqrt{:}\,Z-\sqrt{q}\,Zq-4Pq & \sqrt{:}\,13-\sqrt{q}\,169-144 & \text{or}\ \sqrt{:}\,13-\sqrt{q}\,25 \quad \text{or}\ \sqrt{q}\,8 \\
\sqrt{:}\,Z-\sqrt{q}\,Zq-4Pq & \sqrt{:}\,13-\sqrt{q}\,169-144 & \phantom{\text{or}}\ \sqrt{:}\,13-\sqrt{q}\,25 \quad \phantom{\text{or}}\ \sqrt{q}\,8 \\
\hline
\sqrt{:}\,4Z-\sqrt{q}\,16Zq-64Pq & \sqrt{:}\,52-\sqrt{q}\,2704-2304 & \sqrt{q}\,32
\end{array}$$

$$-20 \quad -\frac{2304}{400}\Big(20\ \sqrt{} \qquad \sqrt{q}\,32 \quad (\textit{Value.}$$

$$\begin{array}{ll}
\sqrt{:}\,\frac{Z}{2} \overset{*}{-} \sqrt{q}\,\frac{Zq-4Pq}{4} & \sqrt{:}\,\frac{13}{2} \overset{*}{-} \sqrt{q}\,\frac{169-144}{4} \\
\sqrt{:}\,\frac{Z}{2}-\sqrt{q}\,\frac{Zq-4Pq}{4} & \sqrt{:}\,\frac{13}{2}-\sqrt{q}\,\frac{169-144}{4} \\
\hline
\sqrt{:}\,\frac{4Z}{2}-\sqrt{q}\,\frac{16Zq-64Pq}{4} & \sqrt{:}\,\frac{52}{2}-\sqrt{q}\,\frac{2704-2304}{4} \\
\hline
\text{Or}\ \sqrt{:}\,2Z-\sqrt{q}\,4Zq-16Pq & \sqrt{:}\,26-\sqrt{q}\,676-576
\end{array}
\quad \text{or} \begin{cases} \sqrt{:}\,\frac{13}{2}-\sqrt{q}\,\frac{25}{4} \\ \sqrt{:}\,\frac{13}{2}-\sqrt{q}\,\frac{25}{4} \end{cases}
\text{or} \begin{cases} \sqrt{q}\,\frac{8}{2}=2 \\ \sqrt{q}\,\frac{8}{2}=2 \end{cases} \ \frac{}{4}$$

$$-10 \quad -\frac{576}{100}\Big(10\ \sqrt{} \qquad \sqrt{q}\,16 \quad (4\ \textit{Value.}$$

All theſe four laſt Examples are wrought according to the 7*th* Direction in Addition of Univerſal Surds, *Chap.* 7. *Book* 3. *Part* 5.

$$\begin{array}{l|l}
\sqrt{:}Z+\sqrt{q}\,Zq-4Pq & \sqrt{:}13+\sqrt{q}\,169-144 \\
\sqrt{:}Z-\sqrt{q}\,Zq-4Pq & \sqrt{:}13-\sqrt{q}\,169-144 \\
\hline
\sqrt{:}2Z+4P & \sqrt{:}\,26+24
\end{array}
\quad \text{or} \left\{\begin{array}{l}\sqrt{:}13+\sqrt{q}25 \\ \sqrt{:}13-\sqrt{q}25\end{array}\right\} \text{or} \left\{\begin{array}{l}\sqrt{q}18 \\ \sqrt{q}\ 8 \\ \hline \sqrt{q}50\end{array}\right.$$

Other Examples explained in Numbers.

$$\begin{array}{l|l}
\sqrt{:}\frac{Z}{2}\overset{*}{+}\sqrt{q}\frac{Zq-4Pq}{4} & \sqrt{:}\frac{13}{2}\overset{*}{+}\sqrt{q}\frac{169-144}{4} \\
\sqrt{:}\frac{Z}{2}-\sqrt{q}\frac{Zq-4Pq}{4} & \sqrt{:}\frac{13}{2}-\sqrt{q}\frac{169-144}{4} \\
\hline
\sqrt{:}\,Z+2P & \sqrt{:}\,13+12
\end{array}
\left\}\text{or}\left\{\begin{array}{l}\sqrt{:}\frac{13}{2}+\sqrt{q}\frac{25}{4} \\ \sqrt{:}\frac{13}{2}-\sqrt{q}\frac{25}{4}\end{array}\right\}\text{or}\left\{\begin{array}{l}\sqrt{q}\frac{18}{2}=3 \\ \sqrt{q}\frac{8}{2}=2 \\ \hline \qquad 5\end{array}\right.\right.$$

The Work of theſe two laſt Examples is like the ſecond Form of Addition of ſquare Surds, *Chap.* 3. *Book* 3. *Part* 5. For firſt the Surds are added, which in the upper Example make $2Z$, in the lower $\frac{2Z}{2}$, that is Z being divided; then they are multiplied, the upper Product is Zq the Siniſter part, and $-Zq+4Pq$ the Dexter part, the lower Product is $\frac{Zq}{4}-\frac{Zq+4Pq}{4}$: for according to the third Conſectary in Multiplication of Simple Surds, and alſo of Simple Irrational *Species*, the Surds multiplied by themſelves have their Characters cancelled: So as $\sqrt{q}Zq-4Pq$ multiplied by it ſelf, ſhall be $Zq-4Pq$; but in the Examples above it is $Zq+4Pq$, becauſe the Signs of the given Numbers were + and −. The Products thus gotten, the Parts found with + and − of like Denomination are ſet aſide; ſo as of the upper Product is left only $4Pq$, of the lower $\frac{4Pq}{4}$, which divided is Pq. The Roots of theſe are $2P$ and P; which multiplied by 2, the Index of the Square, make $4P$ to be added to the upper $2Z$, and $2P$ to the lower Z.

$$\begin{array}{l|l}
\sqrt{:}\sqrt{\frac{Xq+4Pq}{4}}\overset{*}{+}\frac{X}{2} & \sqrt{:}\frac{25+144}{4}\overset{*}{+}\frac{5}{2} \\
\sqrt{:}\sqrt{\frac{Xq+4Pq}{4}}-\frac{X}{2} & \sqrt{:}\frac{25+144}{4}-\frac{5}{2} \\
\hline
\sqrt{:}\sqrt{Xq+4Pq}+2P & \sqrt{:}\,25+144+12 \\
 & \frac{144}{169}\left(\overset{\sqrt{}}{13}+12\right.
\end{array}
\left\}\text{or}\left\{\begin{array}{l}\sqrt{q}\frac{169}{4}+\frac{5}{2} \\ \sqrt{q}\frac{169}{4}-\frac{5}{2}\end{array}\right\}\text{or}\left\{\begin{array}{l}\sqrt{q}\ \frac{18}{2}\ (3 \\ \sqrt{q}\ \frac{8}{2}\ (2 \\ \hline \qquad 5\end{array}\right.\right.$$

Another Example explained with Numbers.

The Total of this Example is miſprinted in *Moore*, being there $\sqrt{X}qq+Pq+2P$; which cannot be, for the Addition is $\frac{4Xq+16Pq}{4}$, and by Diviſion to clear the Fraction, is $Xq+4Pq$; the Multiplication is thus, $\frac{Xq+4Pq}{4}-\frac{Xq}{4}$, and by cancelling the contrary *Species*, is left only $\frac{4Pq}{4}$; which by Diviſion is Pq, whoſe Root is P; this multiplied by 2, is $2P$ to be added to the firſt Addition, as I have ſet it.

Proof of Addition of Compound Irrat. Species.

Addition of Compound Irrational Species is proved by Subſtraction, or by Rational Numbers, like Addition of Compound Surds before; the Work of Particulars by Particulars, and Univerſals by Univerſals: And many of theſe Examples have their Rational Numbers beſides them to ſave farther Inſtance.

CHAP. XVIII. *Subſtraction of Compound Irrational Species.*

Subſtraction of Comp. Irrat. Species.

AS Addition before, ſo *Subſtraction of Compound Irrational Species*, includes the Subſtraction both of Particulars and Univerſals.

Particulars.

Particulars, as the Simple Irrationals take like *Species* from like, and to the Remain their proper Sign is ſubſcribed unleſs the greater be taken from the leſſer, for then the Sign is to be changed: And if the Signs be contrary, though the

Species

Species be alike, then the Sign of the Sum is to be contrary to the Sign of the Subtrahend. Also *Species* unlike and Asymmetral as before, are substracted, by conjoining them with —.

Examples.

Examples.	*Binomials.*	*Residuals.*	*Mixt.*
Greater Numbers.	$Z + \sqrt{q}SR$	$\sqrt{q}B - 8$	$\sqrt{c}Rq + 4D$
Subtrahends.	$R + \sqrt{q}AE$	$\sqrt{q}B - 4$	$\sqrt{c}Rq - 4D$
Remains.	$Z - R + \sqrt{q}SR - \sqrt{q}AE$	4	$8D$

Another Example.

$$\begin{array}{ll} \text{Greater Number.} & \frac{Z}{2} \overset{*}{+} \sqrt{q}\frac{Zq - 4P}{4} \\ \text{Subtrahend.} & \frac{Z}{2} \overset{*}{-} \sqrt{q}\frac{Zq - 4P}{4} \\ \text{Remain.} & \sqrt{q}\frac{4Zq - 16P}{4} \\ & \text{Or } \sqrt{q}Zq - 4P \end{array}$$

In this Example the sinister Parts of the *Data* being both alike and equal, *viz.* $\frac{Z}{2}$, are set aside; the Residue being of contrary Signs, are added together; and afterward Division by 4 being made to discharge the Fraction, the Remain is $\sqrt{q}Zq - 4P$.

Universals.

Universals, like other Compound Irrationals, are substracted with respect to their Signs, and the Sign Universal prefixed, as before discoursed in Surds.

Examples.

Examples.	*Binomials.*	*Residuals.*	*Mixt.*
Greater Numbers.	$\sqrt{:}4Zq + 16Pq$	$\sqrt{:}4Zq - 16Pq$	$\sqrt{:}2Zq + \sqrt{q}4Zqq - 64Pqq$
Subtrahends.	$\sqrt{:}\ Zq + 4Pq$	$\sqrt{:}\ Zq - 4Pq$	$\sqrt{:}\ Zq - 4Pq$
Remains.	$\sqrt{:}\ Zq + 4Pq$	$\sqrt{:}\ Zq - 4Pq$	$\sqrt{:}\ Zq + 4Pq$

The first two of these Examples are wrought according to the *7th* Direction in Substraction of Universal Surds, *Chap.* 8. *Book* 3. *Part* 5. but the last according to the second Form of Substraction of Square Surds, *Chap.* 4. *Book* 3. *Part* 5.

Other Examples explained in Numbers.

Other Examples in Species and Numbers corresponding.

$$\text{Supposing} \begin{cases} A\ 3. & E\ 2. & Z\ 5. & P\ 6. \\ Aq\ 9. & Eq\ 4. & Z\ 13. & X\ 5. \end{cases}$$

$$\begin{array}{ll} \sqrt{:}\frac{Z}{2} \overset{*}{+} \sqrt{q}\frac{Zq - 4Pq}{4} & \sqrt{:}\frac{13}{2} \overset{*}{+} \sqrt{q}\frac{169 - 144}{4} \\ \sqrt{:}\frac{Z}{2} - \sqrt{q}\frac{Zq - 4Pq}{4} & \sqrt{:}\frac{13}{2} - \sqrt{q}\frac{169 - 144}{4} \\ \hline \sqrt{:}Z - 2P & \sqrt{:}13 - 12 \end{array} \Bigg\} \text{or} \begin{cases} \sqrt{:}\frac{13}{2} + \sqrt{q}\frac{25}{4} \\ \sqrt{:}\frac{13}{2} - \sqrt{q}\frac{25}{4} \end{cases} \text{or} \begin{cases} \sqrt{q}\frac{18}{2} = 3 \\ \sqrt{q}\frac{8}{2} = 2 \\ \hline 1 \end{cases}$$

$$\begin{array}{ll} \sqrt{:}\sqrt{\frac{Xq + 4Pq}{4}} \overset{*}{+} \frac{X}{2} & \sqrt{:}\frac{25 + 144}{4} \overset{*}{+} \frac{5}{2} \\ \sqrt{:}\sqrt{\frac{Xq + 4Pq}{4}} - \frac{X}{2} & \sqrt{:}\frac{25 + 144}{4} - \frac{5}{2} \\ \hline \sqrt{:}\sqrt{Xq + 4Pq} - 2P & \sqrt{:}25 + 144 - 12 \end{array} \Bigg\} \text{or} \begin{cases} \sqrt{:}\frac{169}{4} + \frac{5}{2} \\ \sqrt{:}\frac{169}{4} - \frac{5}{2} \end{cases} \text{or} \begin{cases} \sqrt{q}\frac{18}{2} = 3 \\ \sqrt{q}\frac{8}{2} = 2 \\ \hline 1 \end{cases}$$

$$\frac{144}{169}\ \sqrt{(13 - 12}$$

The Work of these two last Examples may be seen in the precedent Chapter of their Addition; between which and their Substraction, all the Difference is to connex the multiplied Root 2P, to the Sum of the Addition of the Sinister part of the *Data*, with the Sign of Substraction —.

Another Example explained with Numbers.

$$\begin{array}{ll} \sqrt{:}\frac{2Zq - 4Pq}{4} \overset{*}{+} \sqrt{q}\frac{4Zqq - 16PqZq}{16} & \sqrt{:}\frac{338 - 144}{4} \overset{*}{+} \sqrt{q}\frac{114244 - 97344}{16} \\ \sqrt{:}\frac{2Zq - 4Pq}{4} - \sqrt{q}\frac{4Zqq - 16PqZq}{16} & \sqrt{:}\frac{338 - 144}{4} - \sqrt{q}\frac{114244 - 97344}{16} \\ \hline \sqrt{:}\ Zq - 4Pq & \sqrt{:}169 - 144 \\ & \sqrt{q}25 = 5 \end{array}$$

The

The Work of this laſt Example in *Moore* is thus to be underſtood: The Addition at firſt being $\frac{4Z,q-8Pq}{4}$, by Diviſion is brought to Z,q—2Pq. The Multiplication is $\frac{4Z,qq-16Z,qPq+16Pqq}{16} - \frac{4Z,qq+16PqZ,q}{16}$; and abating 16 the Denominator, becauſe alike in both Parts, they are reduced to Integers, and by cancelling the contrary *Species*, reſteth only $\frac{16Pqq}{16}$, and by Diviſion Pqq; whoſe Root Pq multiplied by 2, the ſquare Index is 2Pq, which is to be adjoined to the firſt Addition: So will the Remain be √:Z,q—2Pq—2Pq, that is √:Z,q—4Pq. But if the Total of thoſe *Data's* had been required, it would have been √:Z,q—2Pq+2Pq, that is √qZ,q; and by Reduction only Z, and in Numbers thus:

$$\begin{array}{r} 338 \\ -144 \\ \hline 194 \end{array} \qquad \begin{array}{r} 114244 \\ -97344 \\ \hline 16900 \end{array} \left(\sqrt{\ } \; 130 \right. \qquad 16\left(4 \right.$$

$$\frac{194}{4} + \frac{130}{4} = \frac{324}{4} \left(\sqrt{\ } \; \frac{18}{2} \right(9$$

$$\frac{194}{4} - \frac{130}{4} = \frac{64}{4} \left(\frac{8}{2} \right(4$$

13 Total.

5 Difference.

Proof of Subſtraction of Comp. Irrat. Species.

Subſtraction of Compound Irrational Species, Particulars and Univerſals reſpectively, is proved as Subſtraction of Compound Surds, both by Addition and by Rational Numbers. Enough of which may be ſeen by the Examples of this Chapter compared with thoſe in the former of Addition.

CHAP. XIX. *Multiplication of Compound Irrational Species.*

Multiplication of Comp. Irrat. Species.

IN the Multiplication of theſe Compound Irrationals, Particulars agree with the Multiplication of particular Compound Surds; and Univerſals here with Univerſals there.

Particulars.

Particulars therefore are like to be multiplied with like, or elſe reduced thereto, and the Signs + and — ordered as in the Multiplication of Compound Coſſicks.

Examples.

Examples.	*Binomials.*	*Reſiduals.*	*Mixt.*
Multiplicands.	√qB+√qC	√qD—√qB	√cK+√cP
Multipliers.	√qB+√qD	√qB—√qD	√cK—√cP
	B+√qBC	√qBD—B	√cKq+√cKP
	√qBD+√qDC	—D+√qBD	—√cKP—√cPq
Products	B+√qBC+√qBD+√qDC	√q4BD—B—D	√cKq—√cPq

Another Example explained with Numbers.

Another Example out of Moore *explained with Numbers.*

Suppoſing A 3. E 2. Z 5. P 6.

Multiplicand. $\frac{Z}{2} \overset{*}{+} \sqrt{q}\frac{Zq-4P}{4}$

Multiplier. $\frac{Z}{2} \overset{*}{+} \sqrt{q}\frac{Zq-4P}{4}$

$\frac{Zq}{4} \overset{*}{+} \sqrt{q}\frac{Zqq-4ZqP}{16}$

$\sqrt{q}\frac{Zqq-4ZqP}{16} \overset{*}{+} \frac{Zq-4P}{4}$

Product. $\frac{2Zq-4P}{4} \overset{*}{+} \sqrt{q}\frac{4Zqq-16ZqP}{16}$

By breaking the Line into two Segments, the two Squares thereof are $\frac{Zq}{4} + \frac{Zq-4P}{4}$, that is, $\frac{2Zq-4P}{4}$; each of the two Rectangles is $\sqrt{q}\frac{Zqq-4ZqP}{16}$, which added, after the manner of Surds, by their multiplication with 4, make the Product as in the ordinary way of Multiplication here ſet down.

The Explanation by Numbers.

$$\begin{array}{ll}\text{Multiplic.} & \frac{5}{2}\overset{*}{+}\sqrt{q}\frac{25-24}{4} \\ \text{Multiplier.} & \frac{5}{2}\overset{*}{+}\sqrt{q}\frac{25-24}{4}\end{array}\Bigg\}\text{ or }\left\{\begin{array}{l}\frac{5}{2}+\sqrt{q}\frac{1}{4} \\ \frac{5}{2}+\sqrt{q}\frac{1}{4}\end{array}\right\}\text{ or }\left\{\begin{array}{l}\frac{6}{2} \\ \frac{6}{2}\end{array}\right.\left(\begin{array}{l}3 \\ \underline{3} \\ \underline{9}\ \text{Product.}\end{array}\right.$$

$$\frac{25}{4}\overset{*}{+}\sqrt{q}\frac{625-600}{16}$$

$$\sqrt{q}\frac{625-600}{16}\overset{*}{+}\frac{25-24}{4}$$

$$\text{Product.}\quad \frac{50-24}{4}\overset{*}{+}\sqrt{q}\frac{2500-2400}{16}\Bigg\}\text{ or }\Bigg\{\frac{26}{4}+\sqrt{q}\frac{100}{16}\Bigg\}\text{ or }\Bigg\{\frac{26}{4}+\frac{10}{4}\Bigg\}\text{ or }\Bigg\{\frac{36}{4}\Bigg(9$$

Universals. Univerſals Homogeneal, as Surds before, are to be multiplied one into the other alternately, the Siniſter Numbers and *Species* being Figurate, according to the Denomination of the Dexter into which they are multiplied; and Heterogeneals according to the Reduction.

Examples in Homogeneals.

Examples.	*Binomials.*	*Reſiduals.*
Multiplicands.	$\sqrt{:}B+\sqrt{q}H$	$\sqrt{:}D-\sqrt{q}I$
Multipliers.	$\sqrt{:}D+\sqrt{q}K$	$\sqrt{:}B-\sqrt{q}I$
	$BD+\sqrt{q}DqH$ $\sqrt{q}BqK+\sqrt{q}HK$	$DB-\sqrt{q}BqI$ $-\sqrt{q}DqI+\sqrt{q}Iq$
Products.	$\sqrt{:}BD+\sqrt{q}DqH+\sqrt{q}BqK+\sqrt{q}HK$	$\sqrt{:}DB-\sqrt{q}BqI-\sqrt{q}DqI+\sqrt{q}Iq$

Mixt.

Multiplicand.	$\sqrt{:}D+\sqrt{q}P$
Multiplier.	$\sqrt{:}B-\sqrt{q}R$
	$BD+\sqrt{q}BqP$ $-\sqrt{q}DqR-\sqrt{q}PR$
Product.	$\sqrt{:}BD+\sqrt{q}BqP-\sqrt{q}DqR-\sqrt{q}PR.$

Examples in Heterogeneals, Binomial and Reſidual.

Examples in Heterogeneals.

Multiplicands.	$\sqrt{:}B+\sqrt{c}D$	$\sqrt{:}B-\sqrt{c}D$
Multipliers.	$\sqrt{:}B+\sqrt{c}D$	$\sqrt{:}B-\sqrt{c}D$
	$Bq+\sqrt{cc}DqBcc$ $\sqrt{cc}DqBcc+\sqrt{c}Dq$	$Bq-\sqrt{cc}DqBcc$ $-\sqrt{cc}DqBcc+\sqrt{c}Dq$
Products.	$\sqrt{:}Bq+\sqrt{cc}64DqBcc+\sqrt{c}Dq$	$\sqrt{:}Bq-\sqrt{cc}64DqBcc+\sqrt{c}Dq$

The following Examples by the former Suppoſition in this Chapter, are explained by Numbers.

$\sqrt{:}\frac{Z.}{2}$

Examples explained by Numbers.

$$\sqrt{:}\frac{Z}{2}\ast+\sqrt{q}\frac{Zq-4Pq}{4} \qquad \sqrt{:}\frac{13}{2}\ast+\sqrt{q}\frac{169-144}{4}$$

$$\sqrt{:}\frac{Z}{2}-\sqrt{q}\frac{Zq-4Pq}{4} \qquad \sqrt{:}\frac{13}{2}-\sqrt{q}\frac{169-144}{4}$$

$$\frac{Zq}{4}+\sqrt{q}\frac{Zqq-4ZqPq}{16} \qquad \frac{169}{4}+\sqrt{q}\frac{28561-24336}{16}$$

$$-\sqrt{q}\frac{Zqq-4ZqPq}{16}-\frac{Zq-4Pq}{4} \qquad -\sqrt{q}\frac{28561-24336}{16}-\frac{169-144}{4}$$

$$\sqrt{q}\frac{4Pq}{4}\text{ or }\sqrt{q}Pq\text{, or }P \qquad \sqrt{q}\frac{144}{4}\Big(\frac{12\surd}{2}\Big(6$$

$$\sqrt{:}\frac{Z}{2}\ast+\sqrt{q}\frac{Zq-4Pq}{4} \qquad \sqrt{:}\frac{13}{2}\ast+\sqrt{q}\frac{169-144}{4}$$

$$\sqrt{:}\frac{Z}{2}+\sqrt{q}\frac{Zq-4Pq}{4} \qquad \sqrt{:}\frac{13}{2}+\sqrt{q}\frac{169-144}{4}$$

$$\frac{Zq}{4}+\sqrt{q}\frac{Zqq-4ZqPq}{16} \qquad \frac{169}{4}+\sqrt{q}\frac{28561-24336}{16}$$

$$\sqrt{q}\frac{Zqq-4ZqPq}{16}+\frac{Zq-4Pq}{4} \qquad \sqrt{q}\frac{28561-24336}{16}+\frac{169-144}{4}$$

$$\sqrt{:}\frac{2Zq-4Pq}{4}\ast+\sqrt{q}\frac{4Zqq-16ZqPq}{16} \qquad \sqrt{:}\frac{338-144}{4}\ast+\sqrt{q}\frac{114244-97344}{16}$$

$$\begin{array}{r} 338 \\ -144 \\ \hline \frac{194}{4} \end{array} \qquad \begin{array}{r} 114244 \\ -97344 \\ \hline \frac{16900}{16} \end{array}\Big(\surd\,\frac{130}{4}$$

$$\frac{194}{4}+\frac{130}{4}=\frac{324}{4}\Big(\surd\frac{18}{2}\Big(9\ \Big(3\surd$$

$$\frac{194}{4}+\frac{130}{4}=\frac{324}{4}\Big(\frac{18}{2}\Big(9\ \Big(3\surd$$

$$\underline{\ 9\ }\text{ Product.}$$

Multiplication of Compound Irrational Species, both Particulars and Universals, like Multiplication of Compound Surds before, will be proved by Division and by Rational Numbers, as already hath been sufficiently seen. *Proof of Multiplication of Com. Irrat. Species.*

CHAP. XX. *Division of Compound Irrational Species.*

THE *Division of Compound Irrationals*, takes in Particulars and Universals. Particulars are to be divided by a Mixture of Division of *Species* and Compound Surds, variegated as the *Case* requires. *Division of Com. Irrat. Species. Particulars.*

Case 1. When the Divisor is Simple, and the Dividend Compound. *1. Divisor Simple.*

	Divisor.	*Dividend.*	*Quotient.*
Binomial.	$\sqrt{q}B)$	$\sqrt{q}BS+\sqrt{q}BD$	$(\sqrt{q}S+\sqrt{q}D$
Residual.	$\sqrt{q}C)$	$\sqrt{q}BC-\sqrt{q}BD$	$(\sqrt{q}B-\sqrt{q}\frac{BD}{C}$
Mixt.	$3B)$	$6B+\sqrt{q}54BC-\sqrt{c}81BAD$	$(2B+\sqrt{q}6C-\sqrt{c}3AD$

Examples.

These Examples are like the first in Division of Compound Surds.

Case 2. When the Divisor is Compound, and the Dividend Simple. *2. Dividend Simple.*

	Divisor.	*Dividend.*	*Quotient.*
Binomial.	$\sqrt{q}B+\sqrt{q}C)$	$\sqrt{q}D$	$\Big(\frac{\sqrt{q}D}{\sqrt{q}B+\sqrt{q}C}$
Residual.	$\sqrt{q}B-\sqrt{q}C)$	$\sqrt{q}B$	$\Big(\frac{\sqrt{q}B}{\sqrt{q}B-\sqrt{q}C}$
Mixt.	$\sqrt{q}B+\sqrt{q}C-\sqrt{q}D)$	$\sqrt{q}C$	$\Big(\frac{\sqrt{q}C}{\sqrt{q}B+\sqrt{q}C-\sqrt{q}D}$

Examples.

The Quotients of these Examples are set as Fractions, like those in the 5th *Case* of Division of Integral and Rational *Species*; for to divide *Compound Irrational Species*, it is requisite the *Species* be Homogeneal, as well as the Figurate Quantities to be divided.

3 *Divisor reduced to a Simple.* *Case* 3. When the Divisor is Compound, but both Parts being equal, may be reduced to a Simple as the Dividend is.

Examples.

	Divisor.	*Reduced.*	*Dividend.*	*Quotient.*
Binomial.	$\surd qB + \surd qB$)	$\surd q4B$)	$\surd q8Bq$	($\surd q2B$
Residual.	$\surd q4B - \surd qB$)	$\surd qB$)	$\surd qBc$	($\surd qBq$
Mixt.	$\surd q4B - \surd q4B + D$)	D)	$\surd qBD$	($\surd qB$

The Work of these Examples is according to the third *Case* of Division of Compound Surds.

4 *Data Compound.* *Case* 4. When both Divisor and Dividend are Compound.

Examples.

Divisor. *Dividend.* *Quotient.*

Binomial. $\surd qB + \surd qD$) $\surd qBq + \surd qBC + \surd qBD + \surd qDC$ ($\surd qB + \surd qC$

$$\begin{array}{ll} \surd qB + \surd qC & \surd qBq + \surd qBD \\ \hline \surd qBq + \surd qBD & \surd qBC + + \surd qDC \\ \quad \surd qBC + \surd qDC & \\ \hline \end{array}$$

$\surd qBD$

Residual. $\surd qB - \surd qD$) $\surd q4BD - B - D$ ($\surd qD - \surd qB$

$$\begin{array}{ll} \surd qD - \surd qB & \surd qBD - D \\ \hline \surd qBD - D & \surd qBD - B \\ \quad - B + \surd qBD & \\ \hline \end{array}$$

$-\surd cKP$

Mixt. $\surd cK + \surd cP$) $\surd cKq - \surd cPq$ ($\surd cK - \surd cP$

$$\begin{array}{ll} \surd cK - \surd cP & \surd cKq + \surd cKP \\ \hline \surd cKq + \surd cKP & -\surd cKP - \surd cPq \\ \quad -\surd cKP - \surd cPq & \\ \hline \end{array}$$

These Examples are wrought according to the 6th *Case* of Division of Compound Surds.

5. *Heterogeneal.* *Case* 5. When the Quantities are Heterogeneal, reduce them to like Denominations before Division.

Examples.

	Divisor.	*Dividend.*	*Quotient.*
Binomial.	$\surd qB + \surd qD$)	$B + \surd q4BD + D$ Reduced. $\surd qBq + \surd q4BD + \surd qDq$	($\surd qB + \surd qD$
Residual.	$\surd qB - \surd qD$ Reduced. $\surd ccBc - \surd ccDc$)	$\surd ccBc - \surd ccDc$ Reduced. $\surd ccBcc - \surd cc64BcDc + \surd ccDcc$	($\surd ccBc - \surd ccDc$

In these two Examples the Reduction agrees with that of Surds and Irrational *Species*, and the Division according to the second *Case* of Division of the Simple Irrationals in both, is performed after the Reduction, as the next precedent *Case* of this Chapter.

6. *Fractionary.* *Case* 6. When the *Species* given are Fractionary, the Work is mixed after the manner of Fractions and Surds both.

Example.

Divisor. *Dividend.* *Quotient.*

$$\frac{Z}{2} \overset{*}{+} \surd q\frac{Zq-4P}{4}\Big) \frac{2Zq-4P}{4} \overset{*}{+} \surd q\frac{4Zqq-16ZqP}{16}\Big(\frac{4Zq-8P}{4Z} \overset{*}{+} \surd q\frac{16Zqq-64ZqP}{16Zq-64P}$$

Explained in Numbers.

$$\frac{5}{2} \overset{*}{+} \surd q\frac{25-24}{4}\Big) \frac{50-24}{4} \overset{*}{+} \surd q\frac{2500-2400}{16}\Big(\frac{100-48}{20} \overset{*}{+} \surd q\frac{10000-9600}{400-384}$$

The Quotients being depressed by Reduction in *Species*, may be brought to $\frac{Z}{2} \overset{*}{+} \surd q\frac{Zq-4P}{4}$, and in Numbers to $\frac{100-48}{20} \overset{*}{+} \surd q\frac{400}{16}$, or $\frac{52}{20} + \frac{20}{4}$. But as in Multiplication of this Dividend in the foregoing Chapter, the Denominator 2 of the

the Sinister part, was led into the Numerator of the Dexter part, contrary to the manner of other Fractionary Multiplication. So here this Denominator 4 is to be doubled and made Numerator to 20; which $\frac{8}{20}$ added to $\frac{52}{20}$, make $\frac{60}{20}$, that is 3 Integers for the Value of the Root of the Quotient. And this is worthy to be noted in other Examples of like Nature in Particular Irrationals, when as noted by the Asterisque any Quantity Compound is affirmed or denied of a Simple, or a Simple of a Compound. But in others the Division of Fracted Irrationals agreeth with the Division of Fractionary Surds and *Species* mixt: And will appear plainer in this Example, if there be made two Divisions thereof, as in effect there is, *viz.* the Sinister part of Dividend and Divisor for one, and the Dexter part for the other.

Universals. Homogeneal

Division of Universal *Species* Homogeneal, is like the Division of Universal Homogeneal Surds; and according to the *4th Case* of Division of *Species* in this Chapter, only before the Quotient prefix the Sign Universal, and upon every Removal of the Divisor figurate the Sinister Number thereof according to the Dexter Number to which he is applied; and in multiplying the Divisor by the Quotienary *Species*, the Multiplication must be also proper to Universals.

Example.

Divisor. *Dividend.* *Quotient.*
√:B+√qH) √:BD+√qDqH+√qBqK+√qHK (√:D+√qK
D+√qK BD+√qDqH BqK+√qHK
BD+√qDqH
BqK+√qHK

Heterogeneals.

In Heterogeneals, besides Figuration of the Sinister part of the Divisor as aforesaid, if the Quotienary *Species* gotten on the second Removal of the Divisor be of higher Denomination than the next Dexter Number of the Divisor, this Quotienary *Species* ought to be depressed in Quantity by extracting the Root and abating the Index, and this shall be the true Quotienary Number.

Example.

Divisor. *Dividend.* *Quotient.*
DqBcc
√:B+√cD) √:Bq+√cc64DqBcc+√cDq (√:B+√cD
B+√cD Bq+√ccDqBcc
Bq+√ccDqBcc DqBcc+√cDq
√ccDqBcc+√cDq

Here besides figurating the divising B to Bcc, when applied to DqBcc, because Dq will be gotten thereby for the Quotient, which is a Power above D the next Number of the Divisor, therefore the Root D shall be set in the Quotient with the Cube Index.

Example explained by Numbers.

Another Example.

√:B+√qD) √:Bq+√qBqD+√ccBccCq+√cCqDc (√:B+√cC

Explained in Numbers.

√:2+W4) √:4+W16+√ʒΦ4096+√ʒΦ4096 (√:2+ww8.

Proof of Division of Comp. Irrat. Species.

The Proof of Division of these Compound Irrationals, is by their Multiplications, or by Rational Numbers, that is to say, Particular by Particular, and Universal by Universal, as is to be seen before in Division of Surds, and may further be beheld in these two last Examples, where the Dividends are returned, by multiplying the Divisors into the Quotients respectively, and so may others be examined.

Species.	*Numbers supposed.*	
√:B+√cD	√:2+ww8	2+2=4
√:B+√cD	√:2+ww8	2+2=4
Bq+√ccBccDq	4+√ʒΦ4096	W16
√ccBccDq+√cDq	√ʒΦ4096+ww64	
√:Bq+√cc64BccDq+√cDq	√:4+√ʒΦ262144+ww64	
	4 + 8 + 4	
	W16 = 4	

Species.	*Numbers suppofed.*	
√:B+√qD	√:2+ W 4	2+2=4
√:B+√cC	√:2+ww8	2+2=4
Bq+√qBqD	4+W16	W16
√ccBccCq+√ccDcCq	√ʒΦ4096+√ʒΦ4096	
√:Bq+√qBqD+√ccBccCq+√ccDcCq.	4+W16+√ʒΦ4096+√ʒΦ4096	
	4+ 4 + 4 + 4	
	W16 = 4	

CHAP. XXI. *Figuration of Irrational Species.*

Figurate Irrational Species produced.

AS was obferved before in Figuration of Surds; fo here it may be remembred that any *Species* fimply Irrational, multiplied figurately, produceth a Rational: But Compound Irrational *Species* may be fquared, cubed, *&c.* by multiplying them figurately as other Figural Numbers are multiplied; but the *Unciæ* to the Parodical Degrees here, will be the Squares of them in Rational *Species.* See the Examples that follow.

	Simple.		*Particular.*	*Universal.*
Root	√qB	Root	√qB+√qD	√:B+√qD
	√qB		√qB+√qD	√:B+√qD
Square	√qBq		B+√qBD	Bq+√qBqD
	√qB		√qBD+D	√qBqD+√qDq
Cube	√qBc	Square	B+√q4BD+D	√:Bq+√q4BqD+√qDq
			√qB+√qD	√:B+√qD
			√qBc+√q4BqD+√qBDq	Bc+√q4BqqD+√qBqDq
			√qBqD+√q4BDq+√qDc	√qBqqD+√q4BqDq+√qDc
		Cube	√qBc+√q9BqD+√q9BDq+√qDc.	√:Bc+√q9BqqD+√q9BqDq+√qDc

Explained in Numbers, suppofing B 4. *and* D 9.

	Particular.		*Universal.*	
Root	W4+W9	2+3=5 ℨ	√: 4+W9	4+3=7ℨ
	W4+W9	2+3=5	√: 4+W9	4+3=7
	4+W36	25ʒ	16+W144	49√7
	W36+9	5	W144+W81	
Square	4+W144+9	125Φ	√:16+W576+W81	
	4 + 12 + 9=25 ʒ		16+ 24 + 9=49√7	
	W4 + W9		√:4+W9	
	W64+W576+W324		64+W9216+W1296	
	W144+W1296+W729		W2304+W5184+W729	
Cube	W64 + W1296 + W2916+W729		√:64+W20736+W11664+W729	
	8 + 36 + 54 + 27=125Φ		√:64 + 144 + 108 + 27=343√7	

Roots of Irrational Species extracted.

Extraction of the Roots of Irrational *Species* is performed like the Extraction of Surd Roots, at large difcuffed *Chap.* 12. of the precedent part of this third Book, with a mixture of the Extraction of Roots in Coffieks: So as a Retrofpection thither may fave a large Repetition of the Rules here in the different *Cafes* occurrent.

1. Simple.

Cafe 1. If the Surd be Simple, he hath no Root to be otherwife expreffed, than by prefixing the proper Character before the *Species.*

Example.

As to extract the Square Root of B, or the Cube Root of B+D, they are fet thus:

√qB √cB+D

But

But any Rational *Species* set as an Irrational, may have the Root extracted.

As the Square Root of $\sqrt{q}Bq$ is B.

And the Cube Root of $\sqrt{c}Bc$ is B.

Case 2. If any particular Compound *Species* Irradical in the *Species*, have the Sinister *Species* absolute, then prefix before the same the proper Character for the Root to be extracted. *2. Particular Irradical.*

As to extract the Square Root of $B+\sqrt{q}D$, then it is to be set thus: *Example.*

$$\sqrt{q}B+\sqrt{q}D.$$

But if the Sinister *Species* be figurate, then multiply the Index thereof by the Index of the Root to be extracted.

As to extract the Square Root of $\sqrt{q}B+\sqrt{q}D$, it shall be $\sqrt{qq}B+\sqrt{q}D$.

Case 3. If the Compound *Species* be not Irradical, then are they formally Rational or Irrational: And if they be formally Rational, the Root may be extracted after the manner of Cossicks, keeping the Addition and Substraction of the Multiplees as in Surds. But because Operations in *Species* keep the Prime and Original *Species* for the most part throughout the Work, and in all formal Figuration only increase their Quantities; therefore it is easy to see the Root in any Figural Quantity in *Species* of never so high a Power. *3. Particular Radical if formally Rational.*

As in the former Examples in this Chapter of *Example.*

$$\sqrt{q}Bc+\sqrt{q}9BqD+\sqrt{q}9BDq+\sqrt{q}Dc.$$

$$\sqrt{:}\ Bc+\sqrt{q}9BqqD+\sqrt{q}9BqDq+\sqrt{q}Dc.$$

there being no other *Species* used save the Notes of Quantities B and D, it is easy to discern, the Root did consist of B and D.

Case 4. If the Particular or Universal Compound *Species* be not formally Rational, then figurate the Sinister Part as the Dexter, and from thence take the Dexter, add the Square Root of the Difference to the Sinister, and substract it therefrom, half the Sum and half the Difference joined with + shall be the Binomial Root, and with — shall be the Residual Root. *4. Particular or Universal, if not formally Rational.*

As to extract the square Root of $\frac{2Zq-4P}{4} \overset{*}{+} \sqrt{q}\,\frac{4Zqq-16ZqP}{16}$ *Examples.*

Sinister Part squared is $\frac{4Zqq-16ZqP+16P}{16}$. From whence if the

Dexter Part $\frac{4Zq4-16ZqP}{16}$ be substracted, then will the

Remain be 16P, whose Root is 4P; and this Root when

Added to the Sinister part, makes the Sum $\frac{2Zq-8P}{4}$

Substracted therefrom, makes the Difference $\frac{2Zq}{4}$

Half the Sum.	*Half the Difference.*
$\frac{Zq-4P}{4}$	$\frac{Zq}{4}$ or $\frac{Z}{2}$

Binomial Root $\frac{Z}{2}+\sqrt{q}\,\frac{Zq-4P}{4}$

Residual Root $\frac{Z}{2}-\sqrt{q}\,\frac{Zq-4P}{4}$

So

So to extract the Root of $\surd : \frac{2Z_{\cdot}q - 4Pq}{4} * + \surd q \frac{4Z_{\cdot}qq - 16Z_{\cdot}qPq}{16}$

$\frac{4Z_{\cdot}qq - 16Z_{\cdot}qPq + 16Pqq}{16}$ Sinister part squared.

$\frac{4Z_{\cdot}qq - 16Z_{\cdot}qPq}{16}$ Dexter part substracted.

16Pqq Remain. 4Pq Root.

$\frac{2Z_{\cdot}q - 8Pq}{4}$ Sum. $\frac{2Z_{\cdot}q}{4}$ Difference.

$\frac{Z_{\cdot}q - 4Pq}{4}$ Half Sum. $\frac{Z_{\cdot}q}{4}$ or $\frac{Z_{\cdot}}{2}$ Half Difference.

Root $\frac{Z_{\cdot}}{2} + \surd q \frac{Z_{\cdot}q - 4Pq}{4}$ Binomial. $\frac{Z_{\cdot}}{2} - \surd q \frac{Z_{\cdot}q - 4Pq}{4}$ Residual.

Proof of Extraction of Roots of Irrat. Species. These Extractions are to be proved by Production of their Figurals, as those Productions by these Extractions. And also by Rational Numbers, working with them instead of the Irrationals; as by the last Example explained in Numbers will appear.

Species $\surd : \frac{2Z_{\cdot}q - 4Pq}{4} * + \surd q \frac{4Z_{\cdot}qq - 16Z_{\cdot}qPq}{16}$

Numbers $\surd : \frac{338 - 144}{4} * + \surd q \frac{114244 - 97344}{16}$

Sinister Part squared $\frac{114244 - 97344 + 20736}{16}$

Dexter Part substracted $\frac{114244 - 97344}{16}$

Remain 20736 | Root 144

Sum $\frac{338 - 288}{4}$ Half $\frac{169 - 144}{4}$

Difference $\frac{338}{4}$ Half $\frac{169}{4}$ or $\frac{13}{2}$

Binomial Root $\surd : \frac{13}{2} + \surd q \frac{169 - 144}{4}$

Residual Root $\surd : \frac{13}{2} - \surd q \frac{169 - 144}{4}$

338—	144	
338—	144	
2704—	1152+	576
1014	432	576
1014	432	144
114244—	48672+	
	—48672+	20736
114244—	97344+	20736

And thus much may suffice in this place to be spoken of *Species*, and in them of the Simple Elements of both Abstract and Contract Numbers.

Partis sextæ & Libri tertii

FINIS.

ARITHMETICK.

The Fourth BOOK.

CONCERNING
Numbers Proportional, Abstract and Contract.

In Four PARTS.

WHEREIN

Ratio's	are	Deciphered.
Proportions disjunct	are	Dissected.
Proportions continued	are	Computed.
Aequations	are	Enodated.

And their *Comparative* ELEMENTS.

CHAP. I.

Of RATIO'S.

HAVING thus in the three former Books run through the *Simple Elements* of *Numbers*, in their proper Nature, *viz.* *Abstract*, and generally and specially *Contract* : I now come to review them in their common Nature, as they are Relational, and uncover their *Comparative Elements* ; which shew the Comparison of Numbers among themselves, the Description whereof takes up this Chapter, and the Computation the rest of this Book. *Numbers in their common Nature as Relational, and their Comparative Elements, treated of in this Fourth Book.*

Description gives us an Account of the several Species or Kinds of Comparison. Description, *what.*

Computation is an Account of the several Operations belonging to these Species. Computation, *what.*

Comparison of Numbers, shews what Likeness or Relation there is between the Numbers compared ; and is twofold, *Ratio*, or *Proportion*. Comparison *twofold.*

Ratio is a Comparison of Terms, and is, when the Relation or Conference is extended, but to two Numbers or Magnitudes only, as 21 to 12, or 3 to 6, or any such-like : This is sometimes promiscuously called *Proportion*. Ratio, *what. How sometime called.*

Proportion is properly a Comparison of *Ratio's*, and is when the Conference reacheth unto many Numbers : This is sometimes called *Proportionality*, and often *Analogy*. Proportion, *what, and how called.*

Ratio twofold. Every Relation between two Numbers, is either *Ratio* of Equality or Inequality.

Ratio of Equality. *Ratio* of Equality, is when one Number is compared to himself, as 5 to 5, always agreeing in Unity.

Ratio of Inequality. *Ratio* of Inequality, is when one Number is compared to another different from him: Wherein also there is a double Conference, *viz.* *Ratio* of the Greater, and *Ratio* of the Lesser Inequality.

This twofold. *Greater Inequality also twofold.* *Ratio* of the Greater Inequality, is when the greater Number is compared to the Lesser; as 6 to 3, or 5 to 2, *&c.* And this is of two Sorts, either Simple and Prime, or Compound and Conjunct.

Prime, and this, Prime, or Simple *Ratio* of the greater Inequality, is of two sorts.

1. *Containeth the Lesser once and a Part.* The first is, when the greater Number containeth the Lesser more than once, and not twice, but once and a Part more, called generally by the Latin Name, *Superparticularis*, or a Part more: For that one Part of the lesser Number, is the just Difference or Excess betwixt it and the Greater. Specially, and for distinction-sake, each receiveth his Name from that Part it containeth: As if it contain the Half more, then it is called *Sesquialtera*, as 3 to 2.

A farther view of such *Ratio's*.

Examples.

Sesquialtera ———	as 3 to 2.	6 to 4, &c.	$1\frac{1}{2}$	an Half more.
Sesquitertia ———	as 4 to 3.	8 to 6, &c.	$1\frac{1}{3}$	a Third more.
Sesquiquarta ———	as 5 to 4.	10 to 8, &c.	$1\frac{1}{4}$	a Fourth more.
Sesquiquinta ———	as 6 to 5.	12 to 10, &c.	$1\frac{1}{5}$	a Fifth more.
Sesquisexta ———	as 7 to 6.	14 to 12, &c.	$1\frac{1}{6}$	a Sixth more.
Sesquiseptima ———	as 8 to 7.	16 to 14, &c.	$1\frac{1}{7}$	a Seventh more.
Sesquioctava ———	as 9 to 8.	18 to 16, &c.	$1\frac{1}{8}$	an Eighth more.
Sesquinona ———	as 10 to 9.	20 to 18, &c.	$1\frac{1}{9}$	a Ninth more.
Sesquidecima ———	as 11 to 10.	22 to 20, &c.	$1\frac{1}{10}$	a Tenth more.
Sesquiundecima ———	as 12 to 11.	24 to 22, &c.	$1\frac{1}{11}$	an Eleventh more.
&c.				

2. *Containeth the lesser once, and some Parts.* The second Simple sort is, when the Difference is 2, 3, or more Parts of the whole; this is generally called *Superpartiens*, or Parts more, intimating above one Part; but specially every one hath his proper Name, according to his Content, as the former had. For if 5 be compared to 3, then is it called *Superbipartiens-tertias*, because it containeth the Whole, and $\frac{2}{3}$ of the Whole. And so may other Names be given infinitely. Some Examples whereof follow.

Examples.

Superbipartiens.	*Tertias* ———	as 5 to 3.	10 to 6, &c.	$1\frac{2}{3}$	Two Parts more.
	Quintas ———	as 7 to 5.	14 to 10, &c.	$1\frac{2}{5}$	
	Septimas ———	as 9 to 7.	18 to 14, &c.	$1\frac{2}{7}$	
	Nonas ———	as 11 to 9.	22 to 18, &c.	$1\frac{2}{9}$	
	Undecimas ———	as 13 to 11.	26 to 22, &c.	$1\frac{2}{11}$	
	Decimas tertias--	as 15 to 13.	30 to 26, &c.	$1\frac{2}{13}$	
	&c.				
Supertripartiens.	*Quartas* ———	as 7 to 4.	14 to 8, &c.	$1\frac{3}{4}$	Three Parts more.
	Quintas ———	as 8 to 5.	16 to 10, &c.	$1\frac{3}{5}$	
	Septimas ———	as 10 to 7.	20 to 14, &c.	$1\frac{3}{7}$	
	Octavas ———	as 11 to 8.	22 to 16, &c.	$1\frac{3}{8}$	
	Decimas ———	as 13 to 10.	26 to 20, &c.	$1\frac{3}{10}$	
	Undecimas ———	as 14 to 11.	28 to 22, &c.	$1\frac{3}{11}$	
	Decimas tertias--	as 16 to 13.	32 to 26, &c.	$1\frac{3}{13}$	
	&c.				
Superquadrupartiens.	*Quintas* ———	as 9 to 5.	18 to 10, &c.	$1\frac{4}{5}$	Four Parts more.
	Septimas ———	as 11 to 7.	22 to 14, &c.	$1\frac{4}{7}$	
	Nonas ———	as 13 to 9.	26 to 18, &c.	$1\frac{4}{9}$	
	Undecimas ———	as 15 to 11.	30 to 22, &c.	$1\frac{4}{11}$	
	Decimas tertias--	as 17 to 13.	34 to 26, &c.	$1\frac{4}{13}$	
	Decimas quintas-	as 19 to 15.	38 to 30, &c.	$1\frac{4}{15}$	
	&c.				

Super-

Superquintupartiens.	*Sextas* ——	as 11 to 6.	22 to 12, &c.	$1\frac{5}{6}$	Five Parts more.
	Septimas ——	as 12 to 7.	24 to 14, &c.	$1\frac{5}{7}$	
	Octavas ——	as 13 to 8.	26 to 16, &c.	$1\frac{5}{8}$	
	Nonas ——	as 14 to 9.	28 to 18, &c.	$1\frac{5}{9}$	
	Undecimas ——	as 16 to 11.	32 to 22, &c.	$1\frac{5}{11}$	
	Duodecimas ——	as 17 to 12.	34 to 24, &c.	$1\frac{5}{12}$	
	&c.				

Here is to noted, that those *Ratio's* which some have set to fill up the Complement of the other, as *Superbipartiens-Secundas*, *Quartas*, *Sextas*, *Octavas*, &c. and *Supertripartiens-Secundas*, *Tertias*, *Sextas*, *Nonas*, &c. are all Irregular; for properly there are no such *Ratio's*; but those are *Ratio's* that fall under some other Name agreeable thereto: As 6 to 4, called by some *Superbipartiens-Quartas*, is indeed *Sesquialtera*. So 10 to 6, called by some *Superquadrupartiens-Sextas*, is properly no other than *Superbipartiens-Tertias*. The like is to be understood of many others: For as in Common Fractions, it is most regular to set them in their least Terms; so in *Ratio's*: wherefore 10 to 6 shall be accounted $1\frac{2}{3}$, not as $1\frac{4}{6}$.

Compound, otherwise called *Conjunct Ratio's* of the greater Inequality, are of three Kinds. *Compound, and this*

1\. *Containeth the Lesser many times.*

The first is, when the greater Number containeth the Lesser, divers times generally called *Multiplex*, or *Manifold*; and particularly named, according to the Times that the lesser Number is contain'd in the Greater: So that if it contain it twice, then it is called *Dupla*, or *Double*, as 2 to 1, *&c.* A farther Account of their Names follows.

Examples.

Dupla ——	as 2 to 1.	4 to 2, &c.	$\frac{2}{1}$	*Duple*, or Double.
Tripla ——	as 3 to 1.	6 to 2, &c.	$\frac{3}{1}$	*Triple*, or Threefold.
Quadrupla ——	as 4 to 1.	8 to 2, &c.	$\frac{4}{1}$	*Quadruple*, or Fourfold.
Quintupla ——	as 5 to 1.	10 to 2, &c.	$\frac{5}{1}$	*Quintuple*, or Fivefold.
&c.				

2\. *Containeth the Lesser many times, and a Part.*

The second Sort of *Ratio's* of the greater Inequality compound, is named *Multiplex-Superparticularis*; which importeth, that the greater Number containeth the lesser many times, and a Part more, as 5 to 2, which contains 2 twice and a half more, and therefore is called *Dupla-Sesquialtera*. These Kinds of *Ratio's* may be diversly divided, as into *Double*, *Triple*, *Quadruple*, &c. and every of them into their several Subdivisions.

As for Instance.

Examples.

Duplex ——	*Sesquialtera* ——	as 5 to 2.	10 to 4, &c.	$2\frac{1}{2}$
	Sesquitertia ——	as 7 to 3.	14 to 6, &c.	$2\frac{1}{3}$
	Sesquiquarta ——	as 9 to 4.	18 to 8, &c.	$2\frac{1}{4}$
	&c.			
Triplex ——	*Sesquialtera* ——	as 7 to 2.	14 to 4, &c.	$3\frac{1}{2}$
	Sesquitertia ——	as 10 to 3.	20 to 6, &c.	$3\frac{1}{3}$
	Sesquiquarta ——	as 13 to 4.	26 to 8, &c.	$3\frac{1}{4}$
	&c.			
Quadruplex ——	*Sesquialtera* ——	as 9 to 2.	18 to 4, &c.	$4\frac{1}{2}$
	Sesquitertia ——	as 13 to 3.	26 to 6, &c.	$4\frac{1}{3}$
	Sesquiquarta ——	as 17 to 4.	34 to 8, &c.	$4\frac{1}{4}$
&c.	&c.			

3\. *Containeth the Lesser many times, and some Parts.*

The third Sort is called *Multiplex-Superpartiens*, and implieth, that the greater Number containeth the Lesser divers times, and some Parts thereof besides: And are likewise distinguished into *Double*, *Triple*, &c. Some of these, with their Subdivisions, appear in the following Examples.

Dupla

Examples.

Dupla	*Superbipartiens*	*Tertias*	as 8 to 3, &c.	$2\frac{2}{3}$
		Quintas	as 12 to 5, &c.	$2\frac{2}{5}$
		Septimas	as 16 to 7, &c.	$2\frac{2}{7}$
		&c.		
	Supertripartiens	*Quartas*	as 11 to 4, &c.	$2\frac{3}{4}$
		Quintas	as 13 to 5, &c.	$2\frac{3}{5}$
		Septimas	as 17 to 7, &c.	$2\frac{3}{7}$
		&c.		
	Superquadripartiens	*Quintas*	as 14 to 5, &c.	$2\frac{4}{5}$
		Septimas	as 18 to 7, &c.	$2\frac{4}{7}$
		Nonas	as 22 to 9, &c.	$2\frac{4}{9}$
		&c.		
Tripla	*Superbipartiens*	*Tertias*	as 11 to 3, &c.	$3\frac{2}{3}$
		Quintas	as 17 to 5, &c.	$3\frac{2}{5}$
		&c.		
	Supertripartiens	*Quartas*	as 15 to 4, &c.	$3\frac{3}{4}$
		Quintas	as 18 to 5, &c.	$3\frac{3}{5}$
		&c.		
	Superquadripartiens	*Quintas*	as 19 to 5, &c.	$3\frac{4}{5}$
		Septimas	as 25 to 7, &c.	$3\frac{4}{7}$
		&c.		
Quadrupla	*Superbipartiens*	*Tertias*	as 14 to 3, &c.	$4\frac{2}{3}$
		Quintas	as 22 to 5, &c.	$4\frac{2}{5}$
		&c.		
	Supertripartiens	*Quartas*	as 19 to 4, &c.	$4\frac{3}{4}$
		Quintas	as 23 to 5, &c.	$4\frac{3}{5}$
		&c.		
	Superquadripartiens	*Quintas*	as 24 to 5, &c.	$4\frac{4}{5}$
		Septimas	as 32 to 7, &c.	$4\frac{4}{7}$
	&c.	&c.		

Lesser Inequality twofold. *Ratio* of the lesser Inequality, is when the lesser Number is conferred to the Greater, as 3 to 5, or 2 to 6, *&c.*

Containing a Part. These *Ratio's* are also divided into two Sorts, *viz.* either such as contain a Part of the Number only, as one Third, one Fourth, *&c.* as 1 to 3, and 4 to 16, *&c.*

Containing some Parts. Or such as contain many Parts of the greater Number, as Three Quarters, Four Fifths, *&c.* as 3 to 4, and 4 to 5, *&c.*

How differenced in the Names from the other. Both these kinds of *Ratio's* of the lesser Inequality, have the same Names which the *Ratio's* of the greater Inequality had; save that to the beginning of every Name *Sub* is to be adjoined, as *Subdupla*, *Subtripla*, *Subsesquialtera*, &c. Examples of both Sorts follow.

Examples of the first Sort.

Examples of both.

Subdupla	as 1 to 2.	2 to 4, &c.	$\frac{1}{2}$	one Part.
Subtripla	as 1 to 3.	2 to 6, &c.	$\frac{1}{3}$	
Subquadrupla	as 1 to 4.	2 to 8, &c.	$\frac{1}{4}$	
&c.				

Examples of the second Sort.

Subsesquialtera	as 2 to 3.	4 to 6, &c.	$\frac{2}{3}$	many Parts.
Subsesquitertia	as 3 to 4.	6 to 8, &c.	$\frac{3}{4}$	
Subsesquiquarta	as 4 to 5.	8 to 10, &c.	$\frac{4}{5}$	

Ratio's properly conversing with Abstract Numbers, appear in the Front of Relational Numbers, and so are the Subject of the first Part of this 4*th* Book.

Proportion distinguished into Simple and Compound. Proportion, as was said before, is referred to more than two Numbers, and therein may be a Conference of the former several *Ratio's* in their several Terms: But as conversing both with Abstract and Contract Numbers, and so best befitting Arithmetick, the usual Dissection of all Analogy, is into Simple and Compound Proportion.

Simple,

Simple, because the Numbers so compared, need make no use of the Signs + or —. This is also divided into Discontinual and Continual Proportion. *Simple into Discontinual and Continual.*

Discontinual, called also *Disjunct Proportion*, is when there is an Equality of the Difference or *Ratio* between some of the Terms given; but not current through all, either sort seldom exceeding four Terms. *Discontinual twofold.*

This kind of Proportion is double, either Arithmetical or Geometrical.

Arithmetical Discontinual Proportion is, when the Equality of the Difference is not continued alike throughout all the Terms, but the Difference of the first and second Numbers is somewhere distracted. As 4 . 7 : 5 . 8 . where 3, the Difference between 4 and 7, is discontinued between 7 and 5. *Arithmetical, what.*

Hence arose those Proportions called *Musical*, which Mr. *Blundevil*, and some others, make a third Species: And as Mr. *Oughtred*, in Chap. 6. of his *Clavis* tells us is, when in 4 Numbers; As the First is to the Fourth, so is the Difference of the First and Second, to the Difference of the Third and Fourth. As 5. 8. 12. 30. are Musical Proportions, because 5. 30 : : 8 — 5. 30 — 12 : : 3. 18. Also in Species, A, M, N, E, let A . E : : M — A. E — N. Wherefore A E — A N = M E — A E in these Terms duly ordered, the Rule shall be $\frac{AN}{2A-M}=E$ and $\frac{EM}{2E-N}=A$. That is, If the Product of the First and Third be divided by the Excess of the First doubled above the Second, the Quotient shall be a Fourth in Musical Proportion. *Hence arise Musical Proportions. What they are. How to make them.*

Geometrical Discontinual Proportion is, when the *Ratio* is distracted: As 5. 15 : : 6. 18. where the *Subtriple Ratio* between 5 and 15, agreeth not with the *Ratio Cuplasesquialtera* between 15 and 6. *Geometrical Proportion, what.*

This sort of Proportion is either Plain or Figurate. *This twofold.*

Plain or Simple, because the Number found thereby, agreeth in the simple Nature of the *Data*. *Plain.*

Such as these, by the 12, 13, 14, 15, and 16 Definitions of the 5*th* Book of *Euclid*, may be divided into *Alternate*, *Inverse*, *Compounded*, *Divided*, and *Converse*; but as most sutable to *Arithmetick*, they may be divided into *Direct* or *Indirect*. *How sorted by* Euclid. *How here.*

Direct is, when the Term by which the Question is made, (which is the Third) by how much it is greater than the First, by so much it requireth a Fourth Number greater than the Second: And by how much it is lesser, by so much it requireth a Lesser. On this is founded the *Golden Rule Direct*: For of 3, Numbers given, if the Second multiply the Third, and that Product be divided by the First, the Quotient shall be a Fourth Proportional to the three given Numbers. *Direct what, and what founded thereon.*

Example when the Greater requireth a Greater. If 7 give 28, (being *quadrupled*) What shall 9 give? *facit* 36; which is in proportion to 28, as 9 is to 7. *Examples.*

Example when the Lesser requireth a Lesser. If 6 give 4, (being diminished by ⅓) what shall 3 give? *facit* 2; for 2 is in proportion to 4, as 3 is to 6.

Indirect, called also *Reciprocal* or *Reversed Proportion*, is, when the Term by which the Question is made, by how much it is greater than the First, by so much it requireth a Fourth Number lesser than the Second: And by how much it is lesser, by so much it requireth a Greater. And on this is bottomed the *Backward Rule of Three*, or *Golden Rule Reversed*: For of three Numbers given, if the First multiply the Second, and that Product be divided by the Third, the Quotient shall be a Fourth, proportional to the three given Numbers. *Indirect what, how called, and what founded thereon.*

Example when the Greater requireth a Lesser. If 4 give 10, then shall 8 give 5; because as 8 is double to 4, so the Double of 5 is 10. *Examples.*

Example when the Lesser requireth a Greater. If 4 give 10, then shall 2 give 20; for 20 is double to 10, as 4 is to 2.

Figurate Proportions here, are not to be taken so much for any Simple *Ratio* or Proportion that is between Figural Numbers *Homogeneal* or *Heterogeneal*, or any of their Complements or Parodical Degrees mentioned before *in Species*; nor yet only for the Proportional Figural Numbers, found out by the *Direct* or *Indirect* Analogy aforesaid; but the Proportions used about Geometrical Figures; and together with these, such as are discovered by the Operations proper to the Figural Numbers themselves, or depending thereupon. And these receive Names according to the *Index* of the Figural Numbers they deal with; as if Squares, then are they called *Doubled Proportions*; if with Cubes, *Tripled Proportions*; if with Squared-Squares, *Quadrupled Proportions*, &c. Figurate Proportions *how to be understood here.*

All these Plain and Figurate-Geometrical-Discontinual Proportions, with their farther Subdivisions, and the Issues and Operations to them properly belonging, fill up the Second Part of this 4*th* Book, in the Computation thereof.

The other Sort of Proportions before-mentioned, are *Continual.*

Continual *Proportion twofold.* *Continual*, otherwise called *Conjunct Proportion*, is, when three or more Numbers bear like Proportion in their Progression: So as well the Second Number may be referred to the Third, as the First to the Second. And the same Difference, or *Ratio*, shall be between the two last Terms, as between the two First.

This kind of Proportion is also both *Arithmetical* and *Geometrical.*

Arithmetial, *what.* *Arithmetical* Continual Proportion is, when between every two Numbers, or Terms, the Difference or Excess is equal, as 2, 4, 6, 8, &c. where the Excess 2 is continued throughout all the Terms, *&c.*

Geometrical, *what.* *Geometrical* Continual Proportion is, when between every two Numbers, or Terms, the *Ratio* is equal, as 4, 8, 16, 32, &c. where the *Ratio* 2 is continued throughout all the Terms.

Both called Progression. These two Sorts deal with plain Numbers especially, yet the latter in a manner figurates the first Terms; both are called *Progression*, and have their useful Operations computed in the Third Part of this 4*th* Book.

Compound *Proportion how otherwise called.* *Compound* Proportions hold Community especially with Contract Numbers; make use of the Signs + and —, and are those called *Æquations*; that is, Numbers equal to others. This may be called *Proportion of Equality.* And of these there are two principal Sorts.

Æquations *of two sorts.* First, *Pure*, when one Number is compared as equal to another.

Secondly, *Mixt*, (commonly called *Affected*) when one or divers Numbers are compared in Equality to divers others.

Examples of Pure.

Pure { Example in *Geodædicals* – 1 *s.* = 12 *d.*
Example in *Cossicks* — 2♉ = 6ℨ   Root 3.

Examples in *Abstract* Numbers.

Examples of Affected.

Mixt {
4 = 2 + 2   4 = 6 — 2
4 + 2 = 3 + 3   4 — 2 = 3 — 1

Examples in *Contract* Numbers.

1 φ = 2♉ + 3ℨ   1 φ = 4 ♉ — 3 ℨ
1 φ + 1♉ = 9ℨ + 9 N   1 φ — 1 ♉ = 7 ℨ — 3 N   Root 3.
1 φ + 1♉ = 13ℨ − 3N   1 φ — 1 ♉ = 5 ℨ + 8 N

These, as the most profound Part of *Arithmetick*, occupy the last Part of this Fourth Book, and close up the whole Survey of this Numbering Art.

Notes of Ratio's. Touching *Ratio's* is here further to be noted.

1. Antecedent & Consequent. 1*st*, As in *Fractions* there is *Numerator* and *Denominator*: So in *Ratio's* there are two Terms; the first whereof is called the *Antecedent*, and the second the *Consequent.*

2. Ratio's, *how expressed.* 2*dly*, Though *Ratio's* are set commonly one before another, as in the Instances before-mentioned; yet for better conveniency in working, they are also set one over another like Fractions. And by some, to distinguish them from Fractions, instead of the intervening Line, two Pricks are set; and so the *Ratio Sesquialtera* is thus expressed $\overset{3}{\underset{2}{:}}$

3. Ratio's *are Commensurable, or Incommensurable.* 3*dly*, *Ratio's*, as common Fractions before, are either Commensurable, or Incommensurable. For if the two Terms compared have any common Part, that will equally divide them both: then they are Commensurable, as 12 to 21; because 3, a Part of them, is a Common Divisor to both. But on the contrary, if the Numbers have no such Part for a Common Divisor; they are Incommensurable, as 18 to 25: for 25 can evenly be divided by no Number but 5; and 18 cannot be divided equally thereby.

4. Ratio *and Proportion promiscuously used.* 4*thly*, The words *Ratio* and *Proportion* may be found promiscuously used one for the other in good Authors; which the curious cannot stumble at, since they agree in the Genus, for *Ratio* is a Single Proportion, and Proportion but Plural *Ratio's.*

5. *Conjunct* Ratio's *have their Names so.* 5*thly*, In Conjunct *Ratio's*, the Names are Conjunct; as 3 $\frac{1}{4}$ *Triple-Sesquiquarta.*

6*thly*,

6. *6thly*, The Difference between two Numbers, is the Distance of a Number from a Number, and is found by *Substraction*: But the *Ratio* is the containing of a Number in a Number, and is found by *Division*, and therefore to be understood differently.

Difference and Ratio how they differ.

7. *7thly*, The same Difference between Terms may fall out, and the *Ratio* divers. As 6 to 3, the *Ratio* is double, and the Difference 3: But 12 to 9, the *Ratio* is *Sesquitertia*, yet the Difference 3 as before.

Terms alike in Difference, not in Ratio.

8. *8thly*, If one Number shall multiply two Numbers, the Products shall be proportional. And if one Number shall divide two Numbers, the Quotients also shall be proportional. As if 4 multiply 7 and 9, the Products 28 and 36 will be alike proportional: And if 28 and 36 be divided by 4, their Quotients 7 and 9 will be alike proportional.

A Number multiply or divide two others, the Products and Quotients equal.

$$\text{Multiplier } 4\left\{\begin{matrix}7\,.\;28\\9\,.\;36\end{matrix}\right. \qquad \text{Divisor } 4\left)\begin{matrix}28\,(7\\36\,(9\end{matrix}\right. \qquad \frac{9}{7}\left(1\tfrac{2}{7}\right. \qquad \frac{36}{28}\left(1\tfrac{2}{7}\right.$$

Example in Species.

$$A\times\left\{\begin{matrix}B\,.\;BA\\C\,.\;CA\end{matrix}\right. \qquad A\left)\begin{matrix}BA\,.\;(B\\CA\,.\;(C\end{matrix}\right.$$

9. *9thly*, Mr. *Oughtred* in his *Clavis*, Chap. 6. adds; If the Consequents of two *Ratio's* are equal, they are as the Antecedents: but if the Antecedents are equal, they are reciprocal as the Consequents.

What follows if the Terms are equal.

$$\text{As } \frac{7}{1}\,.\,\frac{9}{1} :: 7\,.\,9 \qquad \text{And } \frac{1}{7}\,.\,\frac{1}{9} :: 7\,.\,9$$

10. *10thly*, The *Ratio* of the Antecedent to the Consequent, is compounded either of the *Ratio* of the Antecedent to the Third, and of the Third to the Consequent; or of the *Ratio* of the Third to the Consequent, and of the Antecedent to the Third.

Ratio of the Terms, of what compounded.

$$\text{As } 7\,.\,9 :: \times\left\{\begin{matrix}7\;A\\A\;9\end{matrix}\right. \qquad \text{Also } 7\,.\,9 :: \times\left\{\begin{matrix}A\;9\\7\;A\end{matrix}\right.$$

CHAP. II.

Reduction of RATIO'S.

THE Description of *Relational Numbers* passed in the former Chapter, *Computation* comes next on the Stage.

Computation of Relational Numbers.

Ratio's as they are single Proportions, so they challenge the Simple Elements of Numbers to their Accompt; and as Contract Numbers have their Ortive Numeration before their Original.

Ratio's have their Simple Elements.

The Ortive Numeration of *Ratio's* consists in *Reduction*.

Reduction of *Ratio's*, is to reduce them to their least Terms, or to like Antecedents or Consequents. The first is to be performed as they are of the greater or lesser Inequality.

Reduction of Ratio's, of two sorts.

Ratio's of the greater Inequality are to be divided as Integers; and if any thing remain abbreviated as common Fractions with the Divisor to the least Terms: And then if Occasion be, this Quotient, and the least Terms of the Remain and Divisor so abbreviated, may be reduced into the Form of an improper Fraction.

To reduce Ratio's of the greater Inequality to their least Terms.

As 80 to 32, after Division 16 remaineth; which abbreviated with 32, makes $\frac{1}{2}$ this, and the Quotient 2 is $2\frac{1}{2}$; or reduced like an improper Fraction is $\frac{5}{2}$ and sheweth the *Ratio* of 80 to 32 in its least Terms, is *Dupla-Sesquialtera*.

Example.

Ratio's of the lesser Inequality, are to be abbreviated as common Fractions to their least Terms. As 32 to 80, when abbreviated, is $\frac{2}{5}$ *Subdupla-Sesquialtera*.

To reduce Ratio's of the Lesser Inequality, &c.

Example.

Of

To reduce Ratio's to like Consequents.

Of the other sort of *Reduction*, that to reduce *Ratio's* to like Consequents, is like Reduction of Common Fractions to like Denominators. As $\frac{1}{20}$ and $\frac{1}{30}$

Example.

reduced to like Consequents, is $\frac{30}{600}$ and $\frac{20}{600}$, and abbreviated is $\frac{3}{60}$ and $\frac{2}{60}$.

To reduce Ratio's to like Antecedents.

If *Ratio's* are to be reduced to like Antecedents; then, contrary to the other, their Antecedents are to be multiplied one into another for the common Antecedent; and then croſswiſe every Antecedent is to be multiplied into the other Conſequent, except his own. As to reduce $\frac{20}{1}$ and $\frac{30}{1}$ to one Antecedent, 20 is to be multiplied into 30 for the common Antecedent; and then croſswiſe, 20 into 1, and 30 into 1. So is this Reduction $\frac{600}{30}$ and $\frac{600}{20}$, and by Abbreviation $\frac{60}{3}$ and $\frac{60}{2}$.

Example.

Proof of Reduction of Ratio's.

Reduction of *Ratio's* is proved one part by another, after the manner of other *Reductions*.

CHAP. III.

Addition of RATIO'S.

Geneſis and Analyſis of Ratio's. Continuation and Diminution, what.

RATIO'S have their Original Numeration in their *Geneſis* and *Analyſis*; and their *Geneſis* Prime in *Addition*, (called ſometime *Continuation*) Compound in *Multiplication*: their *Analyſis* Prime in *Subſtraction*, (ſometime called *Diminution*) Compound in *Diviſion* as others before them; but being Comparative herein they differ, for the Operations of the prime Parts of their Numeration are as the Compound Parts of others, and their Compound Parts as Figurals: Wherefore *Addition* of *Ratio's*, in all reſpects, is performed as Multiplication of Fractions. And ſo two *Caſes* are ſufficient.

Addition of Ratio's in two Caſes.

1. Heterological *Terms not reducible.*

Caſe 1. Where the *Heterologal* Terms need no Reduction, multiply Antecedent by Antecedent, for a new Antecedent: And in like manner Conſequent by Conſequent, for a new Conſequent of the Total.

Example.

As if I would add $\frac{2}{3}$ to $\frac{2}{1}$, that is the *Ratio* of 2 to 3, which is *Sub-Seſquialter* to the *Ratio* 2 to 1, which is double: I multiply 2 by 2, and 4 is the new Antecedent of the Total, and 3 by 1. So is 3 the new Conſequent, and the Total *Ratio* is *Seſquitertia*.

Antecedents.

$$\textit{Addends}\ \frac{2}{3} + \frac{2}{1} = \frac{4}{3}\ \text{Total.}$$

Conſequents.

2. Heterologal *Terms reducible.*

Caſe 2. Where the *Heterologal* Terms, or either of them may be reduced, reduce them like Fractions as low as you can; and then multiply the reduced Terms as above.

Example.

As to add $\frac{4}{3}$ to $\frac{3}{2}$, they may be reduced to $\frac{2}{1}$ and $\frac{1}{1}$, and then their *Addition* will make the Total $\frac{2}{1}$.

$$\textit{Addends}\ \begin{matrix}2\\4\\3\\1\end{matrix} + \begin{matrix}1\\3\\2\\1\end{matrix} = \frac{2}{1}\ \text{Total} \qquad \frac{12}{6} \mid \frac{2}{1}$$

Muſical Proportions, how gotten.

Hereby it appeareth that the *Ratio's* commonly called *Harmonica*, or *Harmoniaca Ratio*; and ſometime Muſical Proportions, (which are ſuch as are to be found in Muſical Conſorts) though accompted by ſome a diſtinct Species from the *Ratio's* before deſcribed; yet are nothing elſe but ſeveral *Ratio's* of the former Sorts, only they have other Names, as appeareth in the ſpeculative Part of Muſick. For the *Diapente* is *Seſquialter*. *Diateſſeron*, *Seſquitertia*, *Diapaſon*, *Dupla*. *Diapaſon*

paſon with the *Diapente*, *Tripla*; and the *Tone*, *Seſquioctava*. So that the *Diapaſon* is made of the *Diateſſeron* added to the *Diapente*: And the *Diapente* is made of the *Diateſſeron* added to the *Tone*.

Examples.

	Diapaſon.	*Diateſſeron.*		*Diapente.*		*Diapaſon,*		or *Dupla.*
Ant.	2	4	+	3	added make	12	=	2
Conſ.	1	3		2		6		1

	Diapente.	*Diateſſeron.*		*Tone.*		*Diapente,*		or *Seſquialter.*
Ant.	3	4	+	9	added make	36	=	3
Conſ.	2	3		8		24		2

If many *Ratio's* are to be added together, proceed as in Reduction of Fractions of Fractions; that is, multiply all their Antecedents one into another, and likewiſe all their Conſequents. And if the Terms be reduced before Multiplication, the Total will be in its leaſt Terms. *Many* Ratio's *added.*

Antecedents. *Reduced.* *Example.*

Addends $\frac{3}{2}+\frac{4}{3}+\frac{2}{1}=\frac{24}{6}$ Total. $\quad \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}}+\frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{3}}}+\frac{2}{1}=\frac{4}{1}$

Conſequents.

In *Addition* of *Ratio's* may be obſerved; *Obſervations.*

1. *1ſt*, That *Addition* of *Ratio's*, ſhews how far the *Ratio's* added are diſtant from the *Ratio* of *Equality*. For *Dupla* and *Subdupla* added, ſhall make the *Ratio* of *Equality*. *What* Addition *of* Ratio's *ſhews.*

As $\frac{2}{1}+\frac{1}{2}=\frac{2}{2}$ or $\frac{1}{1}$. So $\frac{3}{5}+\frac{4}{1}=\frac{12}{5}$ Therefore the Complements of $\frac{3}{5}+\frac{4}{1}$ to Unity, which are $\frac{5}{3}+\frac{1}{4}=\frac{5}{12}$ & $\frac{5}{12}+\frac{12}{5}$ make the *Ratio* equal. *Examples.*

Otherwiſe if out of each particular *Ratio* I add or ſubſtract the Remains of the Complement, and make compariſon between the Remains, it will likewiſe ſo appear. For 3 is leſs than 5 by 2, and more than 1 by 2; after the Subſtraction there reſts 1. Likewiſe in the *Ratio* of the Complement ſo working there will alſo remain 1, which is in the *Ratio* of *Equality* with the other 1.

$$\begin{array}{c}12\\ \hline \frac{3}{5}+\frac{4}{1}\\ \hline 5\end{array}\qquad\begin{array}{c}5\\ \hline \frac{5}{3}+\frac{1}{4}\\ \hline 12\end{array}\qquad\begin{array}{c}3\\4\\5\\1\\ \hline 13\end{array}\quad\begin{array}{c}5\\1\\3\\4\\ \hline 13\end{array}\qquad\begin{array}{c}12\\ \hline 5\\ \hline 17\end{array}\quad\begin{array}{c}5\\ \hline 12\\ \hline 17\end{array}\qquad\begin{array}{c}60\\ \hline \frac{12}{5}+\frac{5}{12}\\ \hline 60\end{array}$$

Alſo $-1\frac{2}{5}+3=+1\frac{2}{5}-3$. And $\begin{array}{l}-2+3=+2-3\\ +1=-1\end{array}$

2. *2dly*, That Equal *Ratio's* added to Equal *Ratio's*, make the Total Equal. *When the Total will be Equal.*

As $\frac{2}{2}+\frac{3}{3}=\frac{6}{6}$ or $\frac{1}{1}$. *Example.*

3. *3dly*, That Equal *Ratio's* added to Inequal *Ratio's*, render the Total in the ſame *Ratio* of Inequality the Inequal *Ratio* was before *Addition*. *When the Total is Inequal.*

As $\frac{3}{3}+\frac{3}{2}=\frac{9}{6}$ And $\frac{3}{3}+\frac{2}{3}=\frac{6}{9}$ *Examples.*

Ratio $0+1\frac{1}{2}=1\frac{1}{2}$ *Ratio* $0+\frac{2}{3}=\frac{2}{3}$

Queſtions Reſolved by Addition *of* Ratio's.

Queſtions.

1. The *Ratio* of a Penny to a Farthing is as 4 to 1. The *Ratio* of a Shilling to a Penny is as 12 to 1. What is the *Ratio* of a Shilling to a Farthing? *Anſw.* As 48 to 1, for ſo is the Total of the added *Ratio's*. 1. Ratio *of a Shilling to a Farthing.*

$$\frac{4}{1}+\frac{12}{1}=\frac{48}{1}$$

2. *Of two Horses Draught.* 2. An Horse draws a Weight, but can draw three times as much; and if another Horse be joined to him, How much will both Horses draw betwixt them, when the first draws as much as he can, and the other but three quarters as much as he can?

Answ. Twice as much and a quarter more, as the *Addition* of the given *Ratio's* shews.

$$\frac{3}{1} + \frac{3}{4} = \frac{9}{4}\ (2\tfrac{1}{4}$$

Proof of Addition of Ratio's. *Addition* of *Ratio's* is to be proved by *Substraction*, as in the next Chapter is made plain.

CHAP. IV.

Substraction of RATIO'S.

Substraction of Ratio's *in two Cases.* RATIO'S are substracted as Fractions are divided. So two Cases are sufficient for their *Substraction*.

1. Homological *Terms not reducible.* *Case* 1. When the *Homologal* Terms need no Reduction, multiply crosswise the Antecedent of the *Ratio*, out of which *Substraction* is to be made by the Consequent of the *Ratio* to be substracted, and the Product shall be the remaining Antecedent. And likewise the Consequent of the *Ratio*, from which *Substraction* is to be made by the Antecedent of the *Ratio* to be substracted, and you shall have the Consequent of the Remain.

Examples. As the *Ratio* of $\frac{4}{3}$ substracted from the *Ratio* of $\frac{3}{2}$, the Remain will be $\frac{9}{8}$. And if the *Ratio* of $\frac{3}{2}$ be taken from the *Ratio* of $\frac{4}{3}$, there will remain $\frac{8}{9}$.

$$\frac{3}{2} - \frac{4}{3} = \frac{9}{8}\ \text{Remain.} \qquad \frac{4}{3} - \frac{3}{2} = \frac{8}{9}\ \text{Remain.}$$

2. Homologal *Terms reducible.* *Case* 2. Where the *Homologal* Terms, or either of them, may be reduced, reduce them like Fractions as low as you can; and then multiply the reduced Terms as above.

Example. As to take the *Ratio* of $\frac{4}{3}$ from $\frac{2}{1}$, because 4 and 2 will abbreviate, they are reduced to 2 and 1: And being multiplied, the Remain shall be in its least Terms $\frac{3}{2}$, which otherwise will be $\frac{6}{4}$.

$$\frac{\overset{1}{2}}{1} - \frac{\overset{2}{4}}{3} = \frac{3}{2}$$

Substraction of *many from one, or one from many.* If many *Ratio's* were to be substracted from one, or one *Ratio* from many, then first add together the *Ratio's* exceeding one, and afterward make *Substraction* as above.

Example. As to substract $\frac{2}{1}$ from $\frac{4}{3}$ and $\frac{3}{2}$, I first add $\frac{4}{3}$ and $\frac{3}{2}$ and the Total is $\frac{12}{6}$, or by Reduction $\frac{2}{1}$; From which if $\frac{2}{1}$ be taken, there remaineth the *Ratio* of Equality $\frac{2}{2}$ or $\frac{1}{1}$.

Addition. *Substraction.* Without *Reduction.*

$$\frac{\overset{2}{4}}{\underset{1}{3}} + \frac{\overset{1}{3}}{\underset{1}{2}} = \frac{2}{1} \qquad \frac{\overset{1}{2}}{1} - \frac{\overset{1}{2}}{1} = \frac{1}{1} \qquad \frac{4}{3} + \frac{3}{2} = \frac{12}{6} - \frac{2}{1} = \frac{12}{12}\ \text{or}\ \frac{1}{1}$$

Substraction of *many from many.* If many *Ratio's* are to be substracted from many, proceed in like manner, adding together the *Ratio's* from which *Substraction* is to be made, and also the *Ratio's* to be substracted: And then make *Substraction* as above.

Example. As $\frac{4}{3}$ and $\frac{3}{2}$ to be substracted from $\frac{2}{1}$ and $\frac{3}{4}$, the Remain will be $\frac{3}{4}$

$$\text{For}\ \frac{4}{3} + \frac{3}{2} = \frac{2}{1} \qquad \text{And}\ \frac{2}{1} + \frac{3}{4} = \frac{3}{2} \qquad \text{And}\ \frac{3}{2} - \frac{2}{1} = \frac{3}{4}$$

Some-

Sometime more to puzzle a young Practitioner than otherwise, many *Ratio's* are given to be substracted from many, annexed with the Signs + and —, the which though improper, because, as was noted before, those Signs properly belong to *Æquations*; yet there is more Difficulty apparent than real therein. For it is but to add what is to be added, and substract what is to be substracted of the *Data*. And for a final Resolution, work with the New *Ratio's*. *If Signs be used, how to proceed.*

As if $\frac{2}{1} + \frac{3}{2} - \frac{1}{3}$ were to be substracted from $\frac{4}{3} + \frac{5}{6} - \frac{4}{5}$; then first I add $\frac{2}{1} + \frac{3}{2}$, and from the Total $\frac{3}{1}$ I substract $\frac{1}{3}$ and there resteth $\frac{9}{1}$; Also $\frac{4}{3} + \frac{5}{6}$ added, make $\frac{10}{9}$, from which $\frac{4}{5}$ taken, leaves $\frac{25}{18}$. Lastly, From thence I withdraw the *Ratio* $\frac{9}{1}$, and the ultimate Remain is $\frac{25}{162}$. *Example.*

$$\overset{2}{\frac{4}{3}} + \underset{3}{\frac{5}{6}} = \overset{5}{\frac{10}{9}} - \overset{2}{\frac{4}{5}} = \frac{25}{18} \qquad \overset{1}{\frac{2}{1}} + \underset{1}{\frac{3}{2}} = \frac{3}{1} - \frac{1}{3} = \frac{9}{1}$$

$$\text{And } \frac{25}{18} - \frac{9}{1} = \frac{25}{162} \text{ Ultimate Remain.}$$

In *Substraction* of *Ratio's* may be observed: *Observations.*

1*st*, That Equal *Ratio's*, substracted from Equal *Ratio's*, leave the Remains Equal. *1. When the Remain will be equal.*

$$\text{For } \frac{2}{2} - \frac{3}{3} = \frac{6}{6}$$ *Example.*

2*dly*, That if Equal *Ratio's* be substracted from Inequal *Ratio's*, the Remains will be left in the same *Ratio* of Inequality the Inequal *Ratio* was before *Substraction* made: And is all one as if the *Ratio's* were added. *2. When the Remain will be inequal.*

$$\text{As } \overset{1}{\frac{3}{2}} - \overset{1}{\frac{3}{3}} = \frac{3}{2} \qquad \text{And } \underset{1}{\frac{2}{3}} - \underset{1}{\frac{3}{3}} = \frac{2}{3}$$

$$\textit{Ratio } 1\tfrac{1}{2} - 0 = 1\tfrac{1}{2} \qquad \textit{Ratio } \frac{2}{3} - 0 = \frac{2}{3}$$ *Examples.*

3*dly*, Inequal *Ratio's* substracted from Equal, leave the Remain of the same Inequal Terms the substracted *Ratio* was, but alters the same so, that if before *Substraction* the *Ratio* were of the Greater Inequality, now it shall be of the Lesser; and if before of the Lesser, now of the contrary. *3. Inequal from Equal, what the Remain.*

$$\text{As } \overset{1}{\frac{3}{3}} - \overset{1}{\frac{3}{2}} = \frac{2}{3} \qquad \text{And } \underset{1}{\frac{3}{3}} - \underset{1}{\frac{2}{3}} = \frac{3}{2}$$

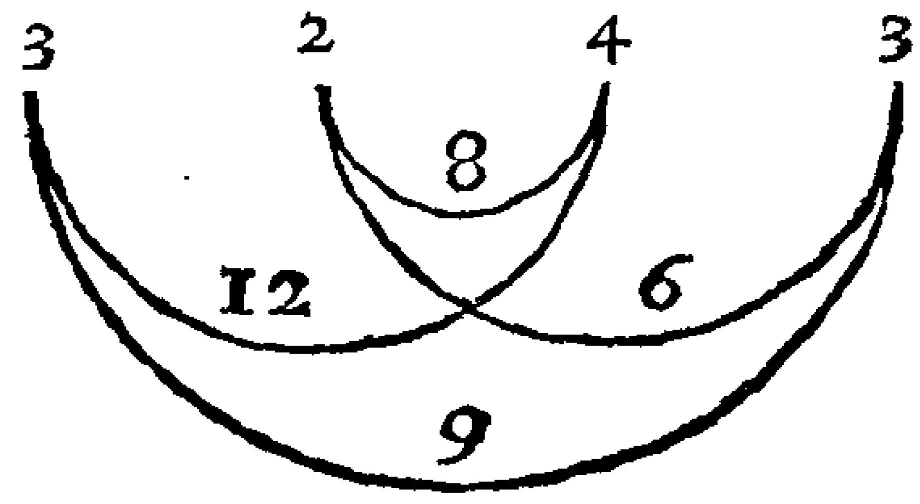
$$\textit{Ratio } 0 - 1\tfrac{1}{2} = \frac{2}{3} \qquad \textit{Ratio } 0 - \frac{2}{3} = 1\tfrac{1}{2}$$ *Examples.*

4*thly*, *Substraction* shews which is the greatest *Ratio* of two given; for after the *Substraction*, if the Remain be of the greater Inequality, then was the *Ratio* from which *Substraction* was made greater than the substracted *Ratio*: But if the Remain be of the lesser Inequality, then understand the contrary. Both these are so evident in the last Examples, that no farther Demonstration thereof is needful. *4. What Substraction of Ratio's shews.*

5*thly*, *Addition* and *Substraction* of *Ratio's* serves to find out new Proportional *Ratio's*; for the new *Ratio's* found by both, are reciprocally proportional. *5. What Addition and Substraction of Ratio's serves for.*

As if $\frac{4}{3}$ be added to $\frac{3}{2}$ without Reduction, the Total will be $\frac{12}{6}$. And if $\frac{3}{2}$ be substracted from $\frac{4}{3}$, the Remain will be $\frac{8}{9}$. Then it is evident that 12 to 8 is as 3 to 2; and 12 to 9, is as 4 to 3: Also 8 to 6, is as 4 to 3; and 9 to 6, is as 3 to 2: So that 8 and 9 have proportion with the other reciprocally, as by the Scheme following may appear: After which Form the Antient Writers of Musick were wont to express both the *Addition* and *Substraction* of *Ratio's*, as Mr. *Oughtred* testifieth, Chap. 10. of his *Clavis*. *Examples.*

The Antient Form of expressing Addition *and* Substraction *of* Ratio's.



$\overset{3}{\underset{2}{:}} + \overset{4}{\underset{3}{:}} = \overset{12}{\underset{6}{:}}$ *Ergo* $\overset{12}{\underset{8}{:}} + \overset{8}{\underset{6}{:}} = \overset{96}{\underset{48}{:}}$ or $\overset{12}{\underset{6}{:}}$ Also $\overset{9}{\underset{6}{:}} + \overset{12}{\underset{9}{:}} = \overset{108}{\underset{54}{:}}$ or $\overset{12}{\underset{6}{:}}$.

$\overset{4}{\underset{3}{:}} - \overset{3}{\underset{2}{:}} = \overset{8}{\underset{9}{:}}$ *Ergo* $\overset{8}{\underset{12}{:}} - \overset{9}{\underset{12}{:}} = \overset{96}{\underset{108}{:}}$ or $\overset{8}{\underset{9}{:}}$ Also $\overset{9}{\underset{6}{:}} - \overset{8}{\underset{6}{:}} = \overset{54}{\underset{48}{:}}$ or $\overset{9}{\underset{8}{:}}$

Questions.

Questions resolved by Substraction *of* Ratio's.

1. Ratio *of a Pound to a Shilling.*

1. The *Ratio* of a Pound to a Penny, is as 240 to 1. The *Ratio* of a Shilling to a Penny, is as 12 to 1. What is the *Ratio* of a Pound to a Shilling? *Answ.* As 20 to 1, for so is the Remain left after *Substraction*.

$$\begin{array}{c}20\\:\\240\\:\\1\end{array} - \begin{array}{c}1\\:\\12\\:\\1\end{array} = \begin{array}{c}20\\:\\1\end{array} \qquad \tfrac{240}{12}\Big(\,20$$

2. *Of the Draught of one Horse.*

2. Suppose two Horses together draw twice as much, and a quarter part more at one time as at another when the Ways are bad: One of which Horses alone can draw a Weight of no greater *Ratio* than 3 to 4. What *Ratio* shall the Weight drawn by the other Horse bear?

Answ. 3 to 1: for if the *Ratio* $\overset{3}{\underset{4}{:}}$ be substracted from the *Ratio* $2\tfrac{1}{4}$, the Remain will be $\overset{3}{\underset{1}{:}}$.

$$\begin{array}{c}3\\:\\9\\:\\4\\:\\1\end{array} - \begin{array}{c}1\\:\\3\\:\\4\\:\\1\end{array} = \begin{array}{c}3\\:\\1\end{array} \qquad \tfrac{36}{12}\Big(\,3$$

Proof of Substraction *of* Ratio's.

Substraction and *Addition* of *Ratio's* are Reciprocal Proofs of each other, as *Multiplication* and *Division* of Fractions, and need no more than these two Questions, being of a contrary Nature to those in *Addition*, compared one with another to make it plain: For the Remain of any *Substraction* of *Ratio's*, added to the *Subtrahend*, after the manner of *Ratio's*, will return the Number from which *Substraction* was made, that is, the Total of their *Addition*.

CHAP. V.

Multiplication of RATIO'S.

Multiplication *of* Ratio's.

THE prime Parts of *Numeration* of *Ratio's*, in their *Addition* and *Substraction*, have been now seen to be like the Compound Parts of *Numeration* in other Numbers. And as before observed, their Compounds Parts of *Numeration*, viz. *Multiplication* and *Division*, are like Figuration of other Numbers.

Rule.

To multiply therefore any *Ratio*, is to multiply the Antecedent and Consequent so often into themselves respectively, as there be Units in the Multiplier.

Examples.

As to double $\overset{3}{\underset{2}{:}}$, the *Ratio Sesquialter*, or multiply it by 2, is to multiply 3 by 3, and 2 by 2: So is the Product $\overset{9}{\underset{4}{:}}$

And to triple the *Ratio Sesquitertia*, I either set down $\overset{4}{\underset{3}{:}}$ three times, and multiply the Antecedents one into another, and so likewise the Consequents: Or multiply 4 cubically, and 3 likewise: Whereby there is produced $\overset{64}{\underset{27}{:}}$.

Factors.	*Product.*	*Factors.*	*Product.*
$\overset{3}{\underset{2}{:}} \times \overset{3}{\underset{2}{:}}$	$= \overset{9}{\underset{4}{:}}$	$\overset{4}{\underset{3}{:}} \times \overset{4}{\underset{3}{:}} \times \overset{4}{\underset{3}{:}}$	$= \overset{64}{\underset{27}{:}}$

In like manner as to multiply any *Ratio* by 2, is to square the same; and to multiply a *Ratio* by 3, is to cube it. So to multiply by any other Number, is to figurate the Terms of the *Ratio* accordingly. *Multiplication figurates the Terms.*

Quest. *Ptolomy* in his *Almagest*, proposes the Diameter of the Sun to the Diameter of the Earth, to be as 11 to 2. And by *Prop.* 18. of *Euclid*'s 12*th* Book, Spheres or Globes have triple Proportion to their Diameters. How much then shall the Sun be bigger than the Earth? *Question of the Bigness of the Sun.*

Answ. 166 times and $\frac{3}{8}$; for the *Ratio* of $\frac{11}{2}$ tripled that is cubed, produceth $\frac{1331}{8}$, which reduced by *Division*, gives $166\frac{3}{8}$ as before. *Answer.*

$$\frac{11}{2} \times \frac{11}{2} \times \frac{11}{2} = \frac{1331}{8} (166\tfrac{3}{8}$$

Multiplication of *Ratio's* is to be proved by their *Division*, as in the next Chapter is to be seen. *Proof of Multiplication of Ratio's.*

CHAP. VI.

Division of RATIO'S.

As *Multiplication* figurates the Terms of the *Ratio*; so on the contrary, *Division* of *Ratio's* is to extract from each Term of the *Ratio* given to be divided, a Root of the second, third, or fourth Quantity, *&c.* according to the Units contained in the Divisor. As to divide a *Ratio* by 2, is to extract the Square Root thereof: And to divide by 3, is to extract the Cube Root, *&c.* *Division of Ratio's is as Extraction of Roots.*

Thus $\frac{9}{4}$ divided by 2, gives in the Quotient $\frac{3}{2}$ *Examples.*
And $\frac{27}{8}$ divided by 3, gives in the Quotient $\frac{3}{2}$
But $\frac{256}{81}$ if divided by 4, makes the Quotient $\frac{4}{3}$

Dividend.

Divisor $\frac{4}{1}$) $\frac{256}{81}$ ($\frac{4}{3}$ Quotient

256	16	4
81	9	3

Quest. If the Body of the Sun, according to *Ptolomy*, be bigger than the Globe of the Earth 166 times and $\frac{3}{8}$; then what *Ratio* is there between their Diameters? *Question of the Diameters of the Sun and the Earth.*

Answ. $\frac{11}{2}$: for $166\frac{3}{8}$ reduced, and the *Ratio* divided by 3, that is, the Cube Root of each Term taken, the Quotient or Root is $\frac{11}{2}$ as aforesaid. *Answer.*

Reduced. $166\frac{3}{8} = \frac{1331}{8}$

Divided. $\frac{3}{1}$) $\frac{1331}{8}$ ($\frac{11}{2}$

1331	11
8	2

Division and *Multiplication* of *Ratio's*, being as Production of Figural Numbers, and Extraction of Roots; it must needs be that the one shall be Proof of the other reciprocally as they were, and hath been largely discoursed before in Figural Numbers. And the last Question being but the Reverse of the Question before in *Multiplication*, will clearly evince without farther Testimony. *Proof of Division of Ratio's.*

Partis primæ Libri quarti

FINIS.

The Second PART of the Fourth BOOK.

CHAP. I.

Of PROPORTIONS Disjunct.

Proportions, *their Computations and Comparative Elements.*

NEXT to the Computation of *Ratio's*, come the Computation of *Proportions*, and their Comparative Elements, to be viewed.

Among *Proportions*, the first that present themselves are Simple, and of them those called *Disjunct.*

Disjunct, where described.

Disjunct *Proportions*, in the first Chapter of this 4*th* Book, were generally described to be *Arithmetical* and *Geometrical.*

Those Arithmetical of little use. *Geometrical excel.*

Those *Arithmetical* (yet of little use in *Arithmetick*) were before remembred to give being to some Musical *Proportions*, and have somewhat mentioned of them hereafter in this Chapter, with others *Geometrical*; but give place to these, as being of much more excellent Use, not only in *Arithmetick*, but in several other Arts and Sciences.

Wherein their Comparative Elements consist.

The Comparative Elements of Geometrical Disjunct *Proportions*, consist in the Invention of new Proportional Numbers, Plain or Figural, according to the *Data* by which they are found.

What they are, vide antea. *Comparative Elements twofold.*

What Plain and Figural *Proportions*, Disjunct and Geometrical, are, the first Chapter of this 4*th* Book hath told us. And that these plain *Proportions* are Direct and Indirect; both which, with their Comparative Elements, may be subdivided into *Primitive* and *Derivative.*

Primitive, *what.*

Primitive give three Numbers to find out a Fourth or new Proportional by a single Operation, and need nothing before Operation save a due and orderly Disposition of the *Data.*

Primitives are again subdivided

How divided, and where handled.

Into
- Common, in the *Rule of Three*
  - *Direct*, Chap. 2.
  - *Indirect*, Chap. 3.
- Peculiar, in *Practice*, Chap. 4.

Derivative *of four sorts.*

Derivative before Operation, need some or other of the Simple Elements of Numbers to fit them for Resolution.

Or, 2. give more than three Numbers.
Or, 3. deal with some particular Subject.
Or, 4. have some peculiar Operation requisite to their Resolution.

Where handled.

Derivatives
- of the first Sort, are Specificks *Direct* and *Indirect*, —— Chap. 5.
- of the second Sort, The Rule of 5 Numbers
  - *Direct*, —— Chap. 6.
  - *Indirect*, —— Chap. 7.
- of the third Sort
  - *Fellowship*, —— Chap. 8.
  - *Alligation*, —— Chap. 9.
  - *Barter* and *Exchange*, —— Chap. 10.
  - *Loss* and *Gain*, —— Chap. 11.
  - *Æquation* of Paiment, —— Chap. 12.
  - *Factorship*, —— Chap. 13.
- of the fourth Sort, *Falshood* or *Position*, —— Chap. 14.

Figural Proportions, where handled.

Figural Disjunct Proportions follow them,
- *Doubled*, —— Chap. 15.
- *Tripled*, &c. —— Chap. 16.

Concerning

Concerning Disjunct Proportions in general, take this farther Account.

Notes on Disjunct Proportions.

1. Terms what, and how set.

1. The *Data*, or Numbers given, are called *Terms*; of which the first is set to the left Hand, and the Residue in order towards the Right.

2. Extreams and Means, what.

2. The least and greatest Terms are called *the Extreams*, and the others *the Means*, or *middle Proportionals*. Also the Sinister Term is called the *Antecedent*, and the Second his *Consequent*. So likewise the Third and Fourth.

3. Arithmetical Disjunct Proportions, *the Total of the Means and Extreams equal. Example.*

3. In Arithmetical *Disjunct Proportions*, the Aggregate of the Extreams shall be equal to the Aggregate of the Means.

As in 4 . 7 : 5 . 8, the Sum of 4 and 8, is equal to the Sum of 7 and 5.

4 . 7 : 5 . 8

12

12

4. Three Terms thereof given, to find a fourth. Example.

4. If of three Terms in Arithmetical *Disjunct Proportion*, a fourth be sought; from the Second added to the Third, the First shall be substracted.

As in the former Example, if 4 . 7 . 5 be given; then from 12, the Sum of 7 and 5, shall 4 be abated; so will the Remain be 8.

For $7 + 5 - 4 = 8$.

5. Geomet. Disjunct Proportions, *the Product of the Extreams and Means equal. Example.*

5. Numbers or Magnitudes in Geometrical *Disjunct Proportion*, have the Aggregate of the greatest and least Terms, greater than the Aggregate of the Residue: but the Product of the Extreams, equal to the Product of the Means.

As 5 . 15 :: 6 . 18, the Total of 5 and 18 is 23, greater than 21, the Total of 15, and 6; but the Product of 5 by 18, which is 90, is equal to the Product of 15 by 6.

Added.	Multiplied.
5 . 15 :: 6 . 18	5 . 15 :: 6 . 18
21	90
23	90

Example in Species.

R . S :: Z . A	R . S :: Z . A

Ergo, R + A shall be greater than S + Z. But $SZ = RA$, and by Consequence $\frac{ZS}{R} = A$ in Numbers $\frac{90}{5} = 18$.

Whence the Rule of Three.

From hence sprang the *Rule of Three*, spoken to in the next Chapter.

6. Four in Geom. Disjunct Proportion, how otherwise Proportional. Alternate.

6. Four Numbers or Magnitudes in Disjunct Geometrical *Proportion* Direct, shall also be proportional; if they be Alternate, Inversed, Compounded, Parted, Conversed and Mixt.

As admit 7 get 28, and accordingly 9, 36, then alternately shall 7 . 9 :: 28 . 36; that is, Antecedent to Antecedent, and Consequent to Consequent; and there the *Ratio* of 7 to 9, is alike to that of 28 to 36.

Inversed.

And Inversed 28 . 7 :: 36 . 9, the *Quadruple Ratio* of 28 to 7, is the same with 36 to 9. Here the Consequent is taken as the Antecedent, and so compared to the Antecedent as the Consequent.

Compounded.

Also Compounded; as 7 and 28 which is 35 is to 28, so shall 9 and 36 that is 45 to 36. This is when the Antecedent and Consequent, together as one, are compared to the Consequent.

Parted.

Again, being divided 7 — 28 . 28 :: 9 — 36 . 36. And if withdrawing the first Term from the Second, the Residue be compared the Second, yet the *Ratio* shall be alike to the *Ratio* of the Remainder of the third Term substracted from the Fourth, compared to the Fourth. For divided *Ratio* is, when the Excess

wherein

wherein the Antecedent exceedeth the Consequent, is compared to the Consequent, as 21 . 28 :: 27 . 36 ; where as well 27 to 36, as 21 to 28, are in the *Ratio Subsesquitertia*.

Conversed. Likewise, if conversed, that is, when the Antecedent is compared to the Excess, wherein the Antecedent exceeds the Consequent, 7 to 35, which is the Sum of the first and second Terms, are in a like *Ratio* as 9 to 45, the Sum of the third and fourth Terms.

Mixt. Moreover, Mixt, as 7 + 28 . 7 — 28 :: 9 + 36 . 9 — 36 ; which in effect, is as 35 . $\overline{21}$ :: 45 . $\overline{27}$, both agreeing in the *Ratio Superbipartiens-Tertias*.

Examples.

An Example in Species explained, with other Numbers.

Data	A . α :: B . β	4 . 16 :: 6 . 24	
Alternate	A . B :: α . β	4 . 6 :: 16 . 24	*Subsesquialter.*
Inversed	α . A :: β . B	16 . 4 :: 24 . 6	*Quadruple.*
Compounded	A + α . α :: B + β . β	4 + 16 . 16 :: 6 + 24 . 24	*Sesquiquarta.*
	A + B . B :: α + β . β	4 + 6 . 6 :: 16 + 24 . 24	*Superbipartiens-Tertias.*
Divided	A — α . α :: B — β . β	4 — 16 . 16 :: 6 — 24 . 24	*Subsesquitertia.*
	A — B . B :: α — β . β	4 — 6 . 6 :: 16 — 24 . 24	*Subtriple.*
Conversed	A . A ± α :: B . B ± β	4 . 4 ± 16 :: 6 . 6 ± 24	*Subquimuple. Subtriple.*
	A . A ± B :: α . α ± β	4 . 4 ± 6 :: 16 . 16 ± 24	*Subduplasesquial. Subdupla.*
Mixt	A + α . A − α :: B + β . B − β	4 + 16 . 4 — 16 :: 6 + 24 . 6 − 24	*Superbipartiens-Tertias.*
	A + B . A − B :: α + β . α − β	4 + 6 . 4 — 6 :: 16 + 24 . 16 − 24	*Quintuple.*

7. Numbers Proportional, the Consequents. 7. Certain Numbers or Magnitudes proportional, it shall be, That as one Antecedent to his Consequent ; so the Sum of the Antecedents to the Sum of the Consequents.

Examples.

Example in Numbers.

As 4 . 16 :: 6 . 24 :: 3 . 12 :: 2 . 8 ; It shall be then,
That 4 . 16 :: 4 + 6 + 3 + 2 . 16 + 24 + 12 + 8.

Example in Species.

As A . a :: B . b :: C . c :: D . d ; Therefore it shall be,
That A . a :: A + B + C + D . a + b + c + d.

8. Antecedents equal the Consequents. 8. When the Antecedents of many *Proportions* are equal, it shall be ; That as one Antecedent, to the Sum of his Consequents : So another Antecedent to the Sum of his Consequents.

Example in Numbers and Species.

Examples.

As	4 . 16 :: 6 . 24	And 4 . 12 :: 6 . 18	And 4 . 10 :: 6 . 15
	A . B :: a . b	A . C :: a . c	A . D :: a . d

Ergo 4 . 16 + 12 + 10 :: 6 . 24 + 18 + 15
A . B + C + D :: a . b + c + d That is, 4 . 38 :: 6 . 57.

9. Four in Arith. Disjunct Proportion added to, or taken from others the Consequents. 9. If 4 Numbers or Magnitudes in Arithmetical *Proportion Disjunct*, be added to, or substracted from 4 others alike Proportional, the Totals and Remains will be Proportionals.

Example.

	3 . 5	9 . 11	Excess 2
	2 . 4	5 . 7	Excess 2
Totals	5 . 9	14 . 18	4
Remains	1 . 1	4 . 4	0

10. Four in Geo. Disjunct Proportion, multiply or divide others the Consequents. 10. If four Numbers, or Magnitudes, in Disjunct Geometrical Proportion, be multiplied or divided by four others respectively Proportional, The Products and Quotients shall be accordingly Proportional : And the *Ratio* in the Products will likewise be multiplied, but in the Quotients divided.

7 . 28

	7	.	28	::	9	.	36	Ratio 4
	8	.	32	::	9	.	36	Ratio 4
Products	56	.	896	::	81	.	1296	16
Quotients	$1\frac{1}{7}$	.	$1\frac{1}{7}$	::	1	.	1	1

Example.

CHAP. II.

The Direct Rule of Three.

IN the Front of the Comparative Elements of Disjunct Geometrical Proportions stands *The Rule of Three*, so called, because three Numbers are given to find out a Fourth Proportional Number; sometime *The Rule of Proportion*, because the Likeness or Agreement of some Numbers within themselves, or each to other, is thereby declared: But most frequently it is called *The Golden Rule* for its Excellency, the Conclusions wrought and obtained thereby being so many, and so profitable and useful, as exceed all Credence in the Unskilful. And to difference this from *The Double Rule of Three*, spoken to hereafter, in the 6*th* and 7*th* Chapters, this is called *The Simple Golden Rule*.

Direct Rule of Three, *why so called.* *Why called the* Rule of Proportion. *Why the* Golden Rule. *Why the* Simple Golden Rule.

This *Rule of Three*, as in the Chapter before noted, is *Direct* and *Indirect*: to the Performance and perfect Understanding of *The Direct Rule of Three*,

Rule of Three, Direct *and* Indirect.

Some things are { *Preparatory.* / *Operatory.* / *Probationary.* }

The *Preparatory* Part, is the Right Disposition and Consideration of the *Data*, or Numbers given. And this is to be had in the Precepts following.

Preparatory *to the Direct.*

1. Place the three given Numbers, as A . B . C, in one or other of the three following Varieties: but the last to the right Hand is of late as brief and best, most in use; that is, to divide the first Term from the Second by a Point, the Second from the Third by four Points, and the Third from the Fourth when found by another single Point. So shall it be read, as A is to B; so is C to D, signifying the Fourth new Proportional when found. And the *Data* so standing, the Number standing in the Place of A, shall be called the first Number or Term; and the Number placed as B, shall be the Second; and the Number in the Place of C, the Third.

1. *How to place the Numbers.*

Common Way.	Old Way.	Late Way.
1 2 3	A ——— B	
A — B — C —		A . B :: C.
	C ———	

2. Let the Number upon which the Question propounded depends, be always set in the third Place, or C, and called the third Number.
3. That Number of the *Data* which is of one Denomination and Nature with the Third, must be set in the Place of A, for the first Number.
4. The right Places found for two of the three given Numbers, of necessity the Number of another Nature or Denomination, left unplaced, shall be set in the second Place instead of B; and so will all the Three be rightly placed.
5. When the Denominations seem to be doubled, as Yards-long, Yards-broad, Pounds-principal, or Pounds-profit, *&c.* the latter Denomination is to be respected in placing the Numbers.
6. The fourth Number, when found, shall be of like Denomination with the Second.

2. *Where to set that on which the Question depends.* 3. *Which must be first.* 4. *Which the second Number.* 5. *Denominations doubled, how to place them.* 6. *Fourth of what Denomination.*

The *Operatory* Part consists in the { Invention of the *Divisor.* / Resolution of the *Question.* }

Operatory *in the Direct.*

To find the Divisor. To find the Divisor after the *Data* are rightly placed, consider whether the fourth Number, which is the Number quesited, must be greater or lesser than the Second. For if greater, then the least of the two Extreams (which are the first and third Numbers) shall be Divisor: But if less, then the greatest of those two Extreams is to be Divisor.

To resolve the Question. To resolve the Question, when the first Number is found to be Divisor, the Rule is called the *Direct Rule of Three*, and is to multiply the second Number by the Third, and divide the Product by the First. The Quotient of this Division shall be the Answer to the Question, and the fourth Proportional: For such Proportion as the third Number beareth to the First, and is greater or less, such Proportion shall this fourth Number bear to the Second.

Example. *Example.* What shall 18 ℔. of Spice [Yards of Cloth, Bushels of Wheat, &c.] cost me, when 3 ℔. [Yards, Bushels, &c.] of the same Commodity costeth me 5 *s*?

The Question thus propounded, I see, by the second Precept, 18 ℔. must be the third Number, because the Question depends thereon; and by the third Precept 3 ℔. shall be the first Number, because it is alike denominate to 18, the third Number. So of necessity 5 *s*. which is the odd denominate Number, shall be set in the second Place, according to the 4*th* Precept; and the Numbers thus rightly placed, stand as here set.

```
    ℔.  s.    ℔.                        ℔. s. ℔.
As  3 . 5 :: 18.          Commonly if  3—5—18?
```

And if wrote at length shall be, If 3 ℔. cost me 5 *s*. what shall 18 ℔?

Then to find the Divisor, consulting with Reason, it is evident, that 18 ℔. shall cost me more than 3 ℔. Wherefore enquiring for a greater Number than 5 *s*. the least Extream, which here is 3, shall be Divisor. And so 5 and 18 must be multiplied together, and the Product 90 divided by 3, the fourth Number and Resolution is obtained, which is found to be 30, and they to be Shillings, denominate as 5 *s*. by the 6*th* Precept.

```
    ℔.  s.    ℔.   s.
As  3 . 5 :: 18 . 30
               5              90
              --              -- (30 s.
              90              3
```

Proof of the Direct Rule of three, by The *Probationary* Part is either to { Reverse the Question, Or, Multiply the Extreams and Means.

Reversing the Question. To reverse the Question: Let the third Number of the one Question, be the First of the other; the former 4*th* the Second of the next Work; and the first of the former, the Third of the Latter; and after Operation, the Fourth of this Latter will be the former second; and so prove both, if there be no Error in the Operations.

Multiplying the Extreams and Means. To multiply the Extreams and Means respectively, according to the Fifth of the foregoing Chapter, the Products being equal, will also prove the Truth of any Question resolved by the *Rule of Three*.

For in the Instance before, If the Question be stated thus;

Example. If 18 ℔. cost 30 *s*. what shall 3 ℔. cost? Seeing a lesser Number than 30 is looked for, 18 the greater of the two Extreams shall be Divisor; which shall divide 90, the Product of 30, into 3. So will 5, the Quotient, be equal to the former second Number, and prove the former Work true.

Also if the Extreams and Means be multiplied in either Operation, they will produce in both Operations 90.

```
    ℔.   s.    ℔. s.
As 18 . 30 :: 3 . 5         3 . 5 :: 18 . 30        18 . 30 :: 3 . 5
           3                     90                       90
    18) -- (5                    90                       90
           90
```

And

And the *Sexcuple Ratio* between 18 and 3, is equal to the *Ratio* of 30 to 5.

Four Numbers in Proportion, how diversly transposed.

Nevertheless though this above be the common Course in reversing the Question, it may be noted further, That where four Numbers are proportional, their Order may be so transposed, that each of those Terms may be last in Proportion. As, first, 1 . 2 :: 3 . 6. Second, 3 . 6 :: 1 . 2. Third, 2 . 1 :: 6 . 3. Fourth, 6 . 3 :: 2 . 1. So as every Proportion doth implicitly contain four Orders, two descending, and two ascending; by one of which Orders, if of four Proportionals any three be given, the other Unknown may be found out.

In Resolution of all Questions proper, Multiplication and Division to be used.

The Benefit thereof.

That nothing may be wanting to compleat the Necessaries of this Chapter, let it be observed, that to the Resolution of every Question propounded, the Multiplication and Division used be proper to the Nature of the Numbers given: And this will save a burdensome Charge of the Memory with many Rules; which is always to be avoided, where one Rule and Method of proceeding is sufficient. As to resolve a Question propounded in Fractions, or other Contract Numbers, it is but to multiply and divide after the manner of Fractions, or other Contract Numbers proper to the *Data, mutatis mutandis*: Whereas most Arithmetical Writers deliver the Rule in Fractions, thus, Multiply the Numerator of the first Fraction, by the Denominator of the second Fraction, and that Product again by the Denominator of the third Fraction, and this Product shall be Divisor. Again, multiply the Denominator of the first Fraction, by the Numerator of the Second, and that Product again by the Numerator of the Third for the Dividend. Much more long and tedious to the Memory, than to multiply the Second by the Third, and divide by the First, as Fractions are to be multiplied and divided.

Example in Fractions.

Examples in Fractions of the Price of ½ an Ell.

If $\frac{3}{4}$ of one Ell cost me $\frac{4}{5}$ of a Pound *Sterling*, What shall $\frac{1}{2}$ of an Ell cost? Here $\frac{3}{4}$ being Divisor, $\frac{4}{5}$ and $\frac{1}{2}$ are multiplied together; and the Product $\frac{2}{5}$ divided by $\frac{3}{4}$, giveth in the Quotient $\frac{8}{15}$ of a Pound, or 10 *s.* 8 *d.*

Ell. *l.* *Ell.* *l.*

As $\frac{3}{4} \cdot \frac{4}{5} :: \frac{1}{2} \cdot \frac{8}{15}$ For $\frac{\overset{2}{4}}{5} \times \frac{1}{\underset{1}{2}} = \frac{2}{5}$ And $\frac{3}{4})\frac{2}{5}(\frac{8}{15}$

The other Way used by several, is thus.

Ell. *l.* *Ell.* *l.*

As $\frac{3}{4} \cdot \frac{4}{5} :: \frac{1}{2} \cdot \frac{8}{15}$

3 × 5 = 15, × 2 = Divisor 30

4 × 4 = 16, × 1 = Dividend 16

$\frac{16}{30} \Big| \frac{8}{15}$

For $\begin{cases} 4 \times 4 \times 1 = 16 \\ 3 \times 5 \times 2 = 30 \end{cases}$

Examples in other Contract Numbers, with their proper Multiplications and Divisions.

Decimals.

Decimals.

Of the Price of 640 Ells.

If 158¼ Ells cost 79 *l.* 2 *s.* 6 *d.* What will 640 Ells cost at that Rate? The first and second Numbers turned into Decimals, are 158,25; and 79,125. Operation being made as aforesaid, the Resolution is 320 *l.* and the whole Work is as here followeth.

	Ells.	*l.*	*Ells.*	*l.*
As	158,25	. 79,125 ::	640 .	320,0

(3)	640	
(2)	3165 000	3165
Index (1) of the	47475 0	50640000 (*l.* 320,0
Quotient.	50640,000	158255
		1582

Astronomicals.

Astronomicals.

Of the Digits eclipsed.

If the Diameter of the Moon be supposed 33′ 28″, and the deficient Scruples at a Lunar Eclipse are found to be 29′ 5″, What are the Digits eclipsed? Astronomers allowing 12 Digits for the whole, 12 shall be the second Number, by the

proper

proper multiplication whereof into the third Number, after Division made there is found 10 Digits, 25′ 41″, and almost 50‴.

$$\text{As } 33' , 28'' . 12 :: 29' , 5'' . 10 \text{ Digits}, 25' , 41'', \&c.$$

$$\begin{array}{r} 29' . 5'' \\ 12^{\circ} \\ \hline 348 . 60 \\ \hline 5^{\circ} . 49' . 00'' \\ \hline \end{array}$$

$$\begin{array}{l} \qquad\qquad \cancel{23} \mid 27 \\ \quad \cancel{14} . \cancel{20} . \cancel{20} \mid 52 \\ \hline) \; \cancel{5}^{\circ} , \cancel{49}' , \cancel{00}'' , \cancel{00}''' , \cancel{00}'''' , \&c. \; (\; 10^{\circ} , 25' , 41'' \\ \hline 5 . 34 . 40 \\ \quad 13 . 56 . 40 \\ \qquad 22 . 52 . 08 \end{array}$$

Logarithms.

Logarithms. *Of Interest-Money.*

If at Simple Interest, 6 *s.* be the Gain of 5 *l.* Principal Money, What in the same space of Time, and at the same Rate, shall 100 *l.* gain? Because Addition of Logarithms is equivalent to Multiplication, and Substraction to Division; The Logarithms of the second and third Numbers added, and the Log. of 5 *l.* the first Number substracted, the Log. for Resolution will be 2,07918,12461, which is the Log. for 120 *s.* or 6 *l.*

$$\begin{array}{ccccc} l. & s. & l. & s. & l. \end{array}$$
$$\text{As } 5 . 6 :: 100 . 120 . \text{or } 6$$

Or, 0,69897,00043. 0,77815,12504 : 2,00000,00000. 2,07918,12461.

Log. of 6.	0,77815,12504		
Log. of 100.	2,00000,00000		
Sum	2,77815,12504	Product.	Log. of 600.
Log. of 5.	0,69897,00043		
Difference	2,07918.12461	Quotient.	Log. of 120.

Cossicks.

Cossicks. *Of the Equality of 30.φ.*

If 16ʓ be equal to 2ſs, what shall 30φ (coming from the same Root) be equal to? Answer, by the following Operation to 3ʓφ and ¼.

As 16ʓ . 2ſs :: 30φ . 3¼ ʓφ.

For 2ſs × 30φ = 60ʓʓʓ. And 16ʓ) 60ʓʓʓ (3¼ʓφ.
Indices 5+3 = 8 Indices 8—2=6

The like for Compound *Cossicks* whole and broken.

Surds.

Surds. *Of the Likeness of 5 + √4.*

If 3+√16 be alike to 4+√9, What shall 5+√4 be like to? By the Operation following, (performed according to the Nature of the *Data*) the Answer appears to be √25+√4.

As 3+√16 . 4+√9 :: 5+√4 . √25+√4.

3—√16	5+√4
9+√144	20+√225
—√144—16	√64+√36
9—16	20+√225+√64+√36
—7	3—√16
—√49	60+√2025+√576+√324
	—√6400—√3600—√1024—√576
	—√1225—√196

—√49)—√1225—√196(√25+√4

The like for *Simple Surds* as well as *Compound*, both Integers and Fractionary.

Species. — *Species. Of 7AD, what required.*

If 8 AB require 56 BC, What ſhall 7 AD require? Operation being made according to the Simple Elements of Species, the Reſolution for Anſwer will be 49 CD.

As 8AB . 56BC :: 7AD . 49CD.
For 56BC × 7AD = 392BCAD. And 8AB)392BCAD(49CD.

The like for other *Species*, Simple, Compound, Integral, Fracted, Rational or Irrational, *mutatis mutandis*.

Proof of Contract like others. And as the Proceſs in the Work of theſe Contract Numbers: So the Proof of the Work, when done, needs no new Inſtitution. For the Reverſe of the Queſtion, or the Multiplication of the Extreams and Means, ſhall prove them all. And over and above, as the Simple Elements of Contract Numbers had their ſingularity of Proof in reducing them to other Numbers: So the Comparative Elements of Contract Numbers may be proved, by taking other Numbers in their ſtead, and comparing the Reſolutions together: As in the laſt Example thus proved is evident.

Example.

Reverſe of the Queſtion.	As 7AD . 49CD	:: 8AB . 56BC.
Extreams in the one,	56BC	49CD
Means in the other,	7AD	8AB
	392BCAD	= 392CDAB

Reduction into other Numbers, ſuppoſing A=1 . B=2 . C=3 . D=4.
Then 8AB . 56BC :: 7AD . 49CD . ſhall be
As 16 . 336 :: 28 . 588 .
For 336 × 28 = 9408. And 16)9408(588.

CHAP. III.

The Indirect Rule of Three.

Indirect Rule of Three. How called. THE next of the Comparative Elements of *Disjunct Proportions Geometrical*, is, *The Indirect Rule of Three*, called ſeverally by Authors, as, *The Reverſed*, *Converſed*, *Inverſed*, *Reciprocal*, and *Backer Rule*; though improperly, except the Firſt and two Laſt.

What herein like to the Direct. In this *Indirect Rule of Three*, all the Preparatory and Probationary Parts, and the finding of the Diviſor, is exactly the ſame with that in the precedent Chapter of *The Direct Rule of Three*, the only Difference is in the Reſolution.

The third Number always Diviſor. To reſolve the Queſtion propounded; the third Number being found to be Diviſor, as it always will in Indirect Proportions, the Rule is, Multiply the firſt and ſecond Numbers together, and divide the Product by the Third. The Quotient of this Diviſion ſhall be the 4*th* Proportional Number, and anſwer the Queſtion. For the Proportion between the third and firſt Numbers, is the ſame between the Second and Fourth.

Queſtion of the Weight of Bread. *Example.* If at 4*s.* the Price of a Buſhel of Wheat, the Penny white Loaf muſt weigh 9 Ounces *Troy*, What ſhall it weigh at 6 *s.* a Buſhel?

The Numbers right placed, according to the Precepts of the former Chapter, to find the Diviſor, I conſider, the dearer the Corn, the leſſer the Loaf: So the Number queſited being leſs than the Second, the greater of the two Extreams, which is 6, the third Number muſt be Diviſor. Therefore 4 multiplied into 9, and the Product 36 divided by 6, giveth 6 Ounces for the Number deſired.

	s.	*Ounces.*		*s.*	*Ounces.*	*Ounces.*
As	4 .	9	::	6 .	6	
		4				36 (6
		36				6

 Proof

Proof of the Indirect Rule of Three.

Proof as in the *Rule of Three Direct*, by

Reverse of the Question. Ratio.

	s.	Ounces.		s.	Ounc.	Extreams.		Means.
As	6	. 6	::	4	. 9	4 . 9	::	6 . 6

6

4)36(9 Ounces. 36 = 36

$2)\frac{6}{4}(\frac{3}{2}$ $3)\frac{9}{6}(\frac{3}{2}$ abbreviated.

$4)6(1\frac{1}{2}$ $6)9(1\frac{1}{2}$ divided.

Proper Multiplication and Division here to be used. The Benefit thereof.

Further as necessary to this Chapter is to be remembred, for the Reason before rendred in the *Direct Rule of Three*, That whensoever any Question resolvable by the *Indirect Rule of Three*, is propounded in Fractions, or other Contract Numbers, the Multiplication and Division proper to the *Data* be used. It being much shorter to remember, to multiply the first and second Numbers after the manner of Fractions, and divide the Product by the Third as Fractions are divided, than to multiply the Numerator of the first Fraction, by the Numerator of the second Fraction; and that Product again by the Denominator of the Third for the Dividend: And again, to multiply the Denominator of the First by the Denominator of the Second; and that Product again by the Numerator of the Third for the Divisor, as some deliver the Rule.

Example in Fractions.

Examples in Fractions. Of keeping Money lent.

If I lend $\frac{3}{4}$ of a Pound to a Friend for 7 Months. And when I come to borrow of him, he can spare me but $\frac{5}{12}$ of a Pound: How long may I keep from paiment thereof again, to equalize the Interest of his Loan to me?

Answ. 12 Months and $\frac{3}{5}$ of a Month.

l. M. l. M.

As $\frac{3}{4} . \frac{7}{1} :: \frac{5}{12} . 12\frac{3}{5}$ For $\frac{3}{4} \times \frac{7}{1} = \frac{21}{4}$ And $\frac{5}{12})\frac{21}{4}(\frac{63}{5}(12\frac{3}{5}$

3 1

The other Way used by several.

l. M. l. M.

As $\frac{3}{4} . \frac{7}{1} :: \frac{5}{12} . 12\frac{3}{5}$

3
7
21
12
42
21
Dividend 252

4
1
4
5
Divisor 20

(1
252(12 $\frac{3}{5}$
20

Example in Decimals.

Decimals. Of the Weight of Bread.

Suppose at 5 *s.* 9 *d.* the Bushel of Wheat, the Penny white Loaf weigh 9 Ounces and 6 Penny Weights; What shall the same weigh, when Wheat is risen to 6 *s.* 6 *d.* the Bushel?

Answ. 8 Ounces, $4\frac{1}{2}$ Penny Weights, and somewhat more.

	s.	Ounces.		s.	Ounces.
As	0,2875 .	0,775	::	0,325 .	0,6855

(4) ,775
(3) 14375
(7) 20125
20125
,2228125

(7)
(3)
(4)

1(25
71871|0
,2228125(,6855
,325
,325
,325
,325

Example

Example in Logarithms.

In case I lend a Friend 320 *l.* for 20 Months: And when I want, he courteously lendeth me 400 *l.* When shall I pay him again, not to trespass on his Kindness? *Logarithms. Of paying Money lent.*

Answ. At 16 Months end.

	l.	*Mon.*		*l.*	*Mon.*
As	320	. 20	::	400	. 16

Or, 2,50514,99783 . 1,30102,99957 : 2,60205,99913 . 1,20411,99827

Log. of 320 .	2,50514,99783		
Log. of 20 .	1,30102,99957		
Sum	3,80617,99740	Product.	Log. of 6400.
Log. of 400 .	2,60205,99913		
Difference	1,20411,99827	Quotient.	Log. of 16.

Example in Species.

If 15 Pioneers, with the help of 3 Boys, can dig a Trench, and build a Wall in 12 Days; and Expedition requires the same to be done in 9 Days: How many Men and Boys are there needful? *Species. Of Workmen to build a Wall.*

Answ. 20 Pioneers and 4 Boys.

As 12 A . 15 B + 3 C :: 9 A . 20 B + 4 C.

For 12 A × 15 B + 3 C = 180 BA + 36 CA.

And 9 A) 180 BA + 36 CA (20 B + 4 C.

The Proof of these, and such others, are as before, by reversing the Question, or multiplying the Extreams and Means; or, moreover, by taking other Numbers instead of these Contracts, and working therewith. All which by the last Example is here instanced. *Proof of these and others.*

Reverse of the Question. As 9A . 20 B + 4 C :: 12 A . 15 B + 3 C. *Example.*

Extreams $\left\{ \begin{array}{c} 20B + 4C \\ 9A \\ \hline 180BA + 36CA \end{array} \right.$ Means $\left\{ \begin{array}{c} 15B + 3C \\ 12A \\ \hline 180BA + 36CA \end{array} \right.$

		Days.		*Labourers.*		*Days.*		*Labourers.*
Omitting the Species.	As	12	.	18	::	9	.	24.

12 × 18 = 216 9) 216 (24

CHAP. IV.

PRACTICE.

THE common Comparative Elements, in the *Direct* and *Indirect Rule of Three* visited, the other Part of the Primitives which is Peculiar, comes to be patent, and this is *Practice*. *Practice.*

Practice is so called, from the frequent Use and general Practice thereof, and is a Compendium or Breviat of the brief Rules and most expeditious Method of resolving the Propositions resolvable by the *Rule of Three*, after the Common Way. So as if any Process therein be shorter than other, it falls under consideration here. *What, and why so called.*

The practical shortning the common Work of the *Rule of Three*, may be comprised under one of these three Heads. *Heads of Practice.*

1. When

1. When the first and third Numbers will abbreviate.

1. When the first and third Numbers may be abbreviated, then reduce them to their least Terms, and by those Terms proceed, as in the *Rule of Three*, to the Resolution of the Question.

Example in the Direct Rule of Three.

Examples. Of the Price of Rods of Wall.

If 30 Rods of Wall making cost 8 *s.* What shall 48 Rods cost? Because 30 and 48 will abbreviate, they may be reduced to 5 and 8 for the first and third Numbers, and the Resolution thereby gotten will be 12 *s.* $\frac{4}{5}$, as by the Common Way.

$6)\frac{30}{48}(\frac{5}{8}$

	Rods.		*s.*		*Rods.*		*s.*
As	5	.	8	::	8	.	$12\frac{4}{5}$

$$\begin{array}{r} 8 \\ 8 \\ \hline 5)\ 64\ (12\tfrac{4}{5} \end{array}$$

Common Way.

As 30 . 8 :: 48 . $12\frac{4}{5}$

$$\begin{array}{r} 48 \\ 8 \\ \hline 30)\ 384\ (12\ s.\ \tfrac{4}{5} \end{array}$$

Example in the Indirect Rule of Three.

Of Labourers to finish a Piece of Work.

If 18 Labourers in 12 Days can finish a Piece of Work; In how many Days shall 24 Labourers finish the same Work? Here 18 and 24, the first and third Numbers, will be reduced to their least Terms, 3 and 4; by which the Resolution will be 9 Days, as by the *Common Way.*

$6)\frac{18}{24}(\frac{3}{4}$

	Lab.		*Days.*		*Lab.*		*Days.*
As	3	.	12	::	4	.	9

$$\begin{array}{r} 3 \\ \hline 4)\ 36\ (9 \end{array}$$

Common Way.

As 18 . 12 :: 24 . 9

$$\begin{array}{r} 18 \\ \hline 96 \\ 12 \\ \hline 24)\ 216\ (9 \end{array}$$

2. When the common Work may be shortned.

2. Though the first and third Numbers will not abbreviate, yet may the common Work of the *Rule of Three* be shortned, if in the *Direct Rule* the third Number be divided by the First; and in the *Indirect Rule*, the first Number be divided by the Third, and the Quotient multiplied by the Second.

Example in the Direct Rule of Three.

Examples. Of Gain by Trade.

If 750 *l.* Stock in merchandising gain 360 *l.* What shall 1250 *l.* Stock gain? *Answ.* 600 *l.* For 1250 divided by 750, gives in the Quotient $1\frac{2}{3}$, which multiplying 360, makes the Product 600.

	l. Stock.		*l. Gain.*		*l. Stock.*		*l. Gain.*
As	750	.	360	::	1250	.	600

$$\begin{array}{r} 1\tfrac{2}{3} \\ \hline 360 \\ 120 \\ 120 \\ \hline 600\,l. \end{array} \qquad \begin{array}{r} 75|0)\ 125|0\ (1\tfrac{2}{3} \\ 75|0 \end{array}$$

Common Way.

As 750 . 360 :: 1250 . 600

$$\begin{array}{r} 360 \\ \hline 75000 \\ 3750 \\ \hline 750)\ 450000\ (600 \end{array}$$

Example in the Indirect Rule of Three.

Of Length, &c. to make an Acre of Land.

If 8 Perches broad require 20 Perches long to make an Acre of Land: What length shall a Piece of Land have that is 2 Perches broad to make an Acre? *Answ.* 80 Perches: For 8 divided by 2, gives 4 in the Quotient; which multiplying 20, makes the Product 80.

	Perches-broad.		*Perches-long.*		*Perches-broad.*		*Perches-long.*
As	8	.	20	::	2	.	80

$\frac{8}{2}(4$ $\qquad$ $\begin{array}{r} 4 \\ \hline 80 \end{array}$ Perches long.

Common Way.

As 8 . 20 :: 2 . 80

$$\begin{array}{r} 8 \\ \hline 2)\ 160\ (80 \end{array}$$

The

The first and third Numbers in these two last Examples, abbreviated as abovementioned, the Work will shew it self thus.

As 750 . 360 :: 1250 . 600 — As 8 . 20 :: 2 . 80

$250)\frac{750}{1250}(\frac{3}{5}$ $3)\frac{5}{1800}(600$ — $2)\frac{8}{2}(\frac{4}{1}$ $\frac{4}{80}$

3. When an Unit is one of the Data.

3. Because one doth neither multiply nor divide. If an Unit among *Geodeticals* be one of the three given Numbers, and the First and Third be of like Denominations, and so need no Reduction as some do, which will be seen among Specificks in the next Chapter; then the Unit is set by, and Operation being made with the other two Numbers of the *Data*, the Work may be shorter than that commonly by the *Rule of Three*, especially when the Unit happens to stand in the first Place, as in the ensuing Process of this Chapter will be proved: Which Operations are all that with some Authors pass by the Name of *Practice*.

This sort of Practice seen in ten Cases.

The ensuing Operations of *Practice* under this third Head, will be comprehended under these ten Cases following.

1. If the Price be any Number of Shillings.

Case 1. If the given Price of one Ell, Yard, Pound-weight, &c. be any certain Number of Shillings, then multiply the given Quantity of Ells, Yards, Pounds, &c. whose Price is desired, by half the Number of Shillings which one shall cost: And from the Product cut off by a Dash of the Pen the right-hand Figure for Primes, (every Unit whereof is in value 2 *s.*) the Residue is Pounds. If a Cipher be in the Dexter Place cut off, it signifies nothing.

Example in Numbers. Even and Odd.

Example 1. If one Ell cost 4 *s.* what shall 401 Ells cost at that Price?

Answ. 80 *l.* 4 *s.* For multiplying 401 by 2, which is the half of 4 the Price given; the Product is 802, of which the Right-hand Figure 2, cut off for Primes, is 4 *s.*

	Ells.		*Common Way.*
At 4 *s. per* Ell, what costs	401 ?		As *Ell.* 1 . *s.* 4 :: *Ells.* 401 . *l.* 80 : *s.* 4
	2		4
$\frac{1}{2}$ of 4 *s.* is 2.	*l.* 80|2 Primes.		160|4
			l. 80:4 *s.*

Odd and Even.

Example 2. If one Ell cost 19 *s.* what will 300 Ells cost?

Answ. 285 *l.* For after Multiplication by $9\frac{1}{2}$, half the Price of 1 Ell, the total Product is 2850; from which the Cipher as insignificant cut off, the Residue is 285.

	Ells.		*Common Way.*
At 19 *s. per* Ell, what costs	300 ?		As *Ell.* 1 . *s.* 19 :: *Ells.* 300 . *l.* 285
	$9\frac{1}{2}$		19
$\frac{1}{2}$ of 19 *s.* is $9\frac{1}{2}$	2700		2700
	150		300
	l. 285|0 Primes.		570|0
			l. 285:0 *s.*

Both odd.

Example 3. If 1 Yard cost 15 *s.* what shall 343 Yards cost?

Answ. 257 *l.* 5 *s.* For multiplying 343 by $7\frac{1}{2}$, the half Price of one Yard, there comes to be cut off $2\frac{1}{2}$ Primes, which is 5 *s.*

	Yards.		*Common Way.*
At 15 *s. per* Yard, what costs	343 ?		As *Yard.* 1 . *s.* 15 :: *Yards.* 343 . *l.* 257 : *s.* 5.
	$7\frac{1}{2}$		15
$\frac{1}{2}$ of 15 *s.* is $7\frac{1}{2}$	2401		1715
	$171\frac{1}{2}$		343
	l. 257|$2\frac{1}{2}$ Prim.		514|5
			l. 257:5 *s.*

2. If the Price be an Aliquot Part of a Pound or Shilling. *Case* 2. If the given Price of one Yard, Ell, Pound-weight, *&c.* be any *Aliquot*, or even Part of a Pound or Shilling: Then divide the Quantity of Yards, Ells, *&c.* whose Price is desired by that *Aliquot* Part; and the Quotient shall be the Resolution in Pounds or Shillings, according to the Part divided by. And if any thing remain upon the Division, every Unit thereof is in Value so much as that Part of a Pound or Shilling which was the Divisor.

The Collection of the Aliquot *Parts of a Pound and Shilling, commonly called* Practice Tables, *here follow.*

Practice *Tables.*

Aliquot Parts of a Pound.

Parts	*Value*	*Parts*	*Value*
	s. d.		*s. d.*
1	20.0	20	1.0
2	10.0	24	0.10
3	6.8	30	0.8
4	5.0	40	0.6
5	4.0	48	0.5
6	3.4	60	0.4
8	2.6	80	0.3
10	2.0	120	0.2
12	1.8	240	0.1
15	1.4	480	0.0½
16	1.3	960	0.0¼

Aliq. Parts of a Shilling.

Parts	*Value*
	d.
1	12
2	6
3	4
4	3
6	2
8	1½
12	1
16	0¾
24	0½
48	0¼

The Tables explained. The Tables are easy to be understood, the Value of every Aliquot Part standing directly against the same on the same Line: As the third Part of a Pound is 6 *s.* 8 *d.* the third Part of a Shilling 4 *d.* the like of all the rest.

How made. The Value of every Part is found, by dividing the Shillings in a Pound by the Parts under 20: The Pence in a Pound by the Parts under 240, *&c.* The like for the Parts of a Shilling, or the lowest Denomination, 960 Farthings in a Pound, and 48 Farthings in a Shilling by the Part desired; and so is the Quotient Farthings, which may be reduced into Pence and Shillings by *Geodetical* Reduction.

Their Use. *Operation by the Parts of a Pound.*

Examples of the first Table. *Example* 1. If one Yard cost 6 *s.* 8 *d.* what shall 348 Yards cost?

Answ. 116 *l.* For 348 divided by 3, or the third part thereof taken, because 6 *s.* 8 *d.* is ⅓ of a Pound, the Quotient will be 116.

Yards.
At 6 *s.* 8 *d.* *per* Yard, what costs 348?

⅓ *l.* 3) 348 (116 *l.*

Common Way.

	Yard.	*d.*		*Yards.*	*l.*
As	1	. 80	::	348	. 116

80
27840 (232|0
12 (*l.* 116

Example 2. If one pound Weight cost 1 *s.* 4 *d.* what shall 8976 Pounds cost?

Answ. 598 *l.* 8 *s.* For dividing 8976 by 15, because 1 *s.* 4 *d.* is the fifteenth part of a Pound: the Quotient is 598, and 6 remaining is $\frac{6}{15}$; which, if $\frac{1}{15}$ be 1 *s.* 4 *d.* is 8 *s.* that is, 6 Shillings and 6 Groats.

At

At 1 s. 4 d. *per* lb, what costs 8976 lb?

Common Way.

As 1 lb . 16 d. :: 8976 lb : 598 l. : 8 s.

$\frac{1}{15}$ l.

```
  1
 142     l.  s.
8976 ( 598 . 8
 15
  15
   15
```

```
    16
 53856
 8976
143616 ( 1196|8
  12     l.598:8 s.
```

Operation by the Parts of a Shilling.

Example 1. If 1 lb cost 3 q. what shall 4048 lb. cost? *Examples of the Second.*

Answ. 12 l. 13 s. For dividing 4048 by 16, which part of a Shilling 3 q. is, the Quotient is 253 s. and by Reduction 12 l. 13 s.

At 3 q. *per* lb, what costs 4048 lb?

Common Way.

As 1 lb . 3 q. :: 4048 lb . 12 l. : 13 s.

$\frac{1}{16}$ s.

```
  8
4048 ( 25|3
 16    l.12:13 s.
  16
   16
```

```
    3    6
12144 ( 3036 ( 25|3
  4      12    l.12:13 s.
```

Example 2. If one Pound cost 2 q. what shall 1227 lb. cost?

Answ. 2 l. 11 s. 1 $\frac{1}{2}$ d. For 2 q. being $\frac{1}{24}$ of a Shilling, and dividing 1227, the Quotient shall be 51 s. and the 3 which remain 3 odd Half-pence.

At 2 q. *per* lb. what costs 1227 lb.?

Common Way.

As 1 lb. . 2 q. :: 1227 lb. : 2 l. : 11 s. : 1 $\frac{1}{2}$ d.

$\frac{1}{24}$ s.

```
 (3
1227 ( 51 s.
 24    l. s. d.
 24    2:11:1½
```

```
   2
 2454
 (2q   (1d.   (1s.
 2454 ( 613 ( 5(1 ( 2 l.
  4      12    20
```

The price of 1 lb. in this last Example, 2 q. being also an Aliquot part of a Pound, (as oftentimes happeneth in the given Price) Operation may be made by the Parts of a Pound. *Price oft-times to be found in both Tables.*

As at 2 q. *per* lb. what costs 1227 lb.?

$\frac{1}{480}$ l.

```
  26             2(3
 122|7 ( 2 l.    267 ( 11 $\frac{1}{8}$ s.
 480              24
```

Case 3. If the given Price of one Yard, Ell, &c. be not any even Part of a Pound or Shilling: Then break the given Price into any Aliquot Parts, and work as before with every of those Parts, and add their Quotients together, with the Remains if any be. Or for the Shillings, work as in the first Case; and for the Pence as in the second Case. *3. If the Price be no Aliquot Part of a Pound or Shilling.*

Example 1. If one hundred Weight cost 13 s. 7 d. what shall 245 hundred Weight cost? *Example 1.*

Answ. 166 l. 7 s. 11 d. as appeareth by both Works so plain, that nothing is needful for Explication.

By

By the first and second Cases.

At 13s.7d. per C. what costs 245 C.?

$\frac{1}{2}$ of 13s. is $6\frac{1}{2}$
$\frac{1}{40}$ l. 0 : 6d.
$\frac{1}{240}$ l. 0 : 1
13 : 7

$6\frac{1}{2}$
1470
$122\frac{1}{2}$
l. 159|$2\frac{1}{2}$ Primes.
159 : 5s. d.
$\frac{245}{40}$ (6 : 2 : 6
$\frac{245}{240}$ (1 : 0 : 5
166 : 7 : 11

By the second Case.

At 13s. 7d. per C. what costs 245 C.?

s. d.
$\frac{1}{2}$ l. 10 : 0
$\frac{1}{6}$ l. 3 : 4
$\frac{1}{80}$ l. 0 : 3
13 : 7

l. s. d.
$\frac{245}{2}$ (122 : 10 : 0
$\frac{245}{6}$ (40 : 16 : 8
$\frac{245}{80}$ (3 : 01 : 3
166 : 7 : 11

Example 2. *Example* 2. If 1 C. cost 11s. 8d. what shall 146 C. cost? *Answ.* 85 l. 3s. 4d.

By the first and second Cases.

At 11s.8d. per C. what costs 146 C.?

$\frac{1}{2}$ of 11s. is $5\frac{1}{2}$
$\frac{1}{30}$ l. 0 : 8d.
11 : 8

$5\frac{1}{2}$
730
73
80|3 Primes.
l. 80 : 6s. d.
$\frac{146}{30}$ (4 : 17 : 4
85 : 3 : 4

By the second Case.

At 11s. 8d. per C. what costs 146 C.?

s. d.
$\frac{1}{3}$ l. 6 : 8
$\frac{1}{4}$ l. 5 : 0
11 : 8

l. s. d.
$\frac{146}{3}$ (48 : 13 : 4
$\frac{146}{4}$ (36 : 10 : 0
85 : 3 : 4

4. *Operation by the Parts of 24.* *Case* 4. If the Aliquot Parts of 24 be taken for the Price given, and Operation made thereby as before by the Aliquot Parts of 20, under the second Case: A brief Resolution of the Question will be had in the Quotient of Division by these Parts; which Quotient shall be Pounds also, because 24 hath but the Cipher cut off from 240, the Pence in a Pound; and therefore the Dexter Figure or Cipher of the Quotient shall be cut off for Primes: And if any thing remain upon the Division, it is always less than one Prime, or 2 s. every Unit being in Value so much as the Divisor contained of 24.

The Aliquot Parts of 24, are known by the single Numbers in this Table standing against them, the uneven Parts having some 2, and some 3 Numbers against them.

A Table of the Parts of 24.

Pence.	*Parts.*	*Pence.*	*Parts.*
1	24	13	2.24
2	12	14	3.4
3	8	15	2.8
4	6	16	3.3
5	12.8	17	3.4.8
6	4	18	2.4
7	8.6	19	2.8.6
8	3	20	2.3
9	4.8	21	2.4.8
10	4.6	22	2.4.6
11	3.8	23	2.3.8
12	2	24	1

The

The Numbers under the Title *Parts* are for Divisors, when such a Number of Pence as stands against them happens to be the given Price of one Ell, Yard, Pound, &c. As if 4 *d.* be the Price given, then shall 6 be Divisor: But if 5 *d.* be the given Price, which is no even Part of 24, then 12 and 8 shall be Divisors, 12 because it is the Part answering to 2 *d.* and 8, because the Part that answers to 3 *d.* which 2 and 3 make 5, and both the Quotients must be added together, as those under the third Case. *The Table explained.*

Operation by the Parts of 24.

Example 1. If 1 ℔. cost 8 *d.* what will 447 ℔. cost? *Example 1.*

Answ. 14 *l.* 18 *s.* For 447 divided by 3, because 8 is the third Part of 24, the Quotient is 149; from which 9 cut off for the Primes, which is 18 *s.* the rest is Pounds.

At 8 *d.* *per* ℔. what cost 447 ℔.?

⅓ of 24 — 447 / 3 (14|9 Primes. — *l.* 14:18 *s.*

By the second Case.

At 8 *d.* *per* ℔. what costs 447 ℔.?

$\frac{1}{30}$ *l.* — (2 — 44(7 / 30 (14 *l.* : 18 *s.*

Example 2. If 1 ℔. cost 9 *d.* what will 4638 ℔. cost? *Example 2.*

Answ. 173 *l.* 18 *s.* 6 *d.* For dividing 4638 by 4 and 8, the Divisors for 9, and adding the Quotients together, the Total is 173 *l.* 9¼ Primes, or 18 *s.* 6 *d.*

At 9 *d.* *per* ℔. what costs 4638 ℔.?

¼ of 24 is 6 *d.*
⅛ of 24 is 3 *d.*
9 *d.*

2(2 — 4638 / 4 (1159½
7(6 — 4638 / 8 (579¾
l. 173|9¼ Pri.
l. 173:18:6 *d.*

By the second Case.

At 9 *d.* *per* ℔. what costs 4638 ℔.?

$\frac{1}{40}$ *l.* 6 *d.*
$\frac{1}{80}$ *l.* 3 *d.*
9 *d.*

2 — 46(38 / 40 (115 *l.* : 19 *s.* : 0 *d.*
(7 — 463(8 / 80 (57 : 19 : 6
173 : 18 : 6

Case 5. If the Price given be above 24 *d.* or 2 *s.* and yet the same may be evenly divided by some Aliquot Part of 24: then see how many such Parts there is in the given Price, and after Division by that Aliquot Part, multiply the Quotient by the other, and cut off the dexter Figure or Cipher as before. *5. If the Price be above 2 s.*

Example 1. If 1 Ell cost 3 *s.* 6 *d.* what shall 400 Ells cost? *Example 1.*

Answ. 70 *l.* For 6 *d.* is an Aliquot Part of 24, and ¼ thereof, and in 3 *s.* 6 *d.* there are 7 Six-pences: Therefore 400 divided by 4, gives 100, which must be multiplied by 7; and from that Product 700, a Cipher cut off leaves 70.

At 3 *s.* 6 *d.* *per* Ell, what costs 400 Ells?

¼ of 24 is 6 *d.*
7
3:6

400 / 4 (100
7
l. 70|0 Primes.

By the second Case.

At 3 *s.* 6 *d.* *per* Ell, what costs 400 Ells?

⅛ *l.* 2 *s.* : 6 *d.*
$\frac{1}{20}$ 1 : 0
3 : 6

400 / 8 (50 *l.*
400 / 20 (20
70 *l.*

Example 2. If 1 Ell cost 2 *s.* 8 *d.* what shall 500 Ells cost? *Example 2.*

Answ. 66 *l.* 13 *s.* 4 *d.* For 4 *d.* is an even Part of 24, and in 2 *s.* 8 *d.* are 8 Groats: Therefore dividing by 6, because 4 is the sixth Part of 24, and multiplying the Quotient by 8, there is 66 *l.* 6⅔ Primes, or 13 *s.* 4 *d.*

Ells.
At 2 *s.* 8 *d.* *per* Ell, what costs 500?

```
              d.        2(2
1/6 of 24 is  4        500 ( 83⅓
              8         6  (  8
             2:8            664
                              2⅔
                         l. 66|6⅔ Primes.
                         l. 66:13:4d.
```

By the second Case.

Ells.
At 2 *s.* 8 *d.* *per* Ell, what costs 500?

```
          s.  d.        2(4    l.  s.  d.
1/8 l.    2 : 6        500 (  62 : 10 : 0
1/12.         2         8
          2 : 8        2
                       50|0 (  4 : 03 : 4
                       12|0
                              66 : 13 : 4
```

6. *If a Fraction be in the Data.* *Case* 6. If besides the whole Quantity given, there be some small Part annexed, as ½ or ⅓, or such-like: After Operation for the whole Quantity, the Price of that Part is to be added.

Example. *Example.* If one Yard cost 3 *s.* 4 *d.* what shall 300½ Yards cost?

Answ. 50 *l.* 1 *s.* 8 *d.* Where after 300 is divided by 6, because 3 *s.* 4 *d.* is ⅙ of a Pound, 1 *s.* 8 *d.* for the half Yard is added to the Quotient.

Yards.
At 3 *s.* 4 *d.* *per* Yard, what costs 300½?

```
1/6 l.     300 ( l.
            6  ( 50   s.  d.
                      1 : 8  for the ½ Yard.
                 50   1 : 8
```

7. *If the Price exceed Pounds.* *Case* 7. If the Price given exceed Pounds, the Pounds are to be multiplied by the Quantity, and the rest of the Price wrought out by some or other of the foregoing Cases: And sometime Geodetical Multiplication is used as the shorter Way.

Example. *Example.* If one Hundred Weight cost 3 *l.* 8 *s.* 4 *d.* what shall 7 C?

Answ. 23 *l.* 18 *s.* 4 *d.* For so by Multiplication of 3 *l.* 8 *s.* 4 *d.* into 7, it will appear.

By Geodetical Multiplication.

C.
At 3 *l.* 8 *s.* 4 *d.* *per Cent.* what costs 7?

```
       7
21:56:28
23:18: 4
```

Otherwise.

C.
At 3 *l.* 8 *s.* 4 *d.* *per* C. what costs 7?

```
3 × 7                  7      7
½ of 8 is 4.           3      4
1/6. l. 4 d.     l.   21     2|8 Primes.
           7/6.        2 : 16
                            2:4
                      23 : 18:4
```

8. *If the Price of an hundred Weight, &c.* *Case* 8. If the Price of an hundred Weight be given, and it be required to know what some certain Number of Pounds will cost, which are not the equal Half or Quarters of the hundred Weight, and so easily reckoned without the Pen: then those Pounds may be broken into Aliquot Parts of 112, (the Weight of one Hundred) and Operation made thereby, like as in the other Cases before.

A Table of the Aliquot Parts of 112.

The Aliquot Parts of 112.

Pounds.	*Parts.*		*Pounds.*	*Parts.*
1	112		14	8
2	56		16	7
4	28		28	4
7	16		56	2
8	14		112	1

Example.

Example. If 1 C. coſt 16 *l.* what will 35 ℔. coſt? *Example of the Uſe of the Table.*

Anſw. 5 *l.* For breaking 35 into Aliquot Parts of 112, that is, 28 and 7; and dividing 16 by 4, which ſtands againſt 28, and by 16 which ſtands againſt 7, among the Aliquot Parts, and adding the Quotients together, the Total will be 5 *l.*

```
                              ℔.                 Common Way.
At 16 l. per C. what coſts 35 ?              ℔.    l.    l.   l.
                                          As 112 . 16 :: 35 . 5
 1/4 } of 112 { 28      16 ( 4 l.                  35
 1/16}         {  7     ---                        ---
              ----       4                          80
                35      16 ( 1                     48       560 ( 5 l.
                        ---                        ---      ---
                        16                         560      112
                        ------
                          5 l.
```

Caſe 9. If the Price of one Pound be given, and it be required to know what the hundred Weight will coſt; or the contrary by the Price of the Hundred to know the Price of 1 ℔. the Rule called the *Billingſgate Rule*, is made uſe of, (being ſo eaſily remembred, that every common Coſtermonger at *Billingſgate* can preſently tell what it comes to) which is thus: For every Farthing in one Pound, take ſo many Groats, and double the Number of Shillings, and this ſhall be the Price of the hundred Weight. So that every Farthing in the Pound increaſes the hundred Weight ſo many times ſeven Groats. *9. The Price of 1 Pound given, to know the Price of the Hundred. Billingſgate-Rule.*

Example. If 1 ℔. coſt 2 *d.* ½, what ſhall 1 C. coſt? *Example.*

Anſw. 1 *l.* 3 *s.* 4 *d.* For in 2 *d.* ½ there being 10 Farthings, the Hundred ſhall coſt 10 Groats, and twice 10 Shillings, which is 23 *s.* 4 *d.*

Caſe 10. If an Unit ſtand in the third Place of the *Data*, and the firſt Number be an Article, then, according to the Ciphers in the Article cut off (by a Perpendicular Line from the higheſt Geodetical Denomination in the ſecond Number) ſo many Figures, and reduce the reſt by common Reduction, and the Numbers that exceed that Line to the Left-hand, ſhall be the Reſolution. *10. If 1 be in the third place.*

Example 1. If 10 Pieces of Cloth coſt 42 *l.* 5 *s.* 5 *d.* what doth one Piece thereof coſt? *Example 1.*

Anſw. 4 *l.* 4 *s.* 6 *d.* ½. Here I cut off 2, of the 42 *l.* becauſe of the Cipher in 10; and that 2 *l.* with the 5 *s.* reduced, makes 45 *s.* of which 4 exceeds the Line; the other 5 *s.* with 5 *d.* is turned into 65 *d.* of which 6 exceeds the Line; and 5 left, the half of 10, is for the Half-penny; or multiplied by 4, is 20, of which the 2 exceeds the Line, and are Farthings.

```
                                                       Common Way.
Pieces.   l. s. d. Piece. l. s. d.           P.   l.   s.  d.   P.  l. s. d.
As 10   . 4|2:5:5 :: 1 . 4:4:6½           As 10 . 42 : 5 : 5 :: 1 . 4:4:6½
           |20                                     20
        ----|--                                    ---
      s. 4|5                                       845
           |12                                      12
        ----|--                                    ----
      d. 6|5                                      1690
           |4                                     845.5          5(6 d.
        ----|--                                 10)1014|5( 1014 ( 8,4 s.
      q. 2|0                                              ----
     ----------                                            12    l. 4:4:6½
```

Example 2. If 100 ℔. of Spice coſt 28 *l.* 10 *s.* 6 *d.* what doth one Pound thereof coſt? *Example 2.*

Anſw. 5 *s.* 8 *d.* 1 *q.* [illegible]. Here, becauſe 100 hath 2 Ciphers, 28 is wholly cut off. So the Price of one ℔. ſhall not reach to one *l.* The Work in the reſt is as the laſt above.

As

					Common Way.
	lb. l. s. d. lb. s. d. q.				lb. l. s. d. lb. s. d. q.
	As 100 . 28 : 10 : 6 :: 1 . 5 : 8 : $1\frac{21}{25}$.				As 100 . 28 : 10 : 6 :: 1 . 5 : 8 : $1\frac{21}{25}$
	20				20
s. 5	70				570
	12				12
d. 8	46				1140
	4				5706
q. 1	$\frac{84}{100}$, or $\frac{21}{25}$.				6846

$$100 \Big) \overline{\underline{273|84}} \Big(\frac{3(1q}{\overset{4}{}\;273}\ \ 4 \Big(\frac{6(8d}{12} \Big(5\ s.$$

Proof of Practice. The Proof of all Proceedings in *Practice*, is by the common Work of the *Rule of Three*, and one Variety by another, as by the Examples before is sufficiently clear.

Variety of Proof. Some prove the Operations of *Practice* by a contrary Question, thus; Mark the Number given for the Price of one Ell, Yard, Pound, *&c.* in the first Question, and how much it wanteth of 20 *s.* and by that which it wanteth, work with the first given Quantity stated into a second Question. And if both Resolutions added together be equal to the Quantity, the Work of both is right.

As if 8 *s.* buy 1 Ell, then will 100 Ells cost 40 *l.* on the contrary, because 8 *s.* wanteth 12 *s.* of 20. if the Price of 100 Ells be enquired at 12 *s.* *per* Ell, the Resolution will be 60 *l.* Now because 40 and 60 make up 100, both Works are found right.

At 8 *s.* *per* Ell, what costs 100 *Ells*?		At 12 *s.* *per* Ell, what costs 100 *Ells*?
4		6
l. 40|0	Proof.	*l.* 60|0
	40 + 60 = 100.	

CHAP. V.

SPECIFICKS.

Specificks. THE last Chapter ended with those Comparative Elements called *Primitive*; therefore this must begin with those called *Derivative*, because they borrow their Being from the former.

Why so called. Foremost, in the order of *Derivatives*, stand *Specificks*; so called, because something is specifical, in or about the Resolution of the Questions propounded, either by reason of the contract Numbers mixed in the *Data*, or the different Denominations of some sorts of Contract Numbers in one or more of the 3 given Numbers; *The Sorts thereof.* or the right Preparation of the *Data*, or the intermixture of Questions or Proportions in the Proposition, and that both in the *Direct* and *Indirect Rule of Three*.

Of these Diversities, the two first Sorts are specifical in respect of the Nature of the Numbers given, *viz.* the first in Fractions and Geodeticals; the second in Geodeticals and Astronomicals: But the two last Sorts are Specifical in respect of the Nature of the Question propounded.

1. If Fractions and Geodeticals be in the Data. 1. When Fractions and Geodeticals are mixed in the *Data*, place an Unit under the Geodeticals; and if any Fraction be mixed with an Integer, reduce the same into an Improper Fraction: then for Resolution of the Question, proceed as in the *Rule of Three*, multiplying and dividing after the manner of Fractions.

Operation by the Direct Rule of Three.

Examples in the Direct Rule of Three. *Example* 1. If 2 Ells cost [illegible] *l.* what shall 1[illegible] Ell cost?

Answ.

Answ. $\frac{7}{12}$ *l.* or 11 *s.* 8 *d.* Here $1\frac{3}{4}$ is to be reduced to $\frac{7}{4}$, and multiplied into $\frac{2}{3}$, as Fractions whose Product is to be divided by $\frac{2}{1}$, or 2 the first Number.

Of the Price of $1\frac{3}{4}$ Ell.

$$\begin{array}{l} \quad\ \ \textit{Ells.}\ \ \textit{l.}\ \ \ \textit{Ells.}\ \ \textit{l.} \\ \text{As } 2 \,.\, \frac{2}{3} :: 1\frac{3}{4} \,.\, \frac{7}{12} \end{array}$$

$$\text{For } \overset{1}{\frac{2}{3}} \times \underset{2}{\frac{7}{4}} = \frac{7}{6} \qquad \text{And } \frac{2}{1})\ \frac{7}{6}\ (\frac{7}{12}\ l.$$

Example 2. If $1\frac{1}{2}$ Ell cost $1\frac{1}{4}$ *l.* what shall $3\frac{3}{4}$ Ells cost?

Of the Price of $3\frac{3}{4}$ Ells.

Answ. $3\frac{1}{8}$ *l.* or 3 *l.* 2 *s.* 6 *d.* The *Data* reduced into improper Fractions, make $\frac{3}{2}$. $\frac{5}{4}$. $\frac{15}{4}$; the rest of the Work needs nothing to explain it.

$$\begin{array}{l} \quad\ \ \frac{3}{\textit{Ell.}}\ \ \frac{5}{\textit{l.}}\ \ \ \frac{15}{\textit{Ells.}}\ \ \textit{l.} \\ \text{As } 1\frac{1}{2} \,.\, 1\frac{1}{4} :: 3\frac{3}{4} \,.\, 3\frac{1}{8} \end{array}$$

$$\text{For } \frac{5}{4} \times \frac{15}{4} = \frac{75}{16} \qquad \text{And } \overset{1}{\underset{1}{\frac{3}{2}}})\ \overset{25}{\underset{8}{\frac{75}{16}}}\ (\frac{25}{8} \text{ or } 3\frac{1}{8}\ l.$$

Operation by the Indirect Rule of Three.

Examples in the Indirect Rule of Three.

Example 1. If at 4 *s.* the Bushel, the Penny White-loaf must weigh 9 Ounces *Troy*: what must it weigh at 5 *s.* $\frac{1}{2}$ *per* Bushel?

Answ. $6\frac{6}{11}$ Ounces. Here $5\frac{1}{2}$ reduced into an Improper Fraction, is $\frac{11}{2}$, under the other Numbers an Unit is placed, and the Operation is as the Indirect Rule of Three in Fractions.

Of the Weight of Bread.

$$\begin{array}{l} \quad\ \textit{s.}\ \ \textit{Ounc.}\ \ \frac{11}{\textit{s.}}\ \ \textit{Ounc.} \\ \text{As } \frac{4}{1} \,.\, \frac{9}{1} :: 5\frac{1}{2} \,.\, 6\frac{6}{11} \end{array}$$

$$\text{For } \frac{4}{1} \times \frac{9}{1} = \frac{36}{1} \qquad \text{And } \frac{11}{2})\ \frac{36}{1}\ (\frac{72}{11}, \text{ or } 6\frac{6}{11} \textit{ Ounces.}$$

Example 2. If $6\frac{6}{11}$ Ounces *Troy* be the Weight of a Loaf, when Wheat is sold for 5 *s.* 6 *d.* the Bushel: what shall the same Loaf weigh when Wheat is sold for 12 *s.* the Bushel?

Another of the Weight of Bread.

Answ. 3 Ounces. Here $6\frac{6}{11}$ is reduced to $\frac{72}{11}$, and $5\frac{1}{2}$ to $\frac{11}{2}$; and under 12 is placed an Unit; and the Multiplication and Division is as before.

$$\begin{array}{l} \quad\ \frac{11}{\textit{s.}}\ \ \frac{72}{\textit{Oun.}}\ \ \ \textit{s.}\ \ \textit{Ounces.} \\ \text{As } 5\frac{1}{2} \,.\, 6\frac{6}{11} :: \frac{12}{1} \,.\, 3 \end{array}$$

$$\text{For } \overset{1}{\underset{1}{\frac{11}{2}}} \times \overset{36}{\underset{1}{\frac{72}{11}}} = \frac{36}{1} \qquad \text{And } \overset{1}{\frac{12}{1}})\ \overset{3}{\frac{36}{1}}\ (\frac{3}{1}, \text{ or } 3 \textit{ Ounces.}$$

2. When the *Data* are Geodeticals or Astronomicals of different Denominations, either the Multiplication and Division used for Resolution of the Questions propounded in such Numbers, must be proper according to the Nature of the Numbers; or else they must be either reduced Geodetically, by common Reduction, into the lowest Denomination, or turned into Decimals.

2. If the Data be of divers Denominations. These reduced or turned into Decimals.

Touching Reduction of the given Numbers, observe:

Their common Reduction is to be noted in four things.

§. 1. If the second Number be reduced, he must be reduced into the lowest Denomination of them given, lower than which he need not be brought. As if the second Number be Pounds, Shillings, and Pence, it must be brought all into Pence: And if Pounds, Shillings, Pence, and Farthings, then all must be reduced into Farthings, &c. 1.

§. 2. If the second Number be reduced, then the fourth Number, when found, shall be of like Denomination to the Second when reduced, whether Shillings, Pence, Farthings, &c. 2.

3. §. 3. If the second Number multiplied into the Third, in the *Direct Rule of Three*, or multiplied into the First in the *Indirect Rule of Three*, be too little to be divided by the Divisor; then to avoid the Work in Fractions, the second Number may be reduced from a gross Denomination into a smaller or subtiler, as from Pounds to Shillings, &c. And likewise the fourth Number, when found, often-times admits of Reduction from a smaller Denomination into a Grosser.

4. §. 4. If the first and third Numbers be of plural Denominations, or single Numbers but of different Denominations, as they must be reduced into the lowest of the given Denominations; so they both must be reduced to one and the same Denomination. As if the one be Pounds and Shillings, and the other Pounds, Shillings and Pence, or Shillings and Pence; both Numbers must be reduced into Pence, &c.

These things observed, the rest of the Work differs not from that in the *Rule of Three* Direct or Indirect for *Geodeticals*, or Direct only in *Astronomicals*, they being all Direct Proportions.

Examples in Geodeticals. What 3 l. will buy.

Operation in Geodeticals, *by the* Direct Rule of Three.

Example. If 4 *l.* 14 *s.* 4 *d.* buy 2 C. 12 ℔. 6 ℥. what shall 3 *l.* buy?

Answ. 1 C. 38 ℔. 5 ℥ $\frac{145}{283}$. Here the first and third Numbers reduced into Pence, are 1132, and 720. The second Number brought into Ounces, is 3782. And after multiplication of 720 into 3782, and division of the Product by 1132, the Quotient is 2405 $\frac{145}{283}$ Ounces, according to the Reduction of the second Number, and may be brought into the grosser Denominations of Pounds and Hundreds. And if the first and third Numbers after Reduction were abbreviated, the Multiplication and Division would be shorter.

l. *s.* *d.* C. ℔. ℥. *l.* C. ℔. ℥.
As 4 : 14 : 4 . 2 : 12 : 6 :: 3 . 1 : 38 : 5 $\frac{145}{283}$.

20	112	20
94 *s.*	236 ℔	60 *s.*
12	16	12
188	1416	120
944	2366	60
1132 *d.*	3782 ℥.	720 *d.*
	720	
	75640	
	26474	
	2723040	

6|5
4592|8
2723040|0 (℥ 2405 (℔. 38 (C. 1
1132 16 112
1132 16
1132 16
1132

What Stock to raise a Profit proposed.

Operation in Geodeticals *by the* Indirect Rule of Three.

Example. If Stock to the Value of 10 *l.* 13 *s.* 3 *d.* raise a considerable Profit in 12 Months: what Stock shall raise the same Profit in 16 Months?

Answ. 7 *l.* 19 *s.* 11 *d.* $\frac{1}{4}$. Here the second Number only needs Reduction into Pence, or may be turned into a Decimal. And because the first and third Numbers will abbreviate, may be wrought as in *Practice*.

Mon. *l.* *s.* *d.* *Mon.* *l.* *s.* *d.*
As 12 . 10 : 13 : 3 :: 16 . 7 : 19 : 11 $\frac{1}{4}$

3 20 4
213
12
426
2133
2559
3
7677

3 3(1 (7|11 d.
7677 (1919 (15|9
4 44
l. 7 : 19 : 11 $\frac{1}{4}$

Mon. *l.* *s.* *d.* *Mon.* *l.* *s.* *d.*
As 12 . 10 : 13 : 3 :: 16 . 7 : 19 : 11 $\frac{1}{4}$
3 . 10,6625 :: 4 . 7,9968 $\frac{1}{4}$
3
4)31,9875(7,9968 $\frac{1}{4}$

	20
s. 19	9375
	12
1	8750
9	375
d. 11	2500
	4
q. 1	0000

Operation in Aſtronomicals.

Example. Suppoſe the Æquation of ♂ be deſired, and his Anomaly given be o Sign, 18 Deg. 12′. 30″. and the Difference of Æquations found by the Aſtronomical Tables, between the 18*th* and 19*th* Degree, be 9′ : 36″ decreaſing, the Queſtion then will ſtand thus : *Example in Aſtronomicals. Of the Æquation of* Mars.

If 1 Degree, or 60′, give 9′ 36″, what ſhall 12′ 30″ ?

And the Proportional Part gained will be 2′, to be ſubſtracted from the given Anomaly, as the following Operations make plain.

By Geodetical Reduction.

As 60 . 9 : 36 :: 12 : 30 . 2

60	60	60
3600	576	750
		576
		4500
		5250
		3750

3600) 432000 (120″ (2′
60

By the Sexagenary Table.

Deg.	′	″	′	″	′
As 1	9	36	12	30	2

12 : 30
1 . 55 . 12‴
4 . 48 . 00⁗
2 . 00 . 00 . 00

By Decimals.

As 60 . 9,6 :: 12,5 . 2
9,6
750
1125
60) 120,00 (2′,00

	s.	d.	′	″	
Anomaly of ♂	0	18	12	30	
Proportional Part			2	00	
Æquation of ♂	0	18	10	30	deſired.

3. When the *Data* are not the ſame three Numbers Reſolution is to be had by, but theſe are included in the Queſtion, and according to the State thereof, by a due preparation of the *Data*, thoſe more covert Numbers are diſcovered, through help of ſome or other of the Simple Elements of Numbers which they call to their Aid. *3. If the Data muſt be prepared.*

In this third Sort of *Specificks*, diligent conſideration muſt be had of the State of the Queſtion, and Nature of the Number queſited thereby to find the three Numbers to work by in the *Rule of Three*, ſince no Rule can be given to reach all Caſes ; but ſometime one, ſometime another, and ſometime more than one of the Simple Elements of Numbers are needful to prepare the *Data*. So as much depends on the Ingenuity of the Operator, as *Ptolomy* once ſaid, *Abs te & à Scientia*. *State of the Queſtion to be diligently heeded, and the reaſon.*

After the Numbers fit for Reſolution are obtained, which muſt be firſt done, (ſeeing without the true *Data* no Queſtion can be truly reſolved) let the Numbers, however propoſed in the Queſtion, (ſometime only to try the Skill of the Reſolver) be duly diſpoſed, according to the Precepts before given in the *Rule of Three*, and then operate by the Direct or Indirect Rule as the Caſe requires ; for all Operation by a wrong Rule will render the Reſult falſe. *Numbers to be duly diſpoſed, however propounded.*

Addition needful. Addition uſeful.

Example 1. Two Poſts, (ſuppoſe *A* and *B*) depart one from another, one directly Eaſtward, and the other directly Weſtward : *A* travelleth 18 Miles a Day, and *B* 30 : How far are they diſtant the third Day after their Departure ? *Q. Of the Diſtance of two Poſts.*

Anſw. 144 Miles. Here 18 and 30, the Travel of both in one Day, are to be added, and the Work with the Total committed to the *Direct Rule of Three*, thus. *Anſwer.*

A. 18

A . 18
B . 30
48

	Day.		*Miles.*		*Days.*		*Miles.*
As	1	.	48	::	3	.	144.

$$\frac{48}{\frac{3}{144}}$$ Miles.

Q. Of the lining of two Cloaks.

Example 2. If one Cloak may be lined with $4\frac{1}{2}$ Yards of Stuff that is yard broad: what Plush will line two of those Cloaks when the Plush is but $\frac{3}{4}$ of a yard broad, and one Cloak will be lined with $\frac{1}{2}$ Yard less than the other?

Answer.

Answ. $11\frac{1}{3}$ Yards of Plush. Here the breadth of both Cloaks is to be added, and the Work with the Total committed to the *Indirect Rule of Three*, thus.

1. Cloak $4\frac{1}{2}$
2. Cloak 4
$8\frac{1}{2}$

	Yard-broad.		Yards long.		Yards broad.		Yards long.
As	$\frac{1}{1}$	.	$8\frac{1}{2}$ (17)	::	$\frac{3}{4}$	.	$11\frac{1}{3}$

$\frac{3}{4}$) $\frac{17}{2}$ ($\frac{34}{3}$ ($11\frac{1}{3}$ Yards long.
2 1

Substraction useful.

Substraction needful.

Q. Of the filling of a Cistern.

Example 1. A Conduit-Pipe rnnning into a Cistern, holding 250 Gallons, poureth in every Hour 24 Gallons; but the Cistern by another Pipe running out thereof, emptieth of it self in every Hour 16 Gallons: in what time shall the Cistern be filled, both the Pipes running?

Answ.

Answ. $31\frac{1}{4}$ Hours. Here the Gallons emptied by the one Pipe, must be substracted from the Gallons filled by the other, that is, 16 from 24; and the Work with the Remain committed to the *Direct Rule of Three*, thus.

Greater Pipe 24
Lesser Pipe 16
8

	Gallons.		*Hour.*		*Gallons.*		*Hours.*
As	8	.	1	::	250	.	$31\frac{1}{4}$

1(2
250 ($31\frac{1}{4}$ Hours.
8

Q. Of the length, &c. to ½ Acre.

Example 2. If 4 Perches in breadth, when 40 is the length of a Piece of Land, make 1 Acre: what length must there be to make one half-Acre, when the breadth is 6 Perches?

Answ.

Answ. $13\frac{1}{3}$ Perches in length. Here, because the Enquiry is but for half an Acre, and 4 was a Proportion of breadth given for a whole Acre; therefore half the 4 shall be taken away, and the Work with the Remain committed to the *Indirect Rule of Three*: Thus,

1. Acre 4
½. Acre 2
2

	Perches broad.		*Perches long.*		*Perches broad.*		*Perches long.*
As	2	.	40	::	6	.	$13\frac{1}{3}$

2
6) 80 ($13\frac{1}{3}$ Perches long.

Multiplication useful.

Multiplication needful.

Q. Of the Powder spent by seven Guns.

Example 1. Suppose 5 Guns being often discharged, spend 60 Barrels of Powder: what will 7 Guns which spend three times as much as the other at every shot?

Answ.

Answ. 252 Barrels. Here 60 is multiplied by 3, and the Work with the Product is committed to the *Direct Rule of Three*: Thus,

Barrels 60
Increased 3
180

	Guns.		*Barrels.*		*Guns.*		*Barrels.*
As	5	.	180	::	7	.	252

7
5) 1260 (252 Barrels.

Example

Example 2. If one have right to pasture on a Common 100 Sheep 40 Days, and he pastureth there four times as many 12 Days: hath he transgressed or not? *Q. Of Pasturage of Sheep.*

Answ. Yes, by the space of two Days. Here 4 is to multiply 100, and the Work with the Product to be committed to the *Indirect Rule of Three*, by which it is resolved he ought to have pastured 400 Sheep but 10 Days. *Answer.*

Sheep 100
Increased 4
400

	Sheep.		*Days.*		*Sheep.*		*Days.*
As	100	.	40	::	400	.	10

$$400\,)\,\overset{100}{4000}\,(\,10 \text{ Days.}$$

Division needful. Division *useful.*

Example 1. Four Guests at Table drank 16 *d.* in Wine: how many Guests that drink but half so much as the former will 18 Pennyworth of Wine serve? *Q. Of Wine drunk.*

Answ. 9 Guests. Here 16 is to be halfed, or divided by 2, and the Work with the Quotient committed to the *Direct Rule of Three:* Thus, *Answer.*

Wine 16 (8
Halfed 2

	d.		*Guests.*		*d.*		*Guests.*
As	8	.	4	::	18	.	9

$$8\,)\,\overset{4}{72}\,(\,9 \text{ Guests.}$$

Example 2. If a Parcel of Hay will maintain 90 Head of Cattel 10 Weeks, and $\frac{1}{3}$ of the Cattel be put out to keeping: how long will the Hay maintain the Rest? *Q. Of Hay to feed Cattel.*

Answ. 15 Weeks. Here 90 is divided by 3, and the Quotient, or the third Part of 90 taken from thence, the Residue is committed to the *Indirect Rule of Three:* Thus, *Answer.*

Cattel 90 (30
Divided 3

90—30=60

	Cattel.		*Weeks.*		*Cattel.*		*Weeks.*
As	90	.	10	::	60	.	15

$$60\,)\,\overset{90}{900}\,(\,15 \text{ Weeks.}$$

Division and Addition needful. Div. *and* Add. *used.*

Example. Certain Reapers in 5 Days can reap 24 Acres of Corn: in how many Days can they, with each Man his Servant, every of which doth half as much Work as his Master, reap 144 Acres? *Q. Of Reapers.*

Answ. In 20 Days. Here the half of 24 being added thereto, the Work is committed to the *Direct Rule of Three:* Thus, *Answer.*

Acres 24 (12 — 24
Halfed 2 — 12
36

	Acres.		*Days.*		*Acres.*		*Days.*
As	36	.	5	::	144	.	20

$$36\,)\,\overset{5}{720}\,(\,20 \text{ Days.}$$

Substraction and Multiplication needful. Sub. *and* Mult. *used.*

Example. *A* having stolen certain Goods, fleeth 40 Miles a Day: *B* setting out 4 Days after him, pursueth 50 Miles a Day: in how many Days may *B* overtake *A*? *Q. Of pursuing a Thief.*

Answ. In 16 Days. Here the Difference between the Journey of *A* and *B* in one Day first taken, and the Day's Journey of *A*, multiplied by the number of Days *B* set out after *A*, the Work is committed to the *Direct Rule of Three:* Thus, *Answer.*

	B . 50	*A* . 40		*Miles.*	*Day.*	*Miles.*	*Days.*
	A . 40	4	As	10 .	1 ::	160 .	16
Differ.	10 Miles.	160					

☞ In like manner Questions may be composed, wherein *Multiplication* and *Addition*, or *Division* and *Substraction*, &c. may be needful: but these are sufficient for Example here.

4. If Questions be intermixt. 4. When there is an Intermixture of Questions or Proportions in the Proposition, the one will be *express*, and the other *implied.*

Expresly. §. 1. When the several Questions are expressed in the Proposition, then proceed according to the State of the Question in the Resolution of either; first with one, and then with the other.

Q. Of the Miles travel to overtake a Thief. *Example* 1. Suppose in the last Proposition it had been demanded, not only in how many Days *B* should overtake *A*, but also after how many Miles travel? Then after the first Work, as above, had resolved *B* to overtake *A* in 16 Days, another Question should be committed to the *Direct Rule of Three*, for the Resolution of this latter Query: Thus,

If *B* travel in 1 Day 50 Miles; how far shall he travel in 16 Days?

Answer. *Answ.* 800 Miles.

	Day.		*Miles.*		*Days.*		*Miles.*
As	1	.	50	::	16	.	800
					50		
					800 Miles.		

Q. Of a Castle besieged. *Example* 2. A Castle besieged hath Victuals enough for 1200 Men 7 Months; but the Captain finding the Siege is like to be long, and that fewer Men will defend it, he would disband some of his Men, and lengthen out his Provision to 12 Months: how many Men shall he retain, and how many disband?

Answer. *Answ.* By the *Indirect Rule of Three*, 700 Men are to be retained; which found, that Number is to be taken from the 1200 propounded, and the Residue, that is 500, to be disbanded.

	Months.		*Men.*		*Months.*		*Men.*
As	7	.	1200	::	12	.	700

12) 8400 (700 Men retained.
(1200 × 7 = 8400) 500 Men disbanded.
1200

Implicitly, with a Mixture of Ratio's, &c. §. 2. When in the Proposition there is such an Intermixture of *Ratio's* or Proportions, that as necessary to the Resolution of the Demand, either the Elements proper to *Ratio's* must be used, or more than one Operation of the *Rule of Three*, though perhaps but three Numbers given; and in such Operations the Quotient of the one Work is to be added to, or substracted from some or other of the *Data*, or the contrary, before Resolution can be had by the other: *Wherein these differ from those that fall under the Rule of five Numbers.* which differs from those Questions falling under the *Rule of five Numbers*, treated of in the two next Chapters, because they may be resolved at one Operation; yet they always give five Numbers; and if they be resolved by two Works, the Quotient of the first is taken whole without Alteration for the next Work, which is not so in these *Specificks*.

Data, 3 Numb. and Add. used.

Data, 3 *Numbers and Addition needful.*

Q. Of two doing a piece of Work. *Example* 1. *A* and *B* are hired to do a Piece of Work, which *A* can do alone in 30 Days, and *B* in 20 Days: in how many Days can they do it together?

Answer. *Answ.* In 12 Days. If I work by *Ratio's*, always where the *Ratio* is manifold, or the contrary, that is, to an Unit, it matters not which of the Terms be made Antecedent: For if the Unit be made Consequent, the *Ratio's* are to be reduced to like Antecedents; and if Antecedent, to like Consequents. Now here being two *Ratio's*, *viz.* that of *A* to the Work, as 30 to 1; and that of *B*, as 20 to 1;

First

First I reduce them to like Antecedents: So are they $\frac{600}{20}$ and $\frac{600}{30}$, and in their least Terms $\frac{60}{2}$ and $\frac{60}{3}$: after Reduction I divide this common Antecedent by 30 added to 20, the Sum of the reduced Consequents, or 60 by 5, that is, $3+2$.

Or otherwise, if I work by the *Rule of Three*, my first Question is, If 30 Days of *A* can do 1 Work: what shall 20 Days of *B*? To which the Answer $\frac{2}{3}$ must be added to the Piece of Work to be done; and that $1\frac{2}{3}$ Work shall be the first Number of the second Question, thus: If $1\frac{2}{3}$ Work come of 20 Days when *B* works with *A*, of what shall 1 Work come? *Variety of Operation.*

By Ratio's.

$$\overbrace{A\,30 \quad B\,20}^{600}$$
$$\quad 1 \qquad\; 1$$

$$\begin{array}{l} 20 \times 30 = 600 \\ 20 + 30 = \;\;50 \end{array} \Big(12 \text{ Days.}$$

By the Rule of Three.

	Days.		*Work.*		*Days.*		*Work.*
As	30	.	1	::	20	.	$\frac{2}{3}$

$$\frac{2|0}{3|0} \text{ Work.}$$

	Work.		*Days.*		*Work.*		*Days.*
As	$1\frac{2}{3}$	.	20	::	1	.	12

$$\frac{5}{3}\Big)\frac{20}{1}\Big(\frac{12}{1} \text{ Days.}$$

Example 2. A Conduit hath three Cocks; if the Cistern be full, and Water run out by the greatest Cock, the Cistern will be empty in three Hours; if it run out by the second Cock, the Water will be all run out in four Hours; but if the Water run out by the least Cock, it will be five Hours e're the Cistern be emptied: In what Time will all the Water be run out if it run by all the Cocks together? *Q. Of emptying a Cistern.*

Answ. In 1 Hour and $\frac{13}{47}$ of an Hour. Here the three *Ratio's* given being reduced to like Antecedents, the common Antecedent will be 60, to be divided by 47 the Sum of the reduced Consequents. *Answer.*

$$\overbrace{\text{Great Cock } 3, \text{ Second Cock } 4, \text{ Least Cock } 5.}^{60}$$

Great Cock 3	Second Cock 4	Least Cock 5
1	1	1
20	15	12

$$\begin{array}{l} \text{Antecedents } 3 \times 4 \times 5 = 60 \\ \text{Consequents } 20+15+12 = 47 \end{array} \Big(1\tfrac{13}{47} \text{ Hour.}$$

Otherwise, if the Work be wrought by the *Rule of Three*, the first Question is, If three Hours running of the great Cock empty the Cistern: what will four Hours running of that Cock? The Answer to which, $1\frac{1}{3}$ Cistern must be added to the Cistern: And this $2\frac{1}{3}$ shall be the first Number of the second Question, thus; If $2\frac{1}{3}$ Cisterns be run out in four Hours, when will 1 Cistern be run out? Then $1\frac{5}{7}$ Hour, the Answer to this Question, shall be the first Number of the third Question; thus; If $1\frac{5}{7}$ Hour empty 1 Cistern, what will 5 Hours do? And $2\frac{11}{12}$ the Answer to this, shall be added again to the 1 Cistern, and the last Question stands thus; If $3\frac{11}{12}$ Cisterns will be emptied in 5 Hours, when shall 1 Cistern be emptied? The Answer to which is $1\frac{13}{47}$ Hour as before. *Variety of Operation.*

		Hours.		*Cistern.*		*Hours.*		*Cistern.*
1.	As	3	.	1	::	4	.	$1\frac{1}{3}$

$$\frac{4}{3}\Big(1\tfrac{1}{3} \text{ Cistern.}$$

		Cisterns.		*Hours.*		*Cistern.*		*Hour.*
2.	As	$2\frac{1}{3}$	.	4	::	1	.	$1\frac{5}{7}$

$$\frac{7}{3}\Big)\frac{4}{1}\Big(\frac{12}{7} \text{ Hour.}$$

		Hour.		*Cistern.*		*Hours.*		*Cisterns.*
3.	As	$1\frac{5}{7}$	.	1	::	5	.	$2\frac{11}{12}$

$$\frac{12}{7}\Big)\frac{5}{1}\Big(\frac{35}{12} \text{ Cisterns.}$$

		Cisterns.		*Hours.*		*Cistern.*		*Hour.*
4.	As	$3\frac{11}{12}$	.	5	::	1	.	$1\frac{13}{47}$

$$\frac{47}{12}\Big)\frac{5}{1}\Big(\frac{60}{47} \text{ Hour.}$$

Quta;

Data, 3 Numb. and Sub. used.

Data 3, *Numbers and Substraction needful.*

Q. Of 1 doing a piece of Work.

Example 1. *A* and *B* do a Piece of Work together in 12 Days, which *A* alone can do in 30 Days: in how many Days can *B* do the same if he work alone?

Answer.

Answ. In 20 Days. Here the two *Ratio's* given are as 12 to 1, and 30 to 1; which being reduced to like Antecedents, this common Antecedent is to be divided by the Difference of their reduced Consequents.

Variety of Operation.

Otherwise to work by the *Rule of Three*, the first Question is; If 30 Days of *A* can do 1 Piece of Work, what shall 12 Days of *A* joined with *B* do? To which the Answer $\frac{2}{5}$ of a Piece of Work shall be taken from the whole Piece, and the Residue, which is $\frac{3}{5}$, shall be the first Number of the second Question, thus; If $\frac{3}{5}$ of the Work shall come of 12 Days, of what comes the whole Work?

By *Ratio's*.

$$\overbrace{\textit{A}\text{ and }\textit{B}\ \ \underset{1}{12}\ \ \textit{A}\ \ \underset{1}{30}}^{360}$$

$$\begin{array}{l} 30 \times 12 = 360 \\ 30 - 12 = 18 \end{array} \Big(20 \text{ Days.}$$

By the Rule of Three.

$$\begin{array}{lcccc} & \textit{Days.} & \textit{Work.} & \textit{Days.} & \textit{Work.} \\ \text{As} & 30 & .\ 1 & ::\ 12 & .\ \frac{2}{5} \end{array}$$

$$6\Big)\frac{12}{30}\Big(\frac{2}{5}\text{ Work.}$$

$$\begin{array}{lcccc} & \textit{Work.} & \textit{Days.} & \textit{Work.} & \textit{Days.} \\ \text{As} & \frac{3}{5} & .\ 12 & ::\ 1 & .\ 20 \end{array}$$

$$\frac{\overset{1}{3}}{5}\Big)\frac{\overset{4}{12}}{1}\Big(\frac{20}{1}\text{ Days.}$$

Q. Of filling a Cistern.

Example 2. Water runneth into a Cistern by a Pipe that can fill it in 8 Hours, and runneth out by another Pipe which can empty the Cistern in 22 Hours: in what Time both running will the Cistern be full?

Answer.

Answ. In 12 Hours and $\frac{4}{7}$ of an Hour. Here the *Ratio's* of 8 to 1, and 22 to 1, reduced to like Antecedents, make $\frac{176}{22}$ and $\frac{176}{8}$, and in their least Terms $\frac{88}{11}$ and $\frac{88}{4}$. This common Antecedent 88, divided by 7, the Difference of the Consequents gives in the Quotient $12\frac{4}{7}$ as before.

$$\overbrace{\text{Filling Pipe } \underset{\underset{22}{1}}{8}\text{ , Emptying Pipe } \underset{\underset{8}{1}}{22}}^{176}$$

$$\begin{array}{l} \text{Antecedents } 8 \times 22 = 176 \\ \text{Consequents } 22 - 8 = 14 \end{array} \Big(\overset{\text{Hours.}}{12\tfrac{4}{7}}, \text{ or } \frac{88}{7}\Big(12\tfrac{4}{7}$$

Variety of Operation.

Otherwise, to work by the *Rule of Three*, the first Question is, If 8 Hours fill 1 Cistern, what shall 22 Hours fill? The Answer whereof $2\frac{3}{4}$ shall be substracted from 1; so will there want $\frac{7}{4}$, which shall be the first Number of the second Question to be wrought as the other in the former Example.

$$\begin{array}{lcccc} & \textit{Hours.} & \textit{Cistern.} & \textit{Hours.} & \textit{Cisterns.} \\ \text{As} & 8 & .\ 1 & ::\ 22 & .\ 2\frac{3}{4} \end{array}$$

$$\frac{\overset{(6}{22}}{8}\Big(2\tfrac{3}{4}\text{ Cisterns.}$$

$$\frac{4}{4} - \frac{11}{4} = \frac{7}{4}$$

$$\begin{array}{lcccc} & \textit{Cistern.} & \textit{Hours.} & \textit{Cistern.} & \textit{Hours.} \\ \text{As} & \frac{7}{4} & .\ 22 & ::\ 1 & .\ 12\frac{4}{7} \end{array}$$

$$\frac{7}{4}\Big)\frac{22}{1}\Big(\frac{88}{7}\Big(12\tfrac{4}{7}\text{ Hours.}$$

Data, above 3 Numb. and Add. used.

Data, *more than* 3 *Numbers and Addition needful.*

Q. Of 4 Mills grinding.

Example 1. Suppose four Mills, whereof the first grindeth 4 Quarters in 3 Hours; the second 5 Quarters in 4 Hours; the third 6 Quarters in 5 Hours; and the fourth 7 Quarters in 6 Hours: in what Time shall they all grind 30 Quarters?

Answ.

Answ. In 6 Hours and $\frac{2}{33}$ of an Hour. The *Ratio's* in this Proposition having both Terms greater than Units, they are to be reduced to like Consequents, and the Sum of the Antecedents divided by the common Consequent. And because there is a Limitation in the Demand of 30 Quarters, the Quotient of this Division is to be the first Number of a Question to be resolved by the *Rule of Three*, with an Unit in the second Place, and the 30 inquired for in the Third. Answer.

	480	450	432	420
Quarters	4	5	6	7
Hours	3	4	5	6
		360		

$$\begin{array}{ll} \text{Antecedents} & 480+450+432+420=1782 \\ \text{Consequents} & 3 \times 4 \times 5 \times 6 = 360 \end{array} \left(4\tfrac{19}{20} \text{ Quarters.} \right.$$

Then, If $4\frac{19}{20}$ Quarters be ground in 1 Hour, in what Time shall 30 Quarters be ground? *Facit* as aforesaid, $6\frac{2}{33}$ Hours.

As $\frac{99}{20}$. 1 :: 30 . $6\frac{2}{33}$ $\qquad \frac{99}{20}\Big)\frac{30}{1}\Big(\frac{200}{33}\Big(6\frac{2}{33}$

The Work by the *Rule of Three*, where the Antecedents and Consequents of the *Ratio's* given are both greater than Units, is different from that where one Term is an Unit in two things. First, In working the several Questions, the Consequents must be made Antecedents, and the Antecedents Consequents. And, 2*ly*, at the last the Consequent is to be divided by the Antecedent, as in the following Operation. *Variety of Operation. The Difference herein from the former.*

1. As $\frac{3}{4}$. 1 :: $\frac{4}{5}$. $1\frac{1}{15}$ $\qquad \frac{3}{4}\Big)\frac{4}{5}\Big(\frac{16}{15}$ 1 added. $2\frac{1}{15}$

2. As $2\frac{1}{15}$. $\frac{4}{5}$ :: 1 . $\frac{12}{31}$ $\qquad \frac{31}{15}\Big)\frac{4}{5}\Big(\frac{12}{31}$

3. As $\frac{12}{31}$. 1 :: $\frac{5}{6}$. $2\frac{11}{72}$ $\qquad \frac{12}{31}\Big)\frac{5}{6}\Big(\frac{155}{72}$ 1 added. $3\frac{11}{72}$

4. As $3\frac{11}{72}$. $\frac{5}{6}$ :: 1 . $\frac{60}{227}$ $\qquad \frac{227}{72}\Big)\frac{5}{6}\Big(\frac{60}{227}$

5. As $\frac{60}{227}$. 1 :: $\frac{6}{7}$. $3\frac{17}{70}$ $\qquad \frac{60}{227}\Big)\frac{6}{7}\Big(\frac{227}{70}$ 1 added. $4\frac{17}{70}$

6. As $4\frac{17}{70}$. $\frac{6}{7}$ :: 1 . $\frac{20}{99}$ $\qquad \frac{297}{70}\Big)\frac{6}{7}\Big(\frac{20}{99}$

This $\frac{20}{99}$ the Quotient of the last Work must be set as $\frac{99}{20}$, and then divided, giveth $4\frac{19}{20}$ as before in the Work by *Ratio's* for the first Number of the Question there stated and resolved.

Example 2. Suppose a Conduit, whose Cistern holdeth 1000 Gallons, hath three Cocks; by the first Cock will run out 20 Gallons in 3 Hours; by the second, 30 Gallons in 7 Hours; and by the third, 40 Gallons in 9 Hours: in what Time will there be run out by all the Cocks $973\frac{1}{3}$ Gallons, and the first Cock be opened half an Hour before the other? *Q. Of emptying a Cistern.*

Answ. In $63\frac{1}{2}$ Hours, or 2 Days 15 Hours and an Half. The Work of the *Ratio's* is as in the last Example, by which will be gotten $15\frac{25}{63}$ Gallons to be run out by all the Cocks running together in 1 Hour: but before the Question can be set therewith to know in what Time $973\frac{1}{3}$ Gallons will be run out, there must be known how much runs out in the half Hour the first Cock runs alone, which will be $3\frac{1}{3}$ Gal- Answer.

lons: this taken from $973\frac{1}{3}$, leaves 970, which shall give 63 Hours, to which the half Hour must be added.

	1260	810	840
Gallons	20	30	40
Hours	3	7	9
		189	

$$\begin{array}{l} \text{Antecedents } 1260+810+840 = \cancel{2}\cancel{9}\cancel{1}0 \\ \text{Consequents } \quad 3 \times 7 \times 9 = \cancel{1}89 \end{array} \Big(15\tfrac{25}{63} \text{ Gallons.}$$

Hours. *Gallons.* *Hour.* *Gallons.*
If 3 run out 20: what shall $\frac{1}{2}$? *facit* $3\frac{1}{3}$

$\frac{3}{1}\Big)\frac{20}{2}\Big(\frac{20}{6}\Big(3\frac{1}{3}$ $\quad 973\frac{1}{3} - 3\frac{1}{3} = 970.$

Gallons. *Hour.* *Gallons.* *Hours.*
If $15\frac{25}{63}$ run out in 1: in what 970? *facit* 63.

$\frac{970}{63}\Big)\frac{970}{1}\Big(\frac{63}{1}$ $\quad$ Added $\frac{1}{2}$; $63\frac{1}{2}$ Hours.

Variety of Operation.

By the *Rule of Three* the Work is as in the last Example till the $15\frac{25}{63}$ Gallons be gotten, and then the other work as above.

As $\frac{3}{20}$: 1 :: $\frac{7}{30}$. $1\frac{5}{9}$ $\qquad$ As $2\frac{5}{9}$. $\frac{7}{30}$ :: 1 : $\frac{21}{230}$

$\frac{3}{20}\Big)\frac{7}{30}\Big(\frac{14}{9}$ $\quad$ 1 added. $2\frac{5}{9}$ $\qquad$ $\frac{23}{9}\Big)\frac{7}{30}\Big(\frac{21}{230}$

As $\frac{21}{230}$. 1 :: $\frac{9}{40}$: $2\frac{13}{28}$ $\qquad$ As $3\frac{13}{28}$: $\frac{9}{40}$ :: 1 : $\frac{63}{970}$

$\frac{21}{230}\Big)\frac{9}{40}\Big(\frac{207}{48}$ $\quad$ 1 added. $3\frac{13}{28}$ $\qquad$ $\frac{97}{28}\Big)\frac{9}{40}\Big(\frac{63}{970}$ $\qquad$ $\frac{970}{63}\Big(15\frac{25}{63}$

Data above 3 Numb. and Sub. used.

Data *more than* 3 *Numbers and Substraction needful.*

Q. Of hunting an Hare.

Example 1. A Gentleman in hunting an Hare, by their tracing in the Snow findeth that the Hare had 60 Lengths of the Hound's Paces before the Hound: And as often as the Hare runneth 8 Paces, the Hound runneth but 6 Paces; but 2 Paces of the Hound are as much as 3 of the Hare's Paces: In how many Paces of the Hound shall he overtake the Hare?

Answer.

Answ. In 540 Paces of the Hound. Here are three Questions included. First, Seeing the 60 Lengths were of the Hound's Paces, how many that made of the Hare's Paces? Secondly, Since 2 Paces of the Hound are equal to 3 of the Hare, that is gain upon the Hare 1 Pace, how many 6 Paces of the Hound shall gain or be equal to? Thirdly, The Gain of 6 Paces of the Hound known, the third Question ariseth, and is resolved by the Quotient of the first Work with the 6 Paces, and the Gain found as hereunder is to be noted.

Hound's *Hare's* *Hound's* *Hare's*
If 2 Paces make 3 Paces: what shall 60 Paces? *facit* 90 Paces.

$2\Big)\frac{60}{180}\Big(90$ Hare's Paces.

If

Hound's *Hare's* *Hound's* *Hare's*
If 2 Paces make 3 Paces : what shall 6 Paces? *facit* 9 Paces.

$$2\overline{)\ \tfrac{6}{18}\ }(9 \text{ Hare's Paces.}$$

Substract 8 Hare's Paces.
Gain 1

Hare's *Hound's* *Hare's*
If 1 Pace be gotten by 6 Paces : in how many Hound's Paces will 90 Paces be gotten? *facit* 540 Hound's Paces, as before.

As 1 . 6 :: 90 . 540.

Example 2. A Gentleman hunteth an Hare; and as often as the Hound runneth 6 Paces, the Hare runneth 8; and 2 Paces of the Hound make 3 of the Hare, and the Hound overtaketh the Hare in 540 Paces of his own : how many Hound's Paces had the Hare before the Hound? Q. *Of hunting an Hare.*

Answ. 60 Paces of the Hound. Here also are 3 Questions to be resolved, yet with this difference from the former Example, that as there Substraction was made from the Quotient of the second Work, here the Quotient of the first Work shall be substracted from the first Number of the second Work; the rest of the Works are similar to the former, as hereunder is evident. Answer.

Hound's *Hare's* *Hound's* *Hare's*
If 6 Paces make 8 Paces : what shall 2 Paces? *facit* $2\frac{2}{3}$ Paces.

$$6\overline{)\ \tfrac{2}{16}\ }(2\tfrac{2}{3} \text{ Hare's Paces.}$$

Hare's Paces.
$3 - 2\frac{2}{3} = \frac{1}{3}$ loss.

Hare's *Hound's* *Hare's* *Hound's*
If 3 Paces make 2 Paces : what shall $\frac{1}{3}$ Pace? *facit* $\frac{2}{9}$ Pace.

$$\tfrac{3}{1}\Big)\tfrac{2}{3}\Big(\tfrac{2}{9} \text{ Hound's Paces.}$$

Paces *Hound's* *Paces* *Hound's*
If 2 Hound come of $\frac{2}{9}$ Pace : of what comes 540 Hound? *facit* 60 Paces.

For $\frac{2}{9} \times \frac{540}{1} = \frac{120}{1}$ (with 9 cancelled to 1, 540 to 60)

And $\frac{2}{1}\Big)\frac{120}{1}\Big(\frac{60}{1}$ Hound's Paces.

Thus, (as was before noted, and might be seen by many other Instances) besides the Rule, due consideration must be had to the Nature of the Question propounded, duly to prepare or dispose the *Data*, and when to add and substract, *&c.* in such kind of Specifical Propositions as these are; to conclude which, one Example more shall be added.

A Worm in a Well 24 Feet deep, creepeth upwards every Day $5\frac{1}{4}$ Feet, and downwards every Night $4\frac{1}{3}$ Feet : in how many Days shall he creep out of the Well? Q. *Of a Worm creeping out of a Well.*

Answ. In $21\frac{19}{21}$ Days. For in 21 Days the Deduction of the downward Motion taken from the upward, yet leaveth $19\frac{1}{4}$ Feet for the Worm to crawl up : And the other $4\frac{3}{4}$ Feet to make up 24, he creepeth up on the 22*d* Day, without any deduction, he being up $\frac{2}{21}$ of a Day before Night. Answer.

Here $4\frac{1}{3}$ taken from $5\frac{1}{4}$, leaveth $\frac{11}{12}$ the Gain upward in 1 Day : Then because the last Day he gets up to the top of the Well, there is no deduction to be made, but 1 Night's deduction is to be taken from 24; and so $4\frac{1}{3}$ taken from 24, there remaineth $19\frac{2}{3}$.

	Foot.		*Day.*		*Feet.*		*Days.*		*Days.*		*Feet.*		*Days.*		*Feet.*
Then as	$\frac{11}{12}$	.	1	::	$19\frac{2}{3}$	.	$21\frac{5}{11}$	And as	$21\frac{5}{11}$	.	$19\frac{2}{3}$	::	21	.	$19\frac{1}{4}$

And

And becauſe $19\frac{1}{4}$ Feet wants $4\frac{3}{4}$ of 24, this muſt be the Motion of the Worm the laſt Day in which he got up to the top of the Well.

	Feet.		*Day.*		*Feet.*		*Day.*
Therefore as	$5\frac{1}{4}$	.	1	::	$4\frac{3}{4}$	.	$\frac{19}{21}$

And this $\frac{19}{21}$ is to be added to the 21 Days in which he got up to the $19\frac{1}{4}$ Feet, ſo both make up $21\frac{19}{21}$ Days as before.

Proof of Specificks Common and Specifical. *Specificks*, beſides their Proof in Common with other Works of the *Rule of Three*, (before ſeen in the foregoing Chapters) have their Specifical Proof.

Of the firſt ſort. Specifical Proof of the firſt ſort, is by turning the Fractions with Integers all into Integers, and working therewith.

This for Queſtions reſolved by the *Direct Rule of Three*, is to multiply the Numerator of the firſt Number of the three given Numbers into the Denominators of the Second and Third, and this Product ſhall be the firſt Number of the Probationary Work. The Numerator of the ſecond given Number ſhall be the ſecond in this Work: And the third Number here ſhall be the Product of the Numerator of the Third, into the Denominator of the Firſt.

But for Queſtions reſolved by the *Indirect Rule of Three*, multiply the Numerator of the firſt of the three given Numbers into the Denominator of the Third, and the Product ſhall be the firſt Number of this probationary Work. The Numerator of the ſecond given Number, ſhall be the ſecond in this Work: And the third Number here ſhall be the Product of the Numerator of the Third into the Denominators of the Firſt and Second.

Examples. Example in two of the former Inſtances, where

Ell		*l.*		*Ells*		*l.*	
$1\frac{1}{2}$	at	$1\frac{1}{4}$,	is as	$3\frac{3}{4}$	to	$3\frac{1}{8}$,	by the *Direct Rule.*

s.		*Ounces*		*s.*		*Ounces*	
$5\frac{1}{2}$	to	$6\frac{6}{11}$,	ſo is	12	to	3,	by the *Indirect Rule.*

The *Data* being reduced into Improper Fractions, ſtand thus.

	Ell		*l.*		*Ells*			*s.*		*Ounc.*		*s.*
1.	$\frac{3}{2}$	.	$\frac{5}{4}$	::	$\frac{15}{4}$.		2.	$\frac{11}{2}$	.	$\frac{72}{11}$	::	$\frac{12}{1}$.

Then 3×4×4=48. Firſt Number.
5. Second Number.
15×2=30. Third Number.

As 48 . 5 :: 30 . $3\frac{1}{8}$.
8 5 Firſt and Third abbrev.

$8)\frac{5}{25}(3\frac{1}{8}$ *l.*

And 11×1=11. Firſt Number.
72. Second Number.
12×2×11=264. Third Number.

As 11 . 72 :: 264 . 3
1 24

$\frac{72}{24}($3 Ounces.

Of the ſecond ſort. Specifical Proof of the ſecond Sort, is to turn the Geodeticals or Aſtronomicals into Decimals, or Decimals into them, and ſo work the Queſtion in both, and compare the Reſolutions together; this may be ſeen in ſome of the Examples before wrought in their proper Place, ſo as farther Inſtance is here purpoſely omitted.

Of the third ſort. Specifical Proof of the third Sort, is by making two Queſtions, and after Reſolution, adding, ſubſtracting, &c. the Quotients that are to be added, ſubſtracted, &c. according to the Nature of the Propoſition.

Example. Example in the firſt of the former Inſtances concerning the two Poſts, where *A* travelling 18 Miles a Day, and *B* on the contrary 30, they were found diſtant the third Day 144 Miles.

	A 18		*B* 30
	3		3
That is	54	+	90

For as 1 . 18 :: 3 . 54
And as 1 . 30 :: 3 . 90
144

Specifical

Specifical Proof of the fourth sort, is by working the Questions by their *Ratio's*, or by the *Rule of Three*; and the Agreement in their Resolutions wrought both ways, declare their Operations right; and because most of the Examples before are so wrought, farther Instances need not here. *Of the fourth sort.*

CHAP. VI.

The Rule of five Numbers Direct.

AFTER *Specificks* follow the next Sort of derived Proportions to be seen in the *Rule of five Numbers*, sometimes called the *Compound Golden Rule*, and sometimes the *Double Rule of Three*: which latter Name is put upon it, because the Proposition may be resolved by a double Operation of the *Simple Golden Rule*, in opposition to which this is called *Compound*. But as the two Questions are Compound in one; so by the *five Numbers* given, from whence it took the first Name, Resolution may be had, and a sixth Proportional found by one Operation. And though they are resolvable by two, yet differ from *Specificks*, as before in the precedent Chapter was, and further in this and the next Chapter may be observed. Rule of five Numbers. *Why called* Compound. *Why the* Double Rule of Three. *Different from* Specificks.

The *Rule of five Numbers* is both *Direct* and *Indirect*; the latter is referred to the next Chapter: to the knowledg of the other are considerable, Rule of five Numbers, Direct *and* Indirect.

Some things $\begin{cases}\text{Preparatory in the right Disposition of the } \textit{Data}.\\ \text{Operatory in the Resolution.}\\ \text{Probationary in the Proof.}\end{cases}$

The Precepts Preparatory, may be these and such-like. *Preparatory to the Direct.*

1. Among the *Data* discharge all the Numbers that are superfluous; for sometime the Proposition giveth five Numbers, when two of them may be discharged as more than needful; and the Numbers reduced to 3, the Question may be resolved by the *Simple Rule of Three*. 1. *All Superfluous to be discharged.*

As, If 100 *l.* in 12 Months gain 8 *l.* what shall 100 *l.* in 16 Months? Here 100 in both Places may be cancelled, and Resolution had without. *Example.*

	Months.		*l.*		*Months.*		*l.*
For as	12	.	8	::	16	.	$10\frac{2}{3}$
	3				4		

$$3\,)\,\frac{4}{32}\,(\,10\tfrac{2}{3}$$

2. Of the 5 given Numbers, let 3 be Conditional and Antecedents, or Suppositious; and the other 2 Interrogative and Consequents, corresponding to some of the former Antecedents. 2. *How many Conditional and Interrogative.*

As in the former Instance 100. 12. 8. are Conditional and Antecedent, the other 100. 16. are Interrogative. And because this latter 100 is Consequential and equal to the former Antecedent, 100 in both Places may be cancelled as before. *Example.*

3. Let the three Conditionals be first placed towards the left Hand; and then the two Terms on which the Question depends, in the 4*th* and 5*th* Places: So as the first Term be the principal Cause of Loss or Gain, Increase or Decrease, Action or Passion. That whose Sirname or Denomination betokeneth the Space of Time, Distance of Place, &c. the second and the third Term the odd Denomination, which is unlike in Nature to all the other Numbers. Then let the fourth Term and first be of one Denomination; likewise the fifth and second. 3. *To place the Numbers aright.*

As in the former Instance, the first 100 being the principal Cause of the Gain, is first set; the 12 Months being the Space of Time, is the second Number; the 8 *l.* Gain is the odd Denomination, and so set in the third Place: Then the latter 100 *l.* Principal-Money answering in the Interrogative Part to the first Number, is *Example.*

is placed in the 4*th* Place: And 16 Months of like Denomination to the second in the 5*th* Place.

4. Denominations double, which regarded.

4. When any of the Numbers seem to be doubly denominate, the latter Denomination is to be regarded in placing the Numbers, as before noted in the *Direct Rule of Three.*

5. Data when need to be reduced.

5. If any of the Numbers be of several Denominations, as Pounds and Shillings, or Pounds, Shillings and Pence, *&c.* reduce the same and his correspondent Number into the lowest of those given Denominations; as the First and Fourth, Second and Fifth: but if the third Number want Reduction, he having no Correspondent but the Sixth not yet found, is only brought into the lowest Denomination given, without altering any of the other *Data:* And according to the Third, or Reduction thereof, shall the Sixth Number when found be denominate.

Sixth when found, how denominate.

6. Terms if abbreviated, shorten the Work.

6. Sometime the Conditional Terms given may be reduced to lesser Proportional Numbers, and working by them will shorten the Work. As in the former Instance, instead of 100 *l.* 12 Months, 8 *l.* may be taken; 50 *l.* 12 Months, 4 *l.* or 25 *l.* 12 Months, 2 *l.* or 5 *l.* 12 Months, 8 *s.* or 5 *l.* 3 Months, 2 *s.* &c.

7. To find if Resolution be by the Direct Rule, or not.

7. To find whether a Question propounded be resolvable by the *Rule of five Numbers Direct*; consider whether the principal Cause of Loss or Gain, Interest or Decrease, Action or Passion, *&c.* be the odd denominate Number in the Proposition: For if so, then is the Resolution by the *Indirect Rule* as in the next Chapter; but if otherwise, by the *Direct*. As in Questions of Money, the *Direct Rule* enquireth always for the Interest, and gives the Principal Money, the *Indirect*, for Stock or Time of Continuance, and gives the Interest.

8. Operation to be as the Nature of the Data *is.*

8. According to the Nature of the *Data*, whether Integers, Fractions, *&c.* so let the Operation be: And if any Proposition be specifical, mix the Work in order to the Resolution accordingly.

Resolution by 1 or 2 Works.

The Premisses observed, the Resolution will be had by one or two Operations.

The Rule for one Operation.

How by one.

Multiply the two first Numbers together for the Divisor, and the three last Numbers one into another for the Dividend; and dividing the one by the other, the Quotient shall be the sixth Proportional, and answer the Question.

The Rule for the Double Work.

How by two.

Take the first, third and fourth Numbers, and work as in the *Direct Rule of Three* for another Proportional: Then take this new Proportional for the second Number of the next Work, and the second and fifth Numbers of the five given Numbers for the first and third Numbers of this second Work, and the Quotient of this last working by the *Rule of Three*, shall answer the Question first propounded.

Q. *Of Interest-Money.*

Example 1. If 400 *l.* be put out for 16 Months, at the Rate of 8 *l. per Centum per Annum*, (accounting 12 Months to the Year) I demand what will the Interest come to?

Answer.

Answ. $42\frac{2}{3}$ *l.* Here the three Conditional Terms are the 100 *l.* that in one Year (or 12 Months) gains 8 *l.* and 100 *l.* the Principal Cause of the Profit or Increase, that shall be the first Number; and the 400 *l.* being also Principal Money the first Consequent, and of like Nature to the 100, shall be set in the fourth place; 12 Months the second Conditional, being the Space of Time in which the Increase groweth, shall be the second Number, and the Consequent 16 Months of like Nature to be 12, shall be in the fifth Place: So as the 8 *l.* Use-Money having none of the five to parallel him, falls into the third Place.

By one Work.

Then to resolve the Question at one Operation, multiply 100 into 12, the Product 1200 is Divisor: And 8 multiplied into 400, and the Product by 16 makes the Dividend 51200, and after Division $42\frac{2}{3}$ found in the Quotient, is denominate like the third Number, and is the Use-Money gained by 400 *l.* in 16 Months, at the Rate aforesaid.

As

	l.	*Months.*	*l.*		*l.*	*Months.*	*l.*
As	100	. 12	. 8	::	400	. 16	. $42\frac{2}{3}$

100
12
12|00

400
8
3200
16
19200
3200
512|00

3(8
512(42$\frac{2}{3}$ *l.*
12

If the Operation be double, then is 100 *l.* 8 *l.* 400 *l.* the three Numbers of the first Work, 32 the Number gotten thereby the second Number of the second Work, and 12 and 16 the first and third Numbers of the same Work: Thus, *By two Works.*

	l.	*l.*		*l.*	*l.*		*Months.*	*l.*		*Months.*	*l.*
As	100	. 8	::	400	. 32	As	12	. 32	::	16	. $42\frac{2}{3}$

4
32

3 4
4
3)128(42$\frac{2}{3}$

If the conditional Terms be reduced to smaller Proportionals, then the Work at one Operation will stand thus. *Example where the Conditionals are reduced.*

	l.	*Mon.*	*s.*		*l.*	*Mon.*	*l.*	*s.*	*d.*
As	5	. 3	. 2	::	400	. 16	: 40	: 13	: 4.

15

32
800
1200
12800

5(5
12800(853
15
15
15
42:13:4

Example 2. If 3 Mowers in 6 Days mow $24\frac{1}{2}$ Acres: how many Acres will 8 Mowers mow in 10 Days? *Q. of Mowers.*

Answ. 108 Acres and $\frac{8}{9}$ of an Acre. **Answer.**

	Mowers.	*Days.*	*Acres.*		*Mowers.*	*Days.*	*Acres.*
As	3	. 6	. $24\frac{1}{2}$	::	8	. 10	. $108\frac{8}{9}$

18

245
1960

1(16
1960(108$\frac{8}{9}$ Acres.
18

Specificks.

Example 1. If 5 Guns in 2 Days spend 60 Barrels of Powder: what will 7 Guns spend in 5 Days, which spend $\frac{1}{3}$ part less than the 5 at every shot? *A specifical Question of Powder spent by 7 Guns.*

Answ. 140 Barrels of Powder.

Barrel 60(20
$\frac{1}{3}$ 3
60——20===40

	Guns.	*Days.*	*Bar.*		*Guns.*	*Days.*	*Bar.*
As	5	. 2	. 40	::	7	. 5	. 140

10

200
1400 1400(140 Barrels.
10

Example 2. If 12 Penny-worth of Wine satisfy 8 Persons at a Meal, when Wine is at 6 *d.* a Quart: how much Wine at 4 *d.* a Quart will suffice 40 Persons that drink twice as much as the other? *Another of Wine drunk.*

Answ. 80 Pennyworth of Wine. Answer.

Wine

Wine	12 *d.*		*Persons.*	*d.quart.*	*d.*		*Persons.*	*d.quart.*	*s.*	*d.*
Increased	2	As	8 .	6 .	24	::	40 .	4 .	6 :	8.
	24		48				96			
							3840		3840 / 48 (80 *d.*	

Proof of the Rule of five Numbers Direct.

The Operations of the *Direct Rule of five Numbers* wrought at once, and by the double Work, seeing they agree in the Resolution, and identify the Answer, prove each other, as by the Resolution of the Gain of 400 *l.* in 16 Months wrought before both ways is made plain.

CHAP. VII.

The Rule of five Numbers Indirect.

Rule of five Numbers Indirect.

THE Knowledg of this *Indirect Rule of five Numbers*, consists chiefly in the Preparatory Part rightly to dispose the *Data*, for which these Precepts are necessary.

1. All superfluous Numbers to be discharged.

1. Discharge all the Numbers superfluous among the *Data*, as before in the *Direct Rule of five Numbers*.

2. How to place the Data.

2. Of the five given Numbers as before, if three be Conditionals and Antecedents, and the other two Interrogative and Consequents, then dispose them thus; Let the odd Denomination, which here will be the principal Cause of Loss or Gain, Increase or Decrease, Action or Passion, *&c.* be the first Term, and the other two Conditionals the Second and Third, as in the *Direct Rule of five Numbers*; and let the Second and Fourth, and Third and Fifth, be of like Denominations.

3. How to place them if there be 4 Conditionals.

3. But if there be four Conditionals or Terms of Explanation; then place the single or odd denominate Term in the first Place as before; the Term on which the Question depends, set in the second Place; the Number countervailed by the first located Term, let be the Third; the Number denominate like the second located Term, place in the fourth Place; and then the other Term of the *Data* countervailing the Fourth, and denominate like the Third located Term, will supply the fifth Place.

4. Sixth when found, how denominate. Other Precepts before useful here. Resolution by 1 or 2 Works.

4. Let the sixth Proportional, when found, be always accounted of like denomination to the First, or the Reduction thereof if reduced. The rest of the 4*th*, 5*th*, 6*th*, 7*th* and 8*th* Precepts of the *Direct Rule of five Numbers*, consider and make use of as occasion shall require.

The *Data* duly disposed, as aforesaid, the Resolution will be had by one or two Operations.

The Rule for one Operation.

How by 1.

Multiply the third and fourth Numbers together for the Divisor, and the other three Numbers one into another for the Dividend; and dividing the one by the other, the Quotient shall be the desired Number, and the sixth Proportional.

The Rule for the double Work.

How by 2.

Take the Fourth for the First, the Fifth for the Second, and the Second for the Third; and the Quotient of this Work by the *Rule of Three*, make the third Number of the second Work: and for the First, take the Third of the *Data*; and for the Second, the First of the *Data*, and the Result of this Work by the *Rule of Three* shall resolve the Question first propounded.

Q. Of Principal Money.

Example 1. If 42⅔ *l.* be Interest for 16 Months, after the Rate of 8 *l. per Centum per Annum*: what Principal Money was delivered to raise that Interest?

Answ. 400 *l.* Here being but three Conditionals, the *Data* is to be disposed according to the second Precept. And because 100 is the principal Cause of Gain in the Conditionals, and the single Denomination, it is first placed, the Year or

12

12 Months being the Space of Time must be Second: And 8 the third Conditional shall occupy the Third Place. Then must 16, because agreeing with the Second, be set in the fourth Place; and $42\frac{2}{3}$ in the fifth Place, being of like denomination with the Third, that is both Use-Money: So the Question stands thus.

If 100 *l.* in 12 Months gain 8 *l.* what Principal-Money in 16 Months will gain $42\frac{2}{3}$ *l?*

By 1 Work. Then to resolve the Question at one Operation, 8 is multiplied into 16, and the Product 128 is Divisor. The other three of the *Data* being multiplied, make the Dividend 51200; which divided by 128, give 400 as the Principal-Money to raise that Interest in the Time proposed.

	l.	*Months.*	*l.*		*Months.*	*l.*	*l.*
As	100 .	12 .	8	::	16 .	$42\frac{2}{3}$.	400.

$$\begin{array}{r} 100 \\ 12 \\ \hline 1200 \\ 42\frac{2}{3} \\ \hline 2400 \\ 4800 \\ 400 \\ 400 \\ \hline 51200 \end{array} \qquad \begin{array}{r} 16 \\ \hline 48 \\ 8 \\ \hline 128 \end{array} \qquad \frac{51200}{128}\Big(400\ l.\ \text{Principal.}$$

By 2 Works. If the Work be double, then is 16 . $42\frac{2}{3}$. 12 . the three Numbers of the first Work; 32 the Number gotten thereby, shall be the third Number of the second Work, and 8 and 100 the other Numbers of that Work: Thus,

	Months.		*l.*		*Months.*	*l.*
As	16	.	$42\frac{2}{3}$	::	12 .	32

$$\begin{array}{r} 42\frac{2}{3} \\ 12 \\ \hline 8 \\ 428 \\ \hline 512 \end{array} \qquad \frac{512}{16}\Big(32\ l.$$

	l.		*l.*		*l.*		*l.*
As	8	.	100	::	32	.	400

$$\begin{array}{r} 100 \\ 32 \\ \hline 8)3200(400\ l. \end{array}$$

One Work by the Indirect, yet 2 both Direct sometimes. Here is worthy to be noted, that in this and several other Instances, though the Resolution at one Operation is by the *Indirect Rule of five Numbers*; yet by the double Work both are sometimes resolved by the *Direct Rule of Three*.

Example, where the Conditionals are reduced. If the Conditional Terms be reduced to lesser Proportionals, the Work at one Operation will stand thus.

	l.		*Mon.*		*l.*		*Mon.*		*l.*		*l.*
As	5	.	3	.	$\frac{1}{10}$	::	16	.	$42\frac{2}{3}$	.	400

$$\begin{array}{r} 5 \\ 3 \\ \hline 15 \\ 42\frac{2}{3} \\ \hline 30 \\ 60 \\ 5 \\ 5 \\ \hline 640 \end{array} \qquad \frac{1}{10} \times \frac{16}{1} = \frac{8}{5} \qquad \frac{8}{5}\Big)\frac{640}{1}\Big(\frac{400}{1}\ l.$$

Q. Of Mowers. *Example* 2. If 3 Mowers in 6 Days mow $24\frac{1}{5}$ Acres: how many Mowers in 10 Days can mow $108\frac{8}{9}$ Acres?

Answer. *Answ.* 8 Mowers.

	Mowers.	Days.	Acres.		Days.	Acres.	Mowers.
As	3 .	6 .	$24\frac{1}{2}$	::	10 .	$108\frac{8}{9}$.	8

6
18
$108\frac{8}{9}$
144
180
16
1960

10
240
5
245

$\frac{1960}{245}$ (8 Mowers.

Specificks.

A specifical Question of Travel.

Example 1. If in 10 Days of 8 Hours long, a Man may journey 200 Miles: in how many Days twice as long may he travel 500 Miles?

Answer.

Answ. In $12\frac{1}{2}$ Days.

Hours 8
Increased 2
16

	Days.	Hours.	Miles.		Hours.	Miles.	Days.
As	10 .	8 .	200	::	16 .	500 .	$12\frac{1}{2}$

8
80
500
400,00

16
32,00

$\frac{400}{32}$ ($12\frac{1}{2}$ Days.

Another of Persons drinking.

Example 2. If 12 pennyworth of Wine satisfy 8 Persons at a Meal, when Wine is at 6 Pence a Quart: how many Persons that drink but half so much Wine at a Meal, will 20 pennyworth of Wine suffice when Wine is at 4 *d.* a Quart?

Answ. 40 Persons.

Wine $\frac{12}{2}$ (6
$\frac{1}{2}$
12—6=6

	Persons.	d.quart.	d.		d.quart.	d.	Persons.
As	8 .	6 .	6	::	4 .	20 .	40

6
48
20
960

4
24

$\frac{960}{24}$ (40 Persons.

Examples of 4 Conditionals.

Where four Conditionals are proposed.

Q. *Of* Paris *Pence.*

Example 1. If 4 *d.* of *Paris* be worth 5 *d. Tournois*, and 5 *d. Tournois* 6 *d.* of *Savoy*: how many Pence *Paris* are 15 *d. Savoy*?

Answer.

Superfluous Numbers discharged.

Answ. 10 *d. Paris.* Here two of the Terms, that is, both the Pence of *Tournois* are superfluous, and so may be omitted, and the Question with three Numbers stand thus.

	d.Savoy.	d.Paris.		d.Savoy.	d.Paris.
As	6 .	4	::	15 .	10

4
6) 60 (10 *d. Paris.*

And so will the Resolution be, if the *Data* be disposed according to the third Precept: Thus,

	d.Paris.	d.Sav.	d.Tourn.		d.Sav.	d.Tourn.	d.Paris.
As	4 .	15 .	5	::	6 .	5 .	10

15
60
5
300

6
30

$\frac{300}{30}$ (10 *d. Paris.*

Q. Of Angels and Crowns.

Example 2. If 2 Angels countervail 20 *s. Sterling*, and 18 *s.* countervail 3 Crowns *French*: how many Angels will countervail 10 Crowns? or how many Crowns will countervail 12 Angels?

In

In these two Questions the *Data* disposed by the third Precept in the former 2 Angels, being the odd Denomination, will stand in the first place, but in the latter 3 Crowns. The Term on which the Question depends, to be set in the second place, must be 10 Crowns in the former, and 12 Angels in the latter. The third Place being filled with the Number countervailing the first located Term, shall be in the former 20 *s.* and in the latter 18 *s.* The Number denominate like him, placed in the Second, to be set in the fourth Place, must be in the former 3 Crowns, and in the latter 2 Angels. Then the remaining Number denominate like the Third, and countervailing the fourth located Terms, will be placed in the fifth Place, which in the former is 18 *s.* and in the latter 20 *s.* *Numbers placed.*

Single Operation. Operation single.

		Angels.	*Crowns.*	*s.*	*Crowns.*	*s.*	*Angels.*
Q. 1.	If	2 .	10 ;	20 ::	3 .	18 ?	*facit* 6.

2 × 10 = 20; 20 × 18 = 360; × 20 = 36,0 ; 20 × 3 = 6,0

$\frac{36}{6}$ (6 Angels.

Double Operation. Double.

	Cr.	*s.*	*Cr.*	*s.*
As	3 .	18 ::	10 .	60

18 × 10 = 180; 3) 180 (60 *s.*

	s.	*Ang.*	*s.*	*Ang.*
As	20 .	2 ::	60 .	6

2 × 60 = 120; 20) 120 (6 Angels.

Single Operation. Single.

		Crowns.	*Angels.*	*s.*	*Angels.*	*s.*	*Crowns.*
Q. 2.	If	3 ;	12 :	18 ::	2 .	20 ?	*facit* 20

3 × 12 = 36; 36 × 20 = 720 ; 18 × 2 = 36

$\frac{720}{36}$ (20 Crowns.

Double Operation. Double.

	Ang.	*s.*	*Ang.*	*s.*
As	2 .	20 ::	12 .	120

20 × 12 = 240 (40; 20); 2) 024 (120 *s.*

	s.	*Cr.*	*s.*	*Cr.*
As	18 .	3 ::	120 .	20

3 × 120 = 360; 18) 360 (20

The Operations of the *Indirect Rule of five Numbers*, are proved as those of the *Direct* for the Resolution by one Operation, and by two is the same, as by the last Examples wrought both ways is sufficiently convincing. *Proof of the Rule of five Numbers Indirect.*

CHAP.

CHAP. VIII.

FELLOWSHIP.

Fellowship how otherwise called.

DERIVATIVES of the third Sort, which deal with some particular Subject, are next to be exhibited; and among them, first, those Operations that converse about *Fellowship*, called also the *Rule of Society* and *Partnership*, because Questions of *Partnership* are resolved thereby.

The Sorts thereof.

Fellowship may be divided into four Sections,
- Without Time, §. 1.
- With Time, §. 2.
- With diversity of Time, §. 3.
- With diversity of Parts, §. 4.

Without Time of two Sorts.

§. 1. *Fellowship* without Time, either propounds the several Disbursments of the particular Partners, and the general Gain or Loss by their Traffique, and requires to know each Man's Part or Share thereof: Or by the general Stock and particular Shares of the Gain or Loss, requires to know their particular Disbursments.

To resolve the First.

To resolve the Propositions of the first Sort, add all the Monies disbursed, or Stock in Partnership together, for the first Number of the *Rule of Three*: Then set the Gain or Loss in the second Place; and each Man's several Stock or Disbursments in the Third: And by so many Operations of the *Rule of Three* as there be Partners, the Desire is obtained.

Q. Of each Partner's Gain.

Example. Three Men, or *A*, *B*, and *C*, consenting to trade together, make a Bank of Money, in which *A* disbursed 100 *l.* *B* 150 *l.* and *C* 250 *l.* And trading therewith, they gained 300 *l.* what is each Man's Part of the Gain?

Answer.

Answ. *A* 60 *l.* *B* 90 *l.* and *C* 150 *l.* Here 100+150+250=500 make the first Number, 300 the Gain the Second, and the several Sums disbursed the Third Numbers, as followeth.

	Stock.
A	100 *l.*
B	150
C	250
	500

As 500 . 300 :: 100 . 60 *l.*
300 × 10 = 3000; 3000 / 50 = 60 *l.* *A.*

As 500 . 300 :: 150 . 90 *l.*
300 × 15 = 4500; 4500 / 50 = 90 *l.* *B.*

As 500 . 300 :: 250 . 150 *l.*
300 × 25 = 7500; 7500 / 50 = 150 *l.* *C.*

To resolve the second Sort.

To resolve the Propositions of the latter Sort, place the Gain or Loss for the first Number; the whole Stock for the Second; and the several Parts of the Gain or Loss for the several third Numbers, and proceed by the *Rule of Three*, as before.

Q. Of each Partner's Stock.

Example. *A*, *B*, and *C*, made a Stock to trade with, and all laid in together 560 *l.* wherewith they gain 150 *l.* of which at the end of their Partnership, *A* took 40 *l.* *B* 50 *l.* and *C* 60 *l.* what had each Man in Stock?

Answer.

Answ. *A* $149\frac{1}{3}$ *l.* *B* $186\frac{2}{3}$ *l.* and *C* 224 *l.*

As

	l.		l.		l.		l.		
As	15 0	.	560	::	4 0	.	$149\frac{1}{3}$		
			4						
			2240					$\frac{2240}{15}\left(149\frac{1}{3}\right.$ l.	*A.*
As	15 0	.	560	::	5 0	.	$186\frac{2}{3}$		
			5						
			2800					$\frac{2800}{15}\left(186\frac{2}{3}\right.$ l.	*B.*
As	15 0	.	560	::	6 0	.	224		
			6						
			3360					$\frac{3360}{15}\left(224\right.$ l.	*C.*

§. 2. *Fellowship with Time*, simply propounds the several Disbursments of the particular Partners at once, and the Time they continue in Stock, with the Gain or Loss in general, and requires to know each Man's Part thereof: Or by the whole Gain or Loss, and Particulars of the Stock, and some particular Partner's Time of Trade, requireth to know the Time of the others Continuance: Or by the Gain or Loss of some particular Partner's Stock and Time according to the *Data*, seeketh the particular Gain or Loss, Stock or Time of another Partner: Or by Mixtures of these, proposeth Propositions mixt. *Fellowship with Time, of four Sorts.*

To resolve the Propositions of the first Sort, multiply severally every Man's Money by the Time it continued in Stock; and the Total of all those Products added together, shall be the first Number of the *Rule of Three*; the Gain or Loss the Second; and the several Products shall be the several third Numbers; and by so many Operations of the *Rule of Three* as there be Partners, Resolution will be had. *To resolve the first.*

Example. A, B, and *C* disburse as followeth; *viz. A* 50 *l. B* 60 *l.* and *C* 80 *l.* And *B* drew away his Money from the Stock at 10 Months end: But *A* and *C* continued the Trade till 16 Months; when the Stock was broken up, there was found 400 *l.* Gain: what is each Man's Share thereof? *Q. Of each Partner's Gain.*

Answ. A $119\frac{27}{67}$ *l. B* $89\frac{37}{67}$ *l.* and *C* $191\frac{3}{67}$ *l.* Answer.

	Stock.		*Time.*		
A	50	×	16	=	800
B	60	×	10	=	600
C	80	×	16	=	1280
					268,0

	l.				l.		
As 268 0	. 400	::	80 0	.	$119\frac{27}{67}$		
	80						
	32000					$\frac{32000}{268}\left(119\frac{27}{67}\right.$ l.	*A.*
As 268 0	. 400	::	60 0	.	$89\frac{37}{67}$		
	60						
	24000					$\frac{24000}{268}\left(89\frac{37}{67}\right.$ l.	*B.*
As 268 0	. 400	::	128 0	.	$191\frac{3}{67}$		
	128						
	51200					$\frac{51200}{268}\left(191\frac{3}{67}\right.$ l.	*C.*

To resolve the Propositions of the second Sort, multiply that Partner's Stock, whose Time is given by the Time for the second Number of the *Rule of Three*: The first Number shall be that Partner's Gain or Loss; And the several third Numbers shall be the other Partner's Gain or Loss respectively. Then proceed as before by the *Rule of Three*, and the Quotients of these Works will be their Money and Time multiplied together; which if divided by their Money respectively, give the Time desired. *To resolve the second Sort*

Example. A, B, and *C* are in Partnership. *A* layeth in 200 *l.* for 10 Months, *B* layeth in 350 *l.* and *C* 100 *l.* and through ill Adventure sustained the Loss of 160 *l.* Whereupon breaking off their Trade, *A* found himself a Loser 80 *l.* *B* 56 *l.* and *C* 24 *l.* The Question is, how long the Money of *B* and *C* continued in Stock? *Q. Of the Time of Trade.*

Anſwer. *Anſw. B* 4 Months, and *C* 6 Months.

Stock. Time. *A Loſs* *B Loſs*

A 200 × 10 = 2000. As 80 . 2000 :: 56 . 1400

56

11200,0 $\frac{11200}{8}$ (1400 / 350 (4 Months *B*.

A Loſs *C Loſs*

As 80 . 2000 :: 24 . 600

24

4800,0 $\frac{4800}{8}$ (600 / 100 (6 Months *C*.

To reſolve the third ſort. To reſolve the Propoſitions of the third Sort, commit the Queſtion to the *Rule of five Numbers*, or the *Double Rule of Three*, *Direct* or *Indirect*, as the Caſe requires.

Q. Of the Stock of 1 Partner. *Example*. *A* and *B* are in Company: *A* putteth in 240 *l.* and gaineth 50 *l.* in 6 Months: what ſhall *B* put in to gain 30 *l.* in 4 Months?

Anſwer. *Anſw.* 216 *l.*

	l.	*Months.*	*l.*		*Months.*	*l.*	*l.*
As	240	. 6 .	50	::	4 .	30 .	216
	6		4				
	1440		2,00				
	30						
	432,00					$\frac{432}{2}$ (	216 *l.*

To reſolve the fourth Sort. Propoſitions various. The Propoſitions of the fourth Sort are many and various: For ſometime the Stock is enquired after, ſometime the Time, and ſometime the Gain or Loſs; ſometime in general, and ſometime particularly of one or more of the Copartners: And ſometime more than one of them is included in the Queſtion; and accordingly the *Data* with much Variety mixed, as in the Examples following.

1. Example, where are { *Data* { Stock, generally. / Time and Gain, particularly. / *Quæſita*, Stocks, particularly.

Q. Of each Partner's Stock. *A*, *B*, and *C*, traffique together with a Stock of 638 *l.* wherewith they gain 90 *l.* And *A* having had his Money in Stock 5 Months, *B* 8 Months, and *C* 7 Months; the Gain was parted, to *A* 18 *l.* *B* 12 *l.* and *C* 60 *l.* what Monies had each in Stock?

Anſwer. *Anſw. A* 168 *l.* *B* 70 *l.* and *C* 400 *l.*

Here, after the particular Gains are ſeverally divided by the Times of Continuance, the Analogy is,

As the Sum of thoſe Quotients to the whole Stock:

So are thoſe Quotients ſeverally to the ſeveral Stocks.

Gains of *A* $\frac{18}{5}$ ($3\frac{3}{5}$ *B* $\frac{12}{8}$ ($1\frac{1}{2}$ *C* $\frac{60}{7}$ ($8\frac{4}{7}$ $3\frac{3}{5} + 1\frac{1}{2} + 8\frac{4}{7} = 13\frac{47}{70}$
Times

As $13\frac{47}{70}$. 638 :: { $3\frac{3}{5}$. 168 . Stock of *A*. / $1\frac{1}{2}$. 70 . Stock of *B*. / $8\frac{4}{7}$. 400 . Stock of *C*.

2. Example, where are { *Data* { Stock of One. / Time and Gain of Another. / *Quæſita*, Stocks ſeverally.

Q. Of each Partner's Stock. In a joint Trade *B* putteth in 200 *l.* more than *A*; *B* continueth his Stock but 5 Months, *A* 7½ Months; they gain alike: what Money had each of them in Stock?

Anſw.

Answ. A 400 *l.* B 600 *l.* Answer.

Here subtracting one Time from the other, the Analogy is;
As the Difference of Times, to the Stock propounded:
So are the Times themselves, to the several Stocks of each other.

$$\text{Times of}\begin{cases} A & 7\tfrac{1}{2} - 5 = 2\tfrac{1}{2} \\ B & 5 \end{cases}$$

$$\text{As } 2\tfrac{1}{2} \;.\; 200 :: \begin{cases} 5 \;.\; 400 \;.\; \text{Stock of } A. \\ 7\tfrac{1}{2} \;.\; 600 \;.\; \text{Stock of } B. \end{cases}$$

3. Example, where are { *Data* { Stocks and Gains, particularly. Time, generally. } *Quæsita*, Times particularly. }

Three are in Company, the Stock of A in their Trade is 168 *l.* of B 70 *l.* of C 400 *l.* they gain 90 *l.* whereof A hath 18 *l.* B 12 *l.* and C 60 *l.* they withdrew their Stock severally; but all the several Times their Monies were in Stock, added together, make 20 Months: how long did each Man continue his Money in Stock? Q. *Of the Time of Trade.*

Answ. A 5 Months, B 8 Months, and C 7 Months. Answer.

Here abbreviate every Man's Gains with his Stock to the least Terms, and take these Parts of any Number that will equally be divided by all the Denominators: Or if no such Number come readily to mind, multiply all the Denominators one into the other; and divide the Product by the respective Denominators, and multiply the Quotients by the several Numerators: And these last Products abbreviate with the Sum of them. Then the Analogy is;
As the Sum of these Parts, is to the whole Time given:
So is every Man's Part severally, to his respective Time.

$$\begin{array}{lcccc} & A. & B. & C. & \\ \text{Gains} & \frac{18}{168}\Big|\frac{3}{28} & \frac{12}{70}\Big|\frac{6}{35} & \frac{60}{400}\Big|\frac{3}{20} & 28 \times 35 \times 20 = 19600 \\ \text{Stock} & & & & \end{array}$$

$$\frac{19600}{28}\Big(700 \times 3 = 2100 \qquad \frac{19600}{35}\Big(560 \times 6 = 3360 \qquad \frac{19600}{20}\Big(980 \times 3 = 2940$$

$$\begin{array}{r|ll} 2100 & 5 & A. \\ 3360 & 8 & B. \\ 2940 & 7 & C. \\ \hline 8400 & 20 & \end{array}$$

$$\begin{array}{lll} & \textit{Parts.} \;\; \textit{Months.} & \textit{Parts.} \;\; \textit{Months.} \\ \text{As } 20 \;.\; 20 :: & \begin{cases} 5 \;.\; 5 \;.\; \text{Time of } A. \\ 8 \;.\; 8 \;.\; \text{Time of } B. \\ 7 \;.\; 7 \;.\; \text{Time of } C. \end{cases} & \end{array}$$

4. Example, where are { *Data* { Stock and Time of one. Time of another. Stock of a Third. Gains of all particularly. } *Quæsita* { Stock of One. Time of Another. } }

A in Company with B and C, putteth in 168 *l.* for 5 Months, B putteth in a Sum of Money for 8 Months, and C 400 *l.* for a certain Time; they gain 90 *l.* whereof A must have 18 *l.* B 12 *l.* and C 60 *l.* how much was the Stock of B? and what Time did the Stock of C continue in the Company? Q. *Of the Stock and Time of Trade.*

Answ. The Stock of B was 70 *l.* And the Time the 400 *l.* of C was in Stock was 7 Months. Answer.

Here the Analogies are; As the Gain of one Partner is to his Stock multiplied by his Time: So is the Gain of the other Partners severally to theirs; Which when found, is to be divided by their Time or Stock respectively.

$$\begin{array}{lr} \text{Stock of } A. & 168 \\ \text{Time} & 5 \\ \hline & 840 \end{array}$$

$$\text{As } 18 \;.\; 840 :: 12 \;.\; \frac{560}{\text{Time } 8}\Big(70 \;.\; \text{Stock of } B.$$

$$\text{As } 18 \;.\; 840 :: 60 \;.\; \frac{2800}{400}\Big(7 \;\text{ Time of } C.$$

5. Example

5. Example, where are: *Data* { Stock and Gain of One. / Time and Gain of Another. / Rate on the Hundred of Both. } *Quæsita* { Stock of One. / Time of Another. }

Q. Of the Stock of one, and Time of another.

A joint Stock of *A* and *B* gained them, after the Rate of 20 *l. per Centum per Annum*, 50 *l.* apiece; *A* had 400 *l.* in Stock, and the Stock of *B* continued but 5 Months: what Money had *B* in Stock, and how long did *A* continue in the Company?

Answ. *A* continued $7\frac{1}{2}$ Months, *B* put into Stock 600 *l.*

Here first finding the Gain of the Stock given by the Rate propounded, the Analogies are,

As the Gain according to the Rate, to 1 Year:
So is the Gain received, to the Time required.
And then, As the Time of one Partner is, to the Stock of the other:
So is the Time of the Partner found, to the Stock of the other sought.

As	100	. 20	:: 400	. 80		for *A*. Then
	l.	*Months.*	*l.*	*Months.*		
As	80	. 12	:: 50	. $7\frac{1}{2}$	.	Time of *A*.
	Time B.	*Stock A.*	*Time A.*	*Stock B.*		
As	5	. 400	:: $7\frac{1}{2}$	. 600	.	Stock of *B*.

6. Example, where are: *Data* { Stock, Time, and part of the Gain of One. / Time and part of the Gain of Another. / Time of a third Partner. } *Quæsita*, Stocks of two Partners.

Q. Of the Stock of two Partners.

Three Partners trade together. *A* continuing his Stock in the Trade 5 Months, must have $\frac{1}{5}$ of the Gain. *B* keepeth his Money in Stock 8 Months. *C* layeth in 400 *l.* for 7 Months, and taketh $\frac{2}{3}$ of the Gain: how much Money had *A* and *B* in the Stock?

Answer.

Answ. *A* 168 *l.* and *B* 70 *l.*

Here, after adding the Parts of Gains given together, and taking the Total out of 1, the Analogies are;

As the Part of Gain of one Partner given, is to his Stock multiplied by his Time: So is the Part of Gain of another: And also so is the Remainder of the Parts taken from 1, to his Stock multiplied by his Time. Which divided by the Time, giveth the Stock of that Partner sought respectively.

$\overset{A}{\frac{1}{5}} + \overset{C}{\frac{2}{3}} = \frac{13}{15}$ Total. $\quad 1 - \frac{13}{15} = \frac{2}{15}$ Remainder.

$$\text{As } \frac{2}{3} \;.\; \overset{(400\times7)}{2800} \;::\; \frac{1}{5} \;.\; \frac{840}{\text{Time of } A\ 5}\Big(\ 168 \text{ Stock of } A.$$

$$\text{As } \frac{2}{3} \;.\; 2800 \;::\; \frac{2}{15} \;.\; \frac{560}{\text{Time of } B\ 8}\Big(\ 70 \text{ Stock of } B.$$

Fellowship with diversity of Time of three sorts.

§. 3. *Fellowship*, with diversity of Time, altereth the Stock and Time of some, or all of the particular Partners continuance; and thereby with the general Gain or Loss, requireth their due Proportions thereof: Or by their particular Gains or Losses, and Times of Occupation with the general Stock, seeketh their particular Disbursments: Or by the Gain or Loss of some particular Partners, Stock and Time, enquireth the particular Gain or Loss, Stock or Time of another Partner, according to the *Data*.

To resolve the first.

To resolve the Propositions of the first Sort, multiply the several Disbursments of the particular Partners, by the respective Times those Disbursements continue in Stock, adding or substracting for every Partner according to the Proposition: These Products added together, shall be the first Number of the *Rule of Three*: And the Sum of the Products of each Partner's Stock, multiplied by his respective Times,

Times shall be the third Numbers. The second Number shall be the Gain or Loss as before.

Example. *A*, *B*, and *C*, trade together for 12 Months: *A* putteth in presently 40 *l.* and 4 Months after 30 *l.* more, and 3 Months after 20 *l.* more. *B* at first putteth in 100 *l.* but 3 Months after taketh away 20 *l.* and 5 Months after 20 *l.* more. *C* layeth down first 60 *l.* and five Months after taketh away 10 *l.* but 3 Months after putteth in 20 *l.* They gain 500 *l.* what is each Man's Part thereof? *Q. Of each Partner's Gain.*

Answ. *A* $164\frac{164}{249}$ *l.* *B* $188\frac{188}{249}$ *l.* and *C* $146\frac{146}{249}$ *l.* Answer.

Stock. *Time.*

$$A\begin{cases} 40 \times 4 = 160 \\ +\ 30 \\ \overline{70} \times 3 = 210 \\ +\ 20 \\ \overline{90} \times 5 = \underline{450} \\ \qquad\qquad 820 \end{cases} \qquad B\begin{cases} 100 \times 3 = 300 \\ -\ 20 \\ \overline{80} \times 5 = 400 \\ -\ 20 \\ \overline{60} \times 4 = \underline{240} \\ \qquad\qquad 940 \end{cases} \qquad C\begin{cases} 60 \times 5 = 300 \\ -\ 10 \\ \overline{50} \times 3 = 150 \\ +\ 20 \\ \overline{70} \times 4 = \underline{280} \\ \qquad\qquad 730 \end{cases}$$

A 820
B 940
C 730
2490

As 2490 . 500 :: 820 . $164\frac{164}{249}$ *l.*
82
41000

$\frac{41000}{249}\left(164\frac{164}{249}\right.$ *l.* *A.*

As 2490 . 500 :: 940 . $188\frac{188}{249}$
94
47000

$\frac{47000}{249}\left(188\frac{188}{249}\right.$ *l.* *B.*

As 2490 . 500 :: 730 . $146\frac{146}{249}$
73
36500

$\frac{36500}{249}\left(146\frac{146}{249}\right.$ *l.* *C.*

To resolve the Propositions of the next Sort; multiply the several Gains or Loss by the respective Times: the Total of these Products place for the first Number; the general Stock for the second Number; and the particular Products for the third Numbers: And proceed as before by the *Rule of Three.* *To resolve the second Sort.*

Example. *A*, *B*, and *C*, together hire certain Pasture-Land for 30 *l.* Rent: And besides their joint Stock of Sheep, equal in Number, *A* feedeth there at first 20 Oxen 80 Days; and taking them away, putteth in 100 other Oxen, which he keepeth there 14 Days. *B* at first putteth in 40 Oxen 50 Days; and removing them, afterward putteth in 16 Oxen for 100 Days. *C* putteth to Pasture only 70 Oxen, and keepeth them there, without alteration, 60 Days: what part of the Rent shall each Man pay? *Q. Of the Pasturage of Cattel, what Rent each shall pay.*

Answ. *A* 8 *l.* 6 *s.* 8 *d.* *B* 10 *l.* and *C* 11 *l.* 13 *s.* 4 *d.* Answer.

Oxen. *Days.*

$$A\begin{cases} 20 \times 80 = 1600 \\ 100 \times 14 = \underline{1400} \\ \qquad\qquad 3000 \end{cases} \qquad B\begin{cases} 40 \times 50 = 2000 \\ 16 \times 100 = \underline{1600} \\ \qquad\qquad 3600 \end{cases} \qquad C\ 70 \times 60 = 4200$$

A 3000
B 3600
C 4200
10800

As 108 . 30 *l.* :: 30 . $8\frac{1}{3}$ *l.*
30
900

$\frac{900}{108}\left(8\frac{1}{3}\right.$ *l.* *A.*

As 108 . 30 :: 36 . 10
36
1080

$\frac{1080}{108}\left(10\right.$ *l.* *B.*

$$\text{As } 108 \;.\; \overset{l.}{30} \;::\; 42 \;.\; \overset{l.}{11\tfrac{2}{3}}$$

$$\begin{array}{r} 42 \\ \hline 1260 \end{array} \qquad \begin{array}{r} 7 \\ \cancel{8}|2 \\ \cancel{1}\cancel{2}\cancel{6}\cancel{0} \\ \hline 108 \end{array} \Big(11\tfrac{2}{3}\ \overset{l.}{} C.$$

To resolve the third Sort.

To resolve the Propositions of the latter sort, where the Time or Stock is diversly given, as before in 2 §. of *Fellowship*, where the Time is simply given: Let the Numbers be orderly disposed, and the Question committed to the *Rule of five Numbers*, *Direct* or *Indirect*, as the Nature of the Question requireth.

Q. Of the Pasturage of Cattel, what one shall pay.

Example. *A* hired Pasturage for 56 Bullocks 150 Days, and was to pay therefore 5 *l.* but meeting with a Market at 90 Days end, selleth off 40, and keeping the Remainder, taketh in 10 Bullocks of *B* at one time for 10 Days, and another time 5 Bullocks for 12 Days; and hireth out Pasturage to *C* for 36 Bullocks for 60 Days: what shall *C* pay for the same?

Answer.

Answ. $1\frac{2}{7}$ *l.* For if 5 *l.* buy 150 Days Pasturage for 56 Bullocks, then will $1\frac{2}{7}$ *l.* buy 60 Days Pasturage for 36 Bullocks, by the *Direct Rule of five Numbers.*

$$\text{As } \overset{Bull.}{56} \;.\; \overset{Days.}{150} \;.\; \overset{l.}{5} \;::\; \overset{Bull.}{36} \;.\; \overset{Days.}{60} \;.\; \overset{l.}{1\tfrac{2}{7}}$$

$$84,00 \qquad \begin{array}{r} 300 \\ \hline 108,00 \end{array} \qquad \begin{array}{r} |24 \\ \cancel{1}\cancel{0}8 \\ \hline 84 \end{array} \Big(\overset{l.}{1\tfrac{2}{7}}$$

If the Question had been, what Cattel to keep.

But if the Paiment of *C* $1\frac{2}{7}$ *l.* had been given, and the Question had been, How many Bullocks he should have depastured there, then the Question had been resolved by the *Indirect Rule of five Numbers.* If 5 *l.* buy 150 Days Pasturage for 56 Bullocks: what will $1\frac{2}{7}$ *l.* buy for 60 Days?

Answer.

Answ. Pasturage for 36 Bullocks.

$$\text{As } \overset{Bull.}{56} \;.\; \overset{Days.}{150} \;.\; \overset{l.}{5} \;::\; \overset{Days.}{60} \;.\; \overset{l.}{1\tfrac{2}{7}} \;.\; \overset{Bull.}{36}$$

$$\begin{array}{r} 150 \\ 56 \\ \hline 900 \\ 750 \\ \hline 8400 \\ 2400 \\ \hline 108,00 \end{array}\ 1\tfrac{2}{7} \qquad \begin{array}{r} 60 \\ 5 \\ \hline 3,00 \end{array} \qquad \begin{array}{r} \cancel{1} \\ \cancel{1}08 \\ \hline 3 \end{array} \Big(36 \text{ Bullocks.}$$

Questions may be various.

Here may be a Mixture of the Propositions, as in the second Section before.

Fellowship, *with diversity of Parts, of 2 sorts.*

§. 4. *Fellowship* with diversity of Parts, is either when a certain Sum is to be divided among several sorts of Partners, so as Parcel of the Partners have their Dividend in proportion to other Parcel of them: Or else when a certain Sum is to be divided among single Partners, according to the *Ratio* of their single parts: This latter Sort is called *Fractionary Fellowship*, and stored with variety of Examples.

Latter, how called.

To resolve the first.

To resolve the Propositions of the first Sort, multiply the Number of each Parcel of Partners by the Proportional Part each Partner is to receive, and the Total of these Products shall be the first Number of the *Rule of Three*; the Sum to be divided the Second; and the several Products the several third Numbers: And proceed as before by the *Rule of Three.*

Q. Of the Expences of Canons and Vicars.

Example. Suppose in a Cathedral were 30 Canons and 40 Vicars, which have allowed to spend 360 *l.* *per Annum*: But every Canon is to have 5 *l.* to the Vicar's 3 *l.* How much then are their Annual Expences severally?

Answer.

Answ. The Canons 200 *l.* and the Vicars 160 *l.*

Canons $30 \times 5 = 150$
Vicars $40 \times 3 = 120$
270

As $270 \, l. \cdot 360 :: 150 \cdot 200 \, l.$

$360 \times 15 = 1800$; $1800 \times 3 = 5400$; $\frac{5400}{27} = 200$ *l.* Canons.

As $270 \cdot 360 :: 120 \cdot 160$

$360 \times 12 = 720$; $720 \times 6 = 4320$; $\frac{4320}{27} = 160$ *l.* Vicars.

To resolve the Propositions of the latter Sort, take a Number that will equally divide by all the Consequents or Denominators; or to find one, multiply all the Denominators or Consequents one into another, or so many of them as be not equal Halfs of some other of them, and divide this ultimate Product, or the Number first taken, by the Consequents severally, and multiply these Quotients by the Antecedents or Numerators: Add all these several Products (or the Quotients where one is Antecedent) into one Total, which with the several Addends abbreviate if it may be: Then with the least Terms, or with the Total, and the several Products or Quotients, if 1 be Antecedent, where they will not be abbreviated, proceed as in the other sort of *Fellowship* to the Work of the *Rule of Three*. What other Necessaries occur are inserted, with the several Examples following. *To resolve the second Sort.*

Example 1. A certain Man bequeathed to charitable Uses 100 *l.* to be divided in such Proportion, that *A* should have $\frac{1}{2}$, *B* $\frac{1}{3}$, and *C* $\frac{1}{4}$: how should the 100 *l.* be divided according to the Intent of the Testator? *Q. Of a Legacy.*

Answ. To *A* $46\frac{2}{13}$ *l.* *B* $30\frac{10}{13}$ *l.* and *C* $23\frac{1}{13}$ *l.* Answer.

Here, and in all Questions of like sort, the *Ratio* of the *Data* is to be minded, for otherwise the Question were impossible, seeing $\frac{1}{2}$ and $\frac{1}{3}$ added to $\frac{1}{4}$, after the manner of Fractions, make more than the Whole, the $\frac{1}{2}$ of 100 *l.* being 50 *l.* the $\frac{1}{3}$ more $33\frac{1}{3}$ *l.* with $\frac{1}{4}$ which is 25 *l.* added together, make in all 108 *l.* 6 *s.* 8 *d.* But the Intent of the Question being, that every time *A* had $\frac{1}{2}$ Pound, or Shilling, *B* should have $\frac{1}{3}$ of a Pound or Shilling, and *C* $\frac{1}{4}$; So as to every 6 Pence *A* had, *B* should have 4 Pence, and *C* 3 Pence: Wherefore taking $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of 12, a Number which will be equally parted by 2, 3, 4. or multiplying those Consequents together, they make 24; which divided by 2, 3, and 4 severally, giveth 12, 8, and 6: these added together (their Antecedents being Units) make 26; which abbreviated with them, make 6, 4, 3, and together 13, like to the Parts of 12. This 13 shall be the first Number, and the other the third Numbers of the *Rule of Three*, as followeth. *The Ratio of the Data to be noted, and the Intent of the Question. How to proceed.*

$\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4}$; $2 \times 4 = 8$; $8 \times 3 = 24$

$\frac{24}{2} = 12$, $\frac{24}{3} = 8$, $\frac{24}{4} = 6$

12	6	*A*
8	4	*B*
6	3	*C*
26	13	

$\frac{12}{2} = 6$, $\frac{12}{3} = 4$, $\frac{12}{4} = 3$; together 13

As $13 \cdot 100 \, l. :: 6 \cdot 46\frac{2}{13} \, l.$ $\quad 100 \times 6 = 600$; $\frac{600}{13} = 46\frac{2}{13}$ *l.* *A.*

As $13 \cdot 100 :: 4 \cdot 30\frac{10}{13}$ $\quad 100 \times 4 = 400$; $\frac{400}{13} = 30\frac{10}{13}$ *l.* *B.*

As $13 \cdot 100 :: 3 \cdot 23\frac{1}{13}$ $\quad 100 \times 3 = 300$; $\frac{300}{13} = 23\frac{1}{13}$ *l.* *C.*

Example

Q. Of a Ship, what each Partner paid.

Example 2. *A*, *B*, *C*, and *D*, bought a Ship for 890 *l.* and were to have their Parts according to these Proportions, *viz.* *A* $\frac{1}{2}$ and $\frac{1}{10}$, *B* $\frac{1}{4}$ and $\frac{1}{10}$, *C* $\frac{1}{6}$, and *D* $\frac{1}{20}$: what must each Man pay?

Answer.

Answ. *A* $457\frac{5}{7}$ *l.* *B* 267 *l.* *C* $127\frac{1}{7}$ *l.* and *D* $38\frac{1}{7}$ *l.*

What to be done when one hath more Parts than one.

Here, and in such Questions where more Parts or Proportions than one belong to one Partner, I may, after the manner of Fractions, add them into one, and work as before, or otherwise: After Division of the Number taken, or the ultimate Product of the Consequents, add the several Quotients belonging to one Partner together for his third Number of the *Rule of Three*; and in this Example, because 60 is a Number that will be equally divided by all the Consequents, as into Halfs, Fourths, Sixths, Tenths and Twentieth Parts, I may take 60 to divide by the Consequents, or in multiplying the Consequents to find a Number, I may omit 2, because the half of 4, and 10 the half of 20, and take only 4, 6, and 20.

$$\overbrace{\tfrac{1}{2}+\tfrac{1}{10}=\tfrac{3}{5}}^{A}\ .\ \overbrace{\tfrac{1}{4}+\tfrac{1}{10}=\tfrac{7}{20}}^{B}\ .\ \overbrace{\tfrac{1}{6}}^{C}\ .\ \overbrace{\tfrac{1}{20}}^{D}$$

Thus, or thus, $\frac{60}{5}(12 \times 3 = 36$ *A.* $\quad \frac{60}{20}(3 \times 7 = 21$ *B.* $\quad \frac{60}{6}(10$ *C.* $\quad \frac{60}{20}(3$ *D.*

36	*A.*
21	*B.*
10	*C.*
3	*D.*
70	

$$A\begin{cases}\frac{60}{2}(30\\ \frac{60}{10}(6\end{cases}\Big\}36 \qquad \text{As } 70 : 890 :: 36 : 457\tfrac{5}{7}. \qquad \begin{array}{r}890\\ 36\\ \hline 5340\\ 2670\\ \hline 3204,0\end{array} \qquad \frac{3204}{7}\left(457\tfrac{5}{7}\right. \ l.\ A.$$

$$B\begin{cases}\frac{60}{4}(15\\ \frac{60}{10}(6\end{cases}\Big\}21 \qquad \text{As } 70 : 890 :: 21 : 267. \qquad \begin{array}{r}890\\ 21\\ \hline 890\\ 1780\\ \hline 1869,0\end{array} \qquad \frac{1869}{7}\left(267\right. \ l.\ B.$$

$$C\quad \frac{60}{6}(10 \qquad \text{As } 70 : 890 :: 10 : 127\tfrac{1}{7}. \qquad \begin{array}{r}890\\ 10\\ \hline 890,0\end{array} \qquad \frac{890}{7}\left(127\tfrac{1}{7}\right. \ l.\ C.$$

$$D\quad \frac{60}{20}(3 \qquad \text{As } 70 : 890 :: 3 : 38\tfrac{1}{7}. \qquad \begin{array}{r}890\\ 3\\ \hline 267,0\end{array} \qquad \frac{267}{7}\left(38\tfrac{1}{7}\right. \ l.\ D.$$

70

Q. Of a Sum to be paid.

Example 3. The Sum of 300 *l.* was to be paid by *A*, *B*, and *C*, in such Proportions, that *A* was to pay 6 *l.* more than $\frac{1}{4}$, *B* $\frac{1}{6}$ and 12 *l.* over, and *C* 8 *l.* less than $\frac{1}{3}$: what must each Man pay?

Answer.

Answ. *A* $102\frac{2}{3}$ *l.* *B* $76\frac{4}{9}$ *l.* and *C* $120\frac{8}{9}$ *l.*

When there is Money overplus, or to be abated.

Here, and in those Questions where there is Overplus-Money, or Money to be abated; From the Sum to be divided, the Overplus-Money is to be substracted, and to that Sum the Abatement must be added, and then proceed as before; and to the respective Portions obtained by the *Rule of Three*, add the Overplus-Money to be added, and substract the other to be substracted.

$$\frac{1}{2}\ \frac{1}{3}\ \frac{2}{3}\quad \underbrace{}_{6}\qquad \frac{6}{2}(3\qquad \frac{6}{3}(2\qquad \frac{6}{3}(2 \times 2 = 4\qquad \begin{array}{ll} 3 & A.\\ 2 & B.\\ 4 & C.\\ \hline 9 & \end{array}$$

A 6 *l.* } +			*l.*			*l.*		
B 12	As	9 .	290	::	3 .	$96\frac{2}{3}$		$\frac{870}{9}(96\frac{2}{3}$ *l.* + 6 = $102\frac{2}{3}$ A.
18			3					
C 8 −			870					
10								
300−10=290	As	9 .	290	::	2 .	$64\frac{4}{9}$		$\frac{580}{9}(64\frac{4}{9}$ *l.* + 12 = $76\frac{4}{9}$ B.
			2					
			580					
	As	9 .	290	::	4 .	$128\frac{8}{9}$		$\frac{1160}{9}(128\frac{8}{9}$ *l.* − 8 = $120\frac{8}{9}$ C.
			4					
			1160					

Example 4. Three Owners of a Ship set her forth; the Charge whereof *A* payeth $\frac{1}{4}$ Part, *B* 300 *l.* and *C* $\frac{9}{16}$ Parts of the Whole: how much Money paid *A* and *C* for their Parts? and what Part of the Ship had *B*? and how much was the whole Charge? *Q. of the Charge to set out a Ship.*

Answ. *A* paid 400 *l.* *C* 900 *l.* *B* had $\frac{3}{16}$ Parts of the Ship, and the whole Charge was 1600 *l.* Answer.

Here, and in others alike, where several Questions are interwoven, several things are to be done in order to their Resolution: For first the Parts of *A* and *C* being given, added together, and the Total substracted from an Unit, or the Whole, discovers the Part of *B*; that known, seeing the Charge thereof is given, the Charge of the other Parts will be had by the *Rule of Three*: which when found, added to the Charge of *B*, answers the whole Charge. *How to proceed when Questions are interwoven.*

Parts of $\overset{A.}{\frac{1}{4}} + \overset{C.}{\frac{9}{16}} = \overset{Sum.}{\frac{13}{16}}$ Unit, or $\frac{16}{16} - \frac{13}{16} = \overset{Remain.}{\frac{3}{16}}$ Part of *B*.

As $\frac{3}{16}$. $\overset{l.}{300}$ :: $\frac{1}{4}$. $\overset{l.}{400}$

$$\frac{3}{16}\Big)\frac{300}{4}\Big(\frac{400}{1}\ \text{Charge of } A \text{ for } \tfrac{1}{4}.$$

Particular Charges.

A for $\frac{1}{4}$. 400 *l.*
B for $\frac{3}{16}$. 300 *l.*
C for $\frac{9}{16}$. 900 *l.*

Total 1600 *l.* Charge.

As $\frac{3}{16}$. 300 :: $\frac{9}{16}$. 900.

$$\frac{3}{16}\Big)\frac{2700}{16}\Big(\frac{900}{1}\ \text{Charge of } C \text{ for } \tfrac{9}{16}.$$

Example 5. Three Merchants have gained 100 *l.* which they divide in such manner, that the $\frac{1}{2}$ Part of *A* was as much as $\frac{5}{8}$ of what *B* had; and $\frac{1}{8}$ Part of *B* was equal to $\frac{1}{2}$ Part of *C*: how much had each Man for his Part? *Q. Of Gain parted.*

Answ. *A* 50 *l.* *B* 40 *l.* *C* 10 *l.* Answer.

Here, and in such others as before, any Number may be taken that will equally be divided by the Denominators or Consequents; and if none come to hand, by their Multiplication as before such a Number may be had: then accounting that for *A*, take the Parts thereof for *B* and *C*; and the other Partners, if more, according to the Proposition: add all these Numbers together for the first Number *The first Number how gotten.*

of the *Rule of Three*, the Number firſt taken, and the Parts thereof ſhall be the ſeveral third Numbers, and the Sum to be divided the Second, as before.

Denominators $2 \times 8 = 16$.

A 16 *l.* then $\frac{1}{2}$ is 8 $= \frac{5}{8}$ B.	And as $\frac{5}{8}$ *B* . 8 *l.* :: 1 *B* . $12\frac{4}{5}$ *l.*	$\frac{5}{8}) \frac{8}{1} (\frac{64}{5}$ B.	
B $12\frac{4}{5}$ then $\frac{1}{8}$ is $1\frac{3}{5} = \frac{1}{2}$ C.	And as $\frac{1}{2}$ *C* . $1\frac{3}{5}$ *l.* :: 1 *C* . $3\frac{1}{5}$ *l.*	$\frac{1}{2}) \frac{8}{5} (\frac{16}{5}$ C.	

C $3\frac{1}{5}$
32

As 32 . 100 *l.* :: 16 . 50 *l.*
16
1600

$\frac{1600}{32}$ (50 *l.* A.

As 32 . 100 :: $12\frac{4}{5}$. 40
$12\frac{4}{5}$
1200
80
1280

$\frac{1280}{32}$ (40 *l.* B.

As 32 . 100 :: $3\frac{1}{5}$. 10
$3\frac{1}{5}$
300
20
320

$\frac{320}{32}$ (10 *l.* C.

Q. Of a Legacy. *Example* 6. A certain Man on his Death-bed, by his laſt Will and Teſtament, bequeathed 360 *l.* to be thus diſpoſed, *viz.* If his Wife, being then with Child, brought forth a Son, then to his Son the $\frac{2}{3}$, and his Wife $\frac{1}{3}$: But if ſhe brought forth a Daughter, then his Wife to have $\frac{2}{3}$, and his Daughter $\frac{1}{3}$. And it happened that the Woman brought forth both a Son and a Daughter. Now the Queſtion is, how the 360 *l.* ſhould be divided, that the Will of the Teſtator ſhould be fulfilled?

Anſwer. *Anſw.* To the Son $170\frac{10}{19}$ *l.* to the Mother $113\frac{13}{19}$ *l.* and to the Daughter $75\frac{15}{19}$ *l.*

How to get the Proportional Numbers. For here, and in ſuch-like Queſtions, Numbers are to be ſought out that bear ſuch Proportion one to another, as the ſeveral Parts propounded, the Intention of the Teſtator being, that the Mother ſhould not have $\frac{1}{3}$ to the Son's Part, and $\frac{2}{3}$ to the Daughter's Part both, ſeeing theſe Parts added together would be $\frac{3}{3}$, and ſo more than the Son's Part: But as the Son was to have $\frac{2}{3}$, ſo the Mother $\frac{1}{3}$, that is proportionally as 3 to 2; whereby the Son is intended to have as much as the Mother, and half as much more; and the like muſt the Mother have to the Daughter. Therefore to find the Proportional Numbers of this Sort, multiply the firſt and laſt Terms or Parts together, as here 2 by 3, which is 6, and this ſhall be the middle Number for the Mother's Portion. Then multiply this middle Number 6 by the leſſer Term 2, and divide the Product 12 by the greater Term 3, and the Quotient ſhall be the leſſer Term, and being here 4, ſhall be for the Daughter's Portion. Again, multiply that middle Number by the greater Term, and divide the Product by the Leſſer, for the greateſt Portion that is here for the Son; ſo 6 multiplied into 3 produceth 18, and this divided by 2 gives 9 in the Quotient. And theſe Numbers found, are the ſeveral third Numbers of the *Rule of Three*, the Total of them the Firſt, and the ſecond Number the Sum to be divided as before.

$2\times3=$ 6 Mother.
$6\times2=\frac{12}{3}(4$ Daughter.
$6\times3=\frac{18}{2}(9$ Son.
19

As 19 . 360 :: 9 . $170\frac{10}{19}$ l.
9
3240

As 19 . 360 :: 6 . $113\frac{13}{19}$
6
2160

As 19 . 360 :: 4 . $75\frac{15}{19}$
4
1440

3240 ÷ 19 = $170\frac{10}{19}$ l. Son.
2160 ÷ 19 = $113\frac{13}{19}$ l. Mother.
1440 ÷ 19 = $75\frac{15}{19}$ l. Daughter
Proof 360

The proper Proof of *Fellowship* (other than such Propositions as are determined by some single Operation of the *Rule of five Numbers*, which have their Proof in common therewith) is to add into one Total the several Quotients that answer the Proposition; which if the Work be right, will return the second Number. As in the last Example $170\frac{10}{19}$ *l.* $113\frac{13}{19}$ *l.* and $75\frac{15}{19}$ *l.* added together, make 360 *l.* the Sum to be divided. *Proof of Fellowship.*

CHAP. IX.

ALLIGATION.

LEAVING *Fellowship*, wherein was a Mixture of Partners and their Stock; the next Subject Derivatives deal with, is the Mixture of Merchandises, as Corn, Wine, Wool, Metals, Medicines, *&c.* under the Title of *Alligation.* *Alligation mixeth divers Merchandises.*

Alligation is of two kinds, Medial and Alternate: Medial properly seeketh a Mean in the Price, Quantity or Quality between the Extreams. And Alternate altereth the placing of the Differences falling out between the mean Price and the Extreams: And both proposeth the Numbers and Quantities Homogeneal, or reduceth them into such, and properly intendeth simple Mixtures, or those done but once; those often repeated being either Figurals, or continued Proportions; of which more in the last Chapter of this Part, and the fifth Chapter of the next, in Section 6. *Two sorts of Alligation.* *Numbers to be Homogeneal.*

Medial Alligation contains six Propositions. *Medial the Propositions.*

Prop. 1. By the Quantities to be mixed, and the particular Prices, to find the Price or Value of some part of the Mixture. *1. To find the Price of part.*

Multiply the Quantities to be mixed severally by their own Prices, and divide the Sum of these Products by the Sum of the Quantities mixed. *Rule.*

Example 1. A Merchant would mix 100 Bushels of Rye at 4 *s.* the Bushel, with 40 Bushels of Barley at 3 *s.* 6 *d.* the Bushel, and 60 Bushels of Wheat at 6 *s.* the Bushel: and know what one Bushel of that Miscellane would be worth. *Q. of Miscellane.*

Answ. 4 *s.* 6 *d.* For the Quantities 100, 40, and 60, added, make 200, and these Quantities multiplied severally by the particular Prices, make 400, 140, and 360; which Products added, make 900; this divided by 200, gives $4\frac{1}{2}$ *s.* as before. *Answer.*

Quantities.	*Bushels.*	*Prices.* *s.*	
Rye	100	× 4	= 400
Barley	40	× $3\frac{1}{2}$	= 140
Wheat	60	× 6	= 360
	2,00		9,00

Analogy.

As 200 . 900 :: 1 . $4\frac{1}{2}$

9 ÷ 2 = $4\frac{1}{2}$ s. Bushel.

Q. *Of the Worth of a Cask of Wine.*

Example 2. Five Casks that hold 60 Gallons apiece, are to be filled with Wine, *viz.* 75 Gallons of Sack at 4 *s. per* Gallon, and 225 Gallons of White-Wine at 2 *s. per* Gallon: what shall one Cask of this mixed Wine be worth?

Answer. *Answ.* 7 *l.* 10 *s.* For after the Value of 1 Gallon of the Mixture is found, as before, to be 2 *s.* 6 *d.* the Value of 60 Gallons, the Content of 1 Cask, is had by Multiplication.

	Quantit. *Gallons.*		*Prices.* *s.*				*Cask.*
Sack	75	×	4	= 300	(1		60 Gallons.
White	225	×	2	= 450	75	2½ Gallon.	2½
	300			750	30		120
							30
							150
							l. 7:10 *s.*

2. *To find the Quantity.*

Prop. 2. By the particular Prices of the Quantities, and Sum paid or received for a Mixture bought or sold; to find what Quantity of each kind was bought or sold.

Rule. Divide the Sum paid or received for the Mixture bought or sold, by the Sum of the particular Prices.

Q. *Of Pieces of Money sent for to the Mint.*

Example 1. A certain Noble-man sent his Servant with the King's Majesties Warrant to the Mint-master for 4000 *l.* and he must bring it in Pieces of 12 *d.* 6 *d.* 4 *d.* 2 *d.* 1 *d.* and he must bring of each sort of Pieces a like Number: how many of each sort must he bring?

Answer. *Answ.* 38400 Pieces of each sort: For 4000 *l.* brought into Pence, because the other Pieces are in Pence, the Result 960000 *d.* divided by 25, that is $12 + 6 + 4 + 2 + 1$, giveth the Answer aforesaid.

Prices.	*Sum paid.*		
12 *d.*	4000 *l.*	22	
6	240	960000	38400 Pieces.
4	160000	25	
2	8000	25	
1	960000 *d.*	25	
25			

Q. *Of Spice sold, how much of a Sort.*

Example 2. A Grocer sold four sorts of Spice, of each a like Quantity, but at several Rates; *viz.* Large Mace at 8 *s.* 4 *d. per* ℔. Cinamon at 6 *s.* 8 *d. per* ℔. Nutmegs at 5 *s. per* ℔. and Ginger at 2 *s. per* ℔. For what he sold, he received 23 *l.* 2 *s.* The Question is, how many Pounds of each sort he sold to make up the said Sum of 23 *l.* 2 *s.*

Answer. *Answ.* 21 ℔. of each sort: For the Prices given added make 22 *s.* and 23 *l.* 2 *s.* reduced into Shillings that they may be Homogeneal, make 462 *s.* which divided by 22 *s.* give 21 ℔ as before.

	Prices.	*Sum received.*		
Large Mace	8 *s.* : 4 *d.*	23 *l.* 2 *s.*	2	℔.
Cinamon	6 : 8	20	462	21 of each Sort.
Nutmegs	5 : 0	460	22	
Ginger	2 : 0	2	22	
	22 : 0	462		

3. *To increase or lessen a Mixture.*

Prop. 3. By the Quantities of a Mixture, to augment or diminish the Mixture proportionally.

Rule. Sum up the Quantities, and then by the *Rule of Three*, as that Sum is to the Augmentation or Diminution: So is the Quantity of each Parcel of the Mixture, to the Quantity of the Mixture desired.

Q. *Of increasing an Ointment.*

Example 1. In the Ointment called *Unguentum album Camphoratum*, there is put to Oil of Roses ℥ 12, white Wax ℥ 3, Ceruse ℥ 6, and Camphire beaten with Oil of Roses ʒ 2; which reduced all into Drams, make 96 . 24 . 48 . 2. and in the Total 170. And if I would make the Quantity to contain 210 Drams:

Drams: how much of each Ingredient must be taken?

Answ. Oil $118\frac{10}{17}$ ʒ, Wax $29\frac{11}{17}$ ʒ, Ceruse $59\frac{5}{17}$ ʒ, Camphire $2\frac{8}{17}$ ʒ. Answer.

As 170 . 210 :: 96 . $118\frac{10}{17}$ Oil.

```
 96
----
1260        1(1
1890        34(0
-----       2016(118 10/17
20160       17
```

As 170 . 210 :: 24 . $29\frac{11}{17}$ Wax.

```
 24
----
 840        (1
 420        16(1
----        504(29 11/17
5040        17
```

As 170 . 210 :: 48 . $59\frac{5}{17}$ Ceruse.

```
  48
 ----
 1680       15(5
  840       1008(59 5/17
 -----      17
 10080
```

As 170 . 210 :: 2 . $2\frac{8}{17}$ Camphire.

```
  2
 ---
 420        (8
            42(2 8/17
            17
```

Example 2. A Pectoral Pouder of 10 ℔. is made up with Sugar ℔ 6, Licorish ℔ 2, Anni-seeds ℔ $1\frac{1}{2}$, and Fennel-seeds ℔ $\frac{1}{2}$: But I would make no more than 4 ℔ of the Pouder; how much of the several Ingredients must be taken? *Q. Of lessening a Pouder.*

Answ. Sugar $2\frac{2}{5}$ ℔, Licorish $\frac{4}{5}$ ℔. Anni-seeds $\frac{3}{5}$ ℔, and Fennel-seeds $\frac{1}{5}$ ℔. Answer.

As 10 . 4 :: 6 . $2\frac{2}{5}$ Sugar.

```
    6
10)24(2 2/5
```

As 10 . 4 :: 2 . $\frac{4}{5}$ Licorish.

```
    2
10)8(4/5
```

As 10 . 4 :: $1\frac{1}{2}$. $\frac{3}{5}$ Anniseeds.

```
   1 1/2
   -----
    4
    2
10)6(3/5
```

As 10 . 4 :: $\frac{1}{2}$. $\frac{1}{5}$ Fennel-seeds.

```
   1/2
10)2(1/5
```

Prop. 4. By the Qualities or Nature of the several Ingredients in a Mixture, to find the Temperament or Emergent Quality of the Mixture. *4. To find the Temperament of a Mixture.*

Dispose the Quantities of the Mixture severally in Rows, setting orderly them their several Qualities; multiply each Quantity by its own Quality. And if the Qualities of the Ingredients be contrary, substract the contrary Qualities so multiplied one from the other, as Hot from Cold, Moist from Dry, &c. and set down the Difference of the Products. Then as the Sum of all the Quantities is, to the Difference of the Products, or the Products where no contrary Qualities are: So is an Unit to the Quality emergent, and always of the same Kind with the greater Product where the Qualities are contrary. Rule.

Example 1. A Goldsmith mixeth 20 Portions of Silver of 6 ℥ fine, with 5 of 8 ℥ fine, 5 of 10 ℥ fine, and 10 Portions of Copper; what Fineness shall the melted Mass be of? *Q. Of the fineness of Metal.*

Answ. $5\frac{1}{4}$ ℥. Answer.

	Quantities. *Portions.*		*Fineness.* ℥.		
Silver	20	×	6	=	120
Silver	5	×	8	=	40
Silver	5	×	10	=	50
Copper	10	×	0	=	00
	40				210

As 40 . 210 :: 1 . $5\frac{1}{4}$

```
21  ℥.
--(5 1/4 Fine.
 4
```

Example 2. If the Species called *Dianthus*, be made according to the *London Dispensatory*, and the Qualities of the Simples taken according to *Sennertus* in his *Institutes*, *Lib.* 5. *Par.* 1. *Chap.* 3. *Parkinson* in his *Herbal*, and other approved Authors: And it be enquired to know the Quality, Emergent or Temperament of the Composition; then disposing orderly the Quantities and Qualities as below, *Q. Of the Quality of Dianthus.*

below,

below, and multiplying respectively the one by the other, and substracting the contrary Qualities one from the other, the Remains left to be divided by the Total of the Quantities, declare the Medicine of a fine Temperament, *viz.* Hot in the first Degree and somewhat above, and Dry not full a Degree.

Answer.

	Ingredients.	*Quantities.*		*Qualities.* hot.	cold.	moist.	dry.	*Products.*			
Dianthus	Rosemary Flowers	℥ 8, or ℈	24 ×	2	0	0	2	48	0	0	48
	Red Roses	℥ 6, or .	18 ×	0	1	0	1	0	18	0	18
	Violets	℥ 6, or .	18 ×	0	1	2	0	0	18	36	0
	Licorish	℥ 6, or .	18 ×	1	0	1	0	18	0	18	0
	Cloves		4 ×	3	0	0	3	12	0	0	12
	Indian Spiknard		4 ×	1	0	0	2	4	0	0	8
	Nutmegs		4 ×	2	0	0	2	8	0	0	8
	Galanga		4 ×	3	0	0	3	12	0	0	12
	Cinamon		4 ×	2	0	0	2	8	0	0	8
	Ginger		4 ×	3	0	0	3	12	0	0	12
	Zedoary		4 ×	2	0	0	2	8	0	0	8
	Mace		4 ×	2	0	0	2	8	0	0	8
	Wood of Aloes		4 ×	2	0	0	2	8	0	0	8
	Cardamoms the Less		4 ×	3	0	0	3	12	0	0	12
	Anniseeds		4 ×	2	0	0	1	8	0	0	4
	Dillseeds		4 ×	2	0	0	3	8	0	0	12
			126					174	36	54	178

Hot. *Cold.*

$174 - 36 = \frac{138}{126}\left(1\tfrac{2}{21}\right.$ Hot. As 126 . 138 :: 1 . $1\tfrac{2}{21}$.

Temperament.

Dry. *Moist.*

$178 - 54 = \frac{124}{126}\left(\tfrac{62}{63}\right.$ Dry. As 126 . 124 :: 1 . $\tfrac{62}{63}$.

When among the Simples there is some Compound.

When in any Composition among the Simples, some compound Ingredient is mixed, then the Temperament of that Compound being first gotten, the other is to be found in like manner. As in Mastick Pills, because among the Simples *Hiera picra* is used, which is a Compound, the Temperament thereof is first found to be Hot in the second Degree, and Dry in the second Degree, and nigh $\frac{3}{4}$ of a Degree more: And then the Temperament of the Pills is found to be almost two Degrees Hot, and above two Degrees and an half Dry.

	Ingredients.	*Quantities.* ℥		*Qualities.* hot	dry.	*Products.*	
Hiera picra simple	Cinamon	6	×	2	2	12	12
	Xylobalsamum	6	×	2	2	12	12
	Roots of *Asarabacca*	6	×	3	3	18	18
	Spiknard	6	×	1	2	6	12
	Mastich	6	×	2	2	12	12
	Saffron	6	×	2	1	12	6
	Aloes $12\frac{1}{2}$ ℥, or	100	×	2	3	200	300
		136				272	372

$\frac{272}{136}\left(2\right.$ Hot.

$\frac{372}{136}\left(2\tfrac{25}{34}\right.$ Dry.

	Ingredients.	*Quantities.* ℥		*Qualities.* hot.	dry.	*Products.*	
Mastich Pills.	Mastich	2	×	2	2	4	4
	Aloes	4	×	2	3	8	12
	Agarick	$1\frac{1}{2}$	×	1	2	$1\frac{1}{2}$	3
	Hiera Simple	$1\frac{1}{2}$	×	2	$2\frac{3}{4}$	3	$4\frac{1}{8}$
		9				$16\frac{1}{2}$	$23\frac{1}{8}$

$\frac{9}{1}\Big)\frac{33}{2}\left(\frac{11}{6}\right(1\tfrac{5}{6}$ Hot.

$\frac{9}{1}\Big)\frac{185}{8}\left(\frac{185}{72}\right(2\tfrac{41}{72}$ Dry.

Prop.

Prop. 5. By the Quantities of a Mixture, to find the particular Quantity of any Ingredient in any Part of the Mixture.

5. To find the Quantity, &c. in a simple Mixture.

If the Mixture be Simple, or but once, then by the *Rule of Three*,

Rule.

As the Total of the Ingredients in the Composition is, to the Quantity of the Dose, (or part of the Mixture proposed): So is the Quantity of the Ingredient proposed in the whole Composition, to the Quantity of the Ingredient in the Dose.

Q. Of Miscellane.

Example. If 700 Bushels of Wheat be mixed with 100 Bushels of Rye: how much Rye is there in one Bushel of that Miscellane?

Answer.

Answ. $\frac{1}{8}$ of a Bushel, that is a Gallon.

	Bushels.
Wheat .	700
Rye .	100
Miscellane .	800

As $800 \,.\, 100 :: 1 \,.\, \frac{1}{8}$

$\frac{1|00}{8|00}$ ($\frac{1}{8}$ Rye.

In a compound Mixture, how best to proceed.

But if the Mixture be compound, that is, often repeated, then the best way is to proceed by figural Proportions, as afterward in the 16*th* Chapter. Otherwise thus, by the *Rule of Three*, proceed to find the Quantity desired after the first Mixture as before. Then proceed accordingly to repeat the like Work upon every Mixture till your Desire be obtained.

Common Way.

Q. Of Wine and Water mixed.

Example. A Merchant hath a Piece of Wine of 128 Gallons, out of which he draweth 16 Gallons, and filleth it up again with Water. Again, he draweth out 16 Gallons, and filleth it up again with Water; and the third time doth the like: how much Wine and Water was at last in the Cask?

Answer.

Answ. $85\frac{3}{4}$ Gallons of Wine, and $42\frac{1}{4}$ Gallons of Water: For by the first Draught there was left but 112 Gallons of Wine in the Cask; which filled up with Water, and 16 Gallons of that Mixture drawn, there was 14 of Wine and 2 of Water drawn out: So upon the second Draught there were but 98 Gallons of Wine in the Cask: Then the Cask filled, there must be 30 Gallons of Water to make up 128. And upon the third Draught there were $12\frac{1}{4}$ Gallons of Wine more drawn out, and $3\frac{3}{4}$ of Water; which $12\frac{1}{4}$ taken from 98, leaves $85\frac{3}{4}$ Gallons of Wine as before, the Residue was Water to fill up the Cask.

	Wine.	
	128	
Wine drawn out	16	Water put in.
Wine remaining	112	at the first Draught.
	14	
Wine remaining	98	at the second Draught.
	$12\frac{1}{4}$	
Wine remaining	$85\frac{3}{4}$	at the third Draught.

Wine. *Water.*
$85\frac{3}{4} + 42\frac{1}{4} = 128$

As $128 \,.\, 16 :: 16 \,.\, 2$

16
96
16
128) 256 (2 Water run out.
14 Wine run out.
16

As $128 \,.\, 30 :: 16 \,.\, 3\frac{3}{4}$

16
180
30
128) 480 ($3\frac{3}{4}$ Water run out.
$12\frac{1}{4}$ Wine run out.
16

Quantity of each in part, how discovered.

Now if the Question had desired to know the Quantity of Wine or Water in any smaller Quantity of the Mixture than the Whole; as suppose in 12 Gallons, then the Analogy is thus: For

Wine;

Wine; As 128 . $85\frac{3}{4}$ :: 12 . $8\frac{5}{128}$

12	
170	(5
859	$\frac{1029}{128}$ ($8\frac{5}{128}$ Gallons in 12.
1029	

Water; As 128 . $42\frac{1}{4}$ :: 12 . $3\frac{123}{128}$

12	
84	123
423	$\frac{507}{128}$ ($3\frac{123}{128}$ Gallons in 12.
507	

6. To find the Quantities mixed, tho unequally.

Prop. 6. By the Total of a Mixture, with the Total Value, and the Values of the ſeveral Ingredients mixed, to find the ſeveral Quantities mixed, though unequally.

Rule.

Multiply the Total of the Mixture by the leaſt Value, ſubſtract the Product from the Total Value, and the Remainer is the firſt Dividend: Then take the ſaid leaſt Value from the greateſt valued Ingredient, and the Remainer is the firſt Diviſor. The Quotient of this Diviſion ſhews the Quantity of the higheſt prized Ingredient, the other is the Complement to the Whole. And when more Ingredients than two are in the Compoſition, the Diviſors are the ſeveral Remains of the leaſt Value taken from the other. The Dividends are the Remains left upon the Diviſions till o remain there; which will be one ſhort of the Number of Ingredients, and this defective Ingredient is to be ſupplied as a Complement. And in *Diviſion* no more muſt be taken in every Quotient, than that there may be left enough for the other Diviſors, and the laſt to leave o remaining.

Example in a Mixture of two Ingredients.

Q. Of Sack mixed.

A Merchant mixeth to the Quantity of 128 Gallons of Sack, which he ſelleth for 37 *l.* 3 *s.* in which Mixture was Malaga at 6 *s.* the Gallon, and Sherry at 4 *s.* the Gallon: how many Gallons of each ſort were in the Mixture?

Anſwer.

Anſw. $115\frac{1}{2}$ Malaga, $12\frac{1}{2}$ Sherry.

Total Mixture. *Total Value.*

128 Gallons { 6 *s.* Malaga. / 4 *s.* Sherry. } 37 *l.* 3 *s.*

128	Gallons	37 *l.* 3 *s.*		(1
4	Diviſor 2	20		$\frac{231}{2}$ ($115\frac{1}{2}$ Malaga.
512		743	Complement	$12\frac{1}{2}$ Sherry.
		512		128
	Dividend	231		

Example, in a Company of three Sexes.

Q. Of Expences of ſeveral Sexes.

Twenty four Perſons, Men, Women, and Children, ſpent 37 *s.* 8 *d.* whereof every Man was to pay 2 *s.* every Woman 18 *d.* and every Child 8 *d.* how many of each Sex were there?

Anſwer.

Anſw. 15 Men, 2 Women, 7 Children.

Company.

Company. 24 *d.* a Man. *Total Expence.*
24 | 18 *d.* a Woman. | 37 *s.* 8 *d.* | 24—8=16 First Divisor.
8 | 8 *d.* a Child. | 12 | 18—8=10 Second Divisor.

192 | 74 | 378 | 452 | 192

First Dividend 260 — $\frac{26(0}{16}$ (15 Men.

Second Dividend $\frac{20}{10}$ (2 Women.

Complement 7 Children.

24

Example in a mixt Sale at four Rates.

A Baker sold 12 Loaves of Bread of four Sorts, for 12 Pence, *viz.* Twopenny-Loaves, Penny-Loaves, Halfpenny-Loaves, and Farthing-Loaves: how many of each Sort were there? Q. *Of Loaves of Bread.*

Answ. 4 Twopenny-Loaves, 2 Penny-Loaves, 2 Halfpenny-Loaves, and 4 Farthing-Loaves. Answer.

Loaves. *q.* *Total Sum.*
8 Twopenny. 12 *d.* | 8 | 7 First Divisor.
12 | 4 Penny. | 4 | 4 | 3 Second Divisor.
1 | 2 Halfpenny. | 48 *q.* | 2 | 1 Third Divisor.
12 | 1 Farthing. | 12 | 1

First Dividend 36 — $\frac{36}{7}$ (4 Twopenny.

Second Dividend $\frac{8}{3}$ (2 Penny.

Third Dividend $\frac{2}{1}$ (2 Halfpenny.

Complement 4 Farthing.

12

In the Questions falling under this Proposition, two things are to be noted. *What to be noted here.*

First, When there is no definite Number of the Species allotted to be had, (as was in the last Example, where the Demand was limited to four Sorts of Loaves) there is no certainty as to the particular Numbers desired: But the Question may oftentimes be resolved by other Numbers than those found out as above. For in the second Example above, if 10 Men, 10 Women, and 4 Children, spend at the Rates aforesaid, the Sum of 37 *s.* 8 *d.* may be paid by them as exactly, as if there be 15 Men, 2 Women, and 7 Children. 1. *If the Number allotted be certain or not.*

Secondly, Sometimes a Question is so propounded, that before a final Resolution thereof, some Operations of the *Rule of Three* must precede. 2. *If other Work must precede Resolution.*

As a Merchant sold 24 ℔ of Pepper, Ginger and Sugar for 48 *s.* *viz.* 4 ℔ of Pepper for 9 *s.* and so much was 6 ℔ of Ginger valued at; and 12 ℔ of Sugar was rated as 9 ℔ of Ginger: how much of each Sort was sold?

Answ. 18 ℔ of Pepper, 2 ℔ of Ginger, and 4 ℔ of Sugar.

	lb	s.		lb	s.			lb	s.		lb	s.	
As	4	. 9	::	1	. $2\frac{1}{4}$	Pepper.	As	6	. 9	::	9	. $13\frac{1}{2}$	Ginger.
	lb	s.		lb	s.			lb	s.		lb	s.	
As	6	. 9	::	1	. $1\frac{1}{2}$	Ginger.	As	12	. $13\frac{1}{2}$	::	1	. $1\frac{1}{8}$	Sugar.

ob.
54 a Pound of Pepper.
36 a Pound of Ginger.
27 a Pound of Sugar.

24
27
168
48
648

s.
48
24
192
96
1152 ob.
648
First Dividend 504

54—27=27 First Divisor.
36—27= 9 Second Divisor.

1
23|8
504 | lb 18 Pepper.
27

Second Dividend 18 | 2 Ginger.
9
Complement 4 Sugar.
24

Alligation Alternate.

Alternate Alligation, to declare the due Proportion of every Ingredient entring the Mixture, doth alter or change the Places of such Excesses or Differences as fall out between the mean Price and the Extreams, ascribing that to the greater Extream which proceeds from the Lesser, and the contrary. And for better direction, Lines (called *Lines of Combination*) are commonly drawn to link or tie together a Number greater than the common Price to one Lesser; from which the Name of *Alligation* was first borrowed, and afterwards became common also to Questions of Mixture resolved by *Medial Alligation*, though there be no such tying or alligating the Numbers together, as in this called *Alternate*; which from the interchanging of the Differences, was added to that of *Alligation*, to distinguish the Species from that of *Medial*.

Lines of Combination, *what*.

Whence the Name of Alligation.

Theorems.

Necessary Theorems *to the Resolution.*

1. Let every greater Extream be linked with one lesser.
2. When either of the Extreams be Single, and the other Extreams be Plural, the single Extream must be linked to all the rest.
3. If both greater and lesser Extreams are not single, then they may be linked so diversly, that sundry Differences may be taken, and diversities of Answers to the Question, yet all true. But if one of the Extreams be single, there can be but one Answer.
4. The Numbers being linked, take the Difference of each Number from the mean or common Price, and place this Difference against the Number he is linked to alternately.
5. Every Number linked with more than one, must have all the Differences of the Numbers he is linked to set against him.
6. Those Differences resolve the Question, when the Price of every of the Ingredients is given, without their Quantities, and the Demand be to mix them so as to sell a certain Quantity at a mean Rate.
7. But when a Quantity of one, with the Prices of all the Ingredients is given, and the Demand is to know the Quantities of the other Ingredients, then the *Rule of Three* is to be used.
8. And when the Price of every Ingredient is given, without any of their Quantities, and the Demand be to make up a certain Quantity to be sold at a mean Rate, Then all the Differences added together shall be the first Number in the *Rule of Three*; the whole Quantity to be mixed shall be the second Number; and each Difference apart the several third Numbers: And so many Sorts mixed, so many Operations of the *Rule of Three*.
9. A Question may be so propounded, as both sorts of *Alligation* are needful to the Resolution.

Examples,

Examples, where the mean Rate is required according to the 6th Theorem. *Mean Rate required.*

A Merchant hath Wheat at 28 *d.* the Bushel, Rye at 20 *d.* *Barley* at 14 *d.* and Oats at 10 *d.* and would mix the same so as a Bushel of the Miscellane may be sold for 16 *d.* how much of each sort must be taken? *Q. Of Miscellane.*

Answ. Because two of the Extreams are greater than 16, the common Price, and two are lesser, the Numbers may be linked two ways, and the Mixture accordingly different: For either to 6 Bushels of Wheat may be taken 2 of Rye, 4 of Barley, and 12 of Oats: or to 2 of Wheat may be taken 6 of Rye, 12 of Barley, and 4 of Oats. Answer.

	d.		
d. 16	28	6	Wheat.
	20	2	Rye.
	14	4	Barley.
	10	12	Oats.
		24	Differences.

	d.		
d 16	28	2	Wheat.
	20	6	Rye.
	14	12	Barley.
	10	4	Oats.
		24	Differences.

Sugar-Cakes are made with Sugar of 14 *d.* the Pound, Flower at 2 *d.* the Pound, and Eggs at 1 *d.* the Pound: what Quantities of each may be taken to make the Paste worth 6 *d.* the Pound? *Q. Of Sugar-Cakes.*

Answ. Eggs and Flower of each 8 lb, of Sugar 9 lb; for the Differences of 1 and 2 from 6, are 4 and 5, which added make 9, belonging to the greater Extream, being single, and so linked to both the lesser Extreams. Answer.

		d.			
Common Price	*d.* 6	1	8	8	Eggs.
		2	8	8	Flower.
		14	4+5	9	Sugar.
				25	Differences.

Examples, where the Quantities of some Ingredients are required, as in the 7th Theorem. *Quantities of some Ingredients required.*

Ten Bushels of Wheat at 28 *d.* the Bushel, is to be mixed with Rye at 20 *d.* Barley at 14 *d.* and Oats at 10 *d.* how many Bushels of those other Sorts may be taken to afford a Bushel of the Miscellane at 16 *d*? *Q. Of Miscellane.*

Answ. The Extreams being Plural, and Numbers the same in the first Example above, according to the Differences situate by the different linking the Numbers, so shall the Answer be by help of the *Rule of Three*, in the manner following; according to both the above-mentioned Varieties. Answer.

	d.			*Analogy.*	
d. 16	28	6	Wheat.		
	20	2	Rye.	As 6 . 2 :: 10 . $3\frac{1}{3}$	Rye.
	14	4	Barley.	As 6 . 4 :: 10 . $6\frac{2}{3}$	Barley.
	10	12	Oats.	As 6 . 12 :: 10 . 20	Oats.

	d.				
d. 16	28	2	Wheat.		
	20	6	Rye.	As 2 . 6 :: 10 . 30	Rye.
	14	12	Barley.	As 2 . 12 :: 10 . 60	Barley.
	10	4	Oats.	As 2 . 4 :: 10 . 20	Oats.

One hundred Quarts of Canary at 12 *d.* the Quart, are to be mixed with Malaga at 9 *d.* and White-wine at 6 *d.* how many Quarts of the two latter must be taken to sell a Quart of the Mixture at 10 *d*? *Q. Of Sack mixed.*

Answ. Of each 40 Quarts: For one of the Extreams being single, there can be no Variety in linking the Numbers. Answer.

d.				
d. 10	12	1+4	5	Canary.
	9	2	2	Malaga.
	6	2	2	White.

Analogy.

As 5 . 2 :: 100 . 40 Malaga.

$5\,)\,\frac{100}{200}\,(\,40$

As 5 . 2 :: 100 . 40 White.

Quantities at a mean Rate required.

Examples, where the Quantities are required at a mean Rate, according to the 8th Theorem.

Q. Of Wool mixed.

A Clothier is to mingle 156 ſtone of Wool of ſeveral Colours, *viz.* Crimſon of 18 *s.* the Stone, Blew of 14 *s.* Green of 11 *s.* and White of 9 *s.* how much of each may be taken to make a Stone of the Mixture worth 12 *s?*

Anſwer.

Anſw. According to the Differences of the Nuumbers diverſly linked, more or leſs may be taken of each ſort: Thus,

s.			*Analogy.*	
s. 12	9	6	As 12 . 156 :: 6 . 78	White.
	11	2	As 12 . 156 :: 2 . 26	Green.
	14	1	As 12 . 156 :: 1 . 13	Blew.
	18	3	As 12 . 156 :: 3 . 39	Crimſon.
		12		

s.				
s. 12	9	2	As 12 . 156 :: 2 . 26	White.
	11	6	As 12 . 156 :: 6 . 78	Green.
	14	3	As 12 . 156 :: 3 . 39	Blew.
	18	1	As 12 . 156 :: 1 . 13	Crimſon.
		12		

Q. Of Silver mixed.

A Goldſmith would mix 90 ℔ of Silver, that the Mixture might hold out 9 ℥ Fine; and taketh of ſundry Sorts of Silver, as ſome of 4 ℥ Fine, ſome of 5, of 6, of 8, with others of 11 and 12 ℥ Fine: how much of each ſort may be taken?

Anſwer.

Anſw. If the 11 ℥ be alligated to 4 and 5, and the 12 ℥ to 6 and 8, then muſt be taken of each of 4 and 5, the quantity of $7\frac{19}{23}$ ℔, and of each of 6 and 8 the Quantity of $11\frac{17}{23}$ ℔, and of 11 ℥ Fine $35\frac{5}{23}$ ℔, and of 12 ℥ Fine $15\frac{15}{23}$ ℔.

℥			*Analogy.*	
s. 9	4	2	2	As 23 . 90 :: 2 . $7\frac{19}{23}$ of 4 ℥
	5	2	2	The like $7\frac{19}{23}$ of 5 ℥
	6	3	3	As 23 . 90 :: 3 . $11\frac{17}{23}$ of 6 ℥
	8	3	3	The like $11\frac{17}{23}$ of 8 ℥
	11	4+5	9	As 23 . 90 :: 9 . $35\frac{5}{23}$ of 11 ℥
	12	1+3	4	As 23 . 90 :: 4 . $15\frac{15}{23}$ of 12 ℥
			23	Fine.

Data diverſly alligated.

Other Varieties of alligating the Data *in the laſt Example.*

9	4	3	3
	5	3	3
	6	3	3
	8	2	2
	11	1	1
	12	3+4+5	12
		Differences	24

9	4	2+3	5
	5	2	2
	6	2	2
	8	2	2
	11	1+3+4+5	13
	12	5	5
		Differences	29

9	4	2+3	5
	5	3	3
	6	2	2
	8	2	2
	11	1+3+5	9
	12	4+5	9
		Differences	30

9	4	3	3
	5	3	3
	6	2+3	5
	8	2+3	5
	11	1+3	4
	12	1+3+4+5	33
		Differences	23

9	4	2	2
	5	2+3	5
	6	2+3	5
	8	3	3
	11	3+4+5	12
	12	1+3+4	8
		Differences	35

9	4	3	3
	5	2+3	5
	6	2+3	5
	8	2+3	5
	11	1+3+4	8
	12	1+3+4+5	13
		Differences	39

Besides these, the Lines of Combination may be otherwise drawn, according as more or less of any one Ingredient is intended to be used: For by *Alternate Alligation* the Quality of a Mixture may be augmented or diminished, or made finer or coarser at pleasure.

Examples, where both Sorts of Alligation *are needful, according to the ninth* Theorem. Alligation of both Sorts used.

A Mint-master hath 10 Penny Weights of Gold of 18 Caracts Fine, 20 of 19 Caracts Fine, 100 of 21 Caracts Fine, and 120 of 22 Caracts Fine; and he would mix them so, that every Penny-weight of the Mixture might be 20 Caracts Fine: whether doth he need to mix any Allay therewith; and if any, how much? Q. *of Gold allayed.*

Answ. First by *Alligation Medial*, the Mixture of the *Data* will be found $21\frac{1}{5}$ Caracts Fine, so as there must be some Allay. Then by *Alligation Alternate* is found, that for every 20 Penny-weights of the Gold must be taken $1\frac{1}{5}$ Penny-weight of Allay: So as for the whole 250 Penny-weights of Gold, must be 15 Penny-weights of Copper, or other Allay. Answer.

Medial.

$$\begin{array}{rcrcr} 10 & \times & 18 & = & 180 \\ 20 & \times & 19 & = & 380 \\ 100 & \times & 21 & = & 2100 \\ 120 & \times & 22 & = & 2640 \\ \hline 25,0 & & & & 530,0 \end{array}$$

$$530 \div 25 = 21\tfrac{1}{5} \text{ Fine.}$$

Alternate.

$$20\begin{cases} 21\frac{1}{5} & 20 \text{ Gold.} \\ 0 & 1\frac{1}{5} \text{ Allay.} \end{cases}$$

As $20 \,.\, 1\frac{1}{5} :: 250 \,.\, 15$

$$\begin{array}{r} 250 \\ 1\frac{1}{5} \\ \hline 250 \\ 50 \\ \hline 300 \end{array}$$

$$20)\,300\,(15 \text{ Allay.}$$

A Physician hath a Medicine compounded of Simples, Hot, Cold, and Temperate; that is to say, 8 ℥ Hot in the third Degree, 1 ℥ Hot in the Second, 1 ℥ Temperate; 2 ℥ Cold in the Second, and 2 ℥ Cold in the Fourth; and would compound the Medicine to make it Hot in $1\frac{1}{2}$ Degree: what Quantities of the Ingredients must be taken? Q. *Of composing a Medicine.*

Answ. By *Medial Alligation*, the Temperament of the Medicine is found to be Hot in the first Degree; and then as the Numbers be linked together by *Alternate Alligation*, the particular Quantities may be taken to make the Medicine of the desired Temperament. Answer.

Medial.

	Quant.		*Qual.*			
Hot	8	×	3	=	24	26
	1	×	2	=	2	
Temp.	1	×	0	=	0	
Cold	2	×	2	=	4	12
	2	×	4	=	8	
	14					38

$\frac{14}{14}$ (1 Hot.

Alternate.

	Hot	3	$3\frac{1}{2} + 5\frac{1}{2}$	9
	Hot	2	$1\frac{1}{2}$	$1\frac{1}{2}$
$1\frac{1}{2}$	Temp.	0	$\frac{1}{2}$	$\frac{1}{2}$
	Cold	2	$1\frac{1}{2}$	$1\frac{1}{2}$
	Cold	4	$1\frac{1}{2}$	$1\frac{1}{2}$
			Differences	14

The Differences happening to equalize the Quantities, the Differences ſerve for the Quantities to be taken without farther work.

Proof of Alligation. *Of the 1, 2, 4 and 6 Propoſitions Medial.*

The uſual Proof of *Alligation* is according to the Species thereof: Thoſe of the Firſt, Second, Fourth and Sixth Propoſitions of *Medial Alligation*, have their Proof by Multiplication of the Quantities mixed by the ſeveral Rates or Qualities thereof before Mixture; and the whole Quantity or Quantities ſo mixed by the new Rate or Quality; which when the Work is right, will be both equal.

As in the firſt Example of the firſt Propoſition.

			s.				
							Buſhels 200 whole Quantity.
							$4\frac{1}{2}$ *s.* new Rate.
Rye	100	×	4	=	400		800
Barley	40	×	$3\frac{1}{2}$	=	140		100
Wheat	60	×	6	=	360		
	200				900 *s.*	=	900 *s.* or 45 *l.*

And in the laſt Example of the ſecond Propoſition.

Mace.	*Cinamon.*	*Nutmegs.*	*Ginger.*		
21 ℔.	21 ℔.	21 ℔.	21 ℔.	21 ℔.	Quantity of each.
$8\frac{1}{3}$ *s.*	$6\frac{2}{3}$ *s.*	5 *s.*	2 *s.*	22 *s.*	Total Price.
168	126	105	42	42	
7	14			42	
175	140			462	

175 + 140 + 105 + 42 = 462 or 23 *l.* 2 *s.*

Of the third Medial, and of Alternate.

Thoſe of the third Propoſition in *Medial Alligation*, and thoſe alſo of *Alligation Alternate*, generally are proved as Operations in *Fellowſhip* before, by adding all the Quotients together, to return the ſecond Number or Total of the Quantities mixed. Neverthelcſs thoſe of *Alligation Alternate* may be proved, and it ſeems the better way, as others of the firſt Propoſition in *Alligation Medial*, by multiplying each Quotient by the Rate or Quality thereof before Mixture, to agree with the whole Quantity mixed, multiplied by the new Rate or Quality.

As in the firſt Example of the third Propoſition.

	Oil.		*Wax.*		*Ceruſe.*		*Camphire.*		*Total Quantity.*
Quotients	$118\frac{10}{17}$	+	$29\frac{11}{17}$	+	$59\frac{5}{17}$	+	$2\frac{8}{17}$	=	210 Second Number.

And in the firſt Example of the Work by the 8th Theorem.

	White.		*Green.*		*Blew.*		*Crimſon.*		*Total Quantity.*
Quotients	78	+	26	+	13	+	39	=	156 Second Number.

And by the Rates thus.

							156 Total Quantity.
							12 *s.* New Rate.
White	78	×	9 *s.*	=	702		312
Green	26	×	11	=	286		156
Blew	13	×	14	=	182		
Crimſon	39	×	18	=	702		
	156				1872	=	1872 or 93 *l.* 12 *s.*

Those of the fifth Proposition, and others depending on Proportions, admit of Proof with them by reversing the Question. *Of the fifth Medial.*

As in the first Example of the fifth Proposition.

Bushel of Miscellane		*Rye.*		*Bushels of Miscellane.*		*Rye.*
If 1	contain	$\frac{1}{8}$:	How much Rye is contain'd in	800?	*Answ.*	100.

$$\text{As } 1 \;.\; \tfrac{1}{8} \;::\; 800 \;.\; 100 \qquad \frac{800}{8}\Big(100$$

Those resolved by both Sorts of *Alligation*, *Medial* and *Alternate*, have their Proofs respectively where any Difference is. *Of Both.*

CHAP. X.

Barter and Exchange.

BOTH *Barter* and *Exchange* agreeing essentially, are placed together in this Chapter: For *Barter* is but an Exchange of Wares or Merchandises one for another: And Exchange a *Barter* of one sort of Money or Merchandise for another; or the same Merchandise by the Accompt, Weight or Measure of another Countrey. *Barter and Exchange, how they agree.*

Barter, (vulgarly called *Truck* and *Scosing*) and the Concerns thereof relating to the Exchange of one Commodity for another, so as the Merchant may save his own, have Part in Money, or get some Overplus by the Bargain, may be comprised under the 10 following *Cases*. *Barter how called.* *10 Cases thereof.*

Case 1. If the Price of both Commodities be given, to know how much of one Commodity may be given for any Quantity of the other. *1. One Quantity for another.*

By the *Rule of Three*, get the Total Value of the Quantity to be exchanged, and afterward by another Operation of the same Rule, get the Quantity desired. Rule.

Example. *A* and *B* barter; *A* hath 24 Broadcloths, at 10 *l.* the Piece; *B* hath Wheat at 5 *s.* the Bushel: how much Wheat will pay for the Cloth? Q. *Of Wheat for Cloth.*

Answ. First the Value of the Broadcloths is found to be 240 *l.* then at 5 *s.* the Bushel, 240 *l.* will buy 960 Bushels: And so much Wheat ought *A* to have, or else will lose by the Bargain. Answer.

	Cloth.		*l.*		*Cloths.*		*l.*			*s.*		*Bushel.*		*l.*		*Bushels.*
As	1	.	10	::	24	.	240		As	5	.	1	::	240	.	960
			24											20		
			240										5)	4800	(	960

Case 2. If the Price in ready-Money, and *Barter*-Price of one be given, to know the *Barter*-Price of the other, and how much at that Price of the one Commodity may be given for any Quantity of the other. *2. Barter-price of one for the other.*

See what is gained on the Shilling, Pound, Hundred, *&c.* by the one Party: then by the *Rule of Three* the Gain or Overplus of the other is found, as also the quesited Quantity. Rule.

Example. *A* and *B* barter, *A* hath Raisins at 30 *s.* *per* C. ready Money; but in *Barter* will sell for 40 *s.* *per* C. *B* hath Sugar at 12 *d.* *per* lb. ready Money, but would gain proportionally to the other: how therefore must *B* rate his Sugar in *Barter*? and how many lb. of Sugar may he give for 4 C. of Raisins? Q. *Of Sugar for Raisins.*

Answ. The Gain found by the Question to be 10 *s.* in 40 *s.* the first Question stands thus: If 30 *s.* gain 10 *s.* what shall 1 *s.* which is the ready-Money Price of the Sugar? and by the Work 4 *d.* is gained: So must the *Barter*-price of the Sugar be 16 *d.* whereby the first Question is answered. Then if 16 *d.* buy 1 lb. of Sugar, 8 *l.* (the *Barter*-Price of 4 C. of Raisins) shall buy 120 lb. of Sugar; which answereth the second Question. Answer.

As

As 30 s. . 10 s. :: 1 s. . $0\frac{1}{3}$ s. or As 30 s. . 40 s. :: 1 s. . $1\frac{1}{3}$ s.

	s.		s. d.
A Ready-Money Price	30	*B* Ready-Money Price	1 . 0
Barter Price	40	Barter Price	1 . 4
Difference	10	Difference	0 . 4

As 1 C. . 40 s. :: 4 C. . 160 s. As $1\frac{1}{3}$ s. . 1 ℔. :: 160 s. . 120 ℔. Sugar.

3. Ready-Money Price of one for another. *Case* 3. If the Price in Ready-Money, and *Barter* of one Party with the *Barter*-price of the other be given, to know the Ready-Money Price of the other, and how much at that Price of the one will countervail a Quantity of the other Commodity.

Rule. Find by the *Rule of Three* the Ready-Money Price desired; and by another Work of the same Rule the Quantity sought.

Q. Of Salt for Wine. *Example.* *A* hath Salt at 4 *s.* the Bushel Ready-Money; but in *Barter* will have 4 *s.* 6 *d.* and will exchange with *B* for Wine at 18 *l.* the Tun: how is the Wine rated in Ready-Money? and how much Salt shall *B* have for 3 Tuns of Wine?

Answer. *Answ.* By resolving the first Question, 16 *l.* is found to be the Ready-Money Price of the Wine: And by resolving the second Question, for 3 Tuns of Wine *B* shall receive 240 Bushels of Salt.

As $4\frac{1}{2}$ s. . 4 s. :: 18 l. . 16 l. Tuns 3 × 18 l. = 54 l. As $4\frac{1}{2}$ s. . 1 Bushel. :: 54 l. . 240 Bushels.

$$18 \times 20 = 360;\quad 360 \times 4 = 1440$$

$$\frac{9}{2}\Big)\frac{1440}{1}\Big(\frac{2880}{9}\Big(\frac{32,0}{16{:}0\ s.}$$

$$54 \times 20 = 1080$$

$$\frac{9}{2}\Big)\frac{1080}{1}\Big(\frac{2160}{9}\Big(240 \text{ Bushels.}$$

4. Gains on the 100, which the most. *Case* 4. If the Rates, both in ready Money and *Barter*, of both Parties be given; to know the Gains of each upon the Hundred, and which is the greatest Gainer.

Rule. After the Gains of each Party on the 100 is found by the *Rule of Three*; substract the one from the other, and the Difference shall be the Gain of one Party above the other.

Q. Of Figs for Ginger. *Example.* *A* and *B* will *barter*: *A* hath Figs at 24 *s. per* C. ready Money; but in *Barter* will have 30 *s.* *B* hath Ginger at 4 *l.* 5 *s. per* C. but in *Barter* will have 4 *l.* 15 *s. per* C. how much did each gain on the 100 by the *Barter*? and which must have Money of the other to ballance the *Barter*, and how much?

Answer. *Answ.* *A* will be found to gain 25 *l.* on the 100, and *B* but 11 *l.* 15 *s.* $\frac{5}{17}$; it follows therefore *A* is the greatest Gainer. And to ballance the *Barter*, *B* must have of *A* one half of the Difference, 13 *l.* 4 *s.* 8 *d.* $\frac{8}{17}$.

As 24 s. . 6 s.gain. :: 100 l. . 25 l.gain.

$$100 \times 20 = 2000;\quad 2000 \times 6 = 12000$$

$$24\Big)12000\Big(\frac{50,0}{25{:}0}\ l.\ A \text{ gains } per\ 100$$

	l.	s.	d.	
A .	25 :	0 :	0	gains.
B .	11 :	15 :	$3\frac{9}{17}$	gains.
	13 :	4 :	$8\frac{8}{17}$	Difference.
	6 :	12 :	$4\frac{4}{17}$	half.

As $4\frac{1}{4}$ l. . $\frac{1}{2}$ l.gain. :: 100 l. . $11\frac{13}{17}$ l.gain.

$$\frac{17}{4}\Big)\frac{100}{2}\Big(\frac{200}{17}\Big(11\tfrac{13}{17}\ l.\ B \text{ gains } per\ 100$$

Case

Case 5. If the Rates, both in ready Money and *Barter*, of one Party be given, and he will have a part of his *Barter*-Price in ready Money; to know how the other Party may rate his Goods to be equal in the *Barter*. 5. *Barter-Price and ready Money of one for the other.*

Substract the demanded Part from the *Barter*-Price, and the other Price, and with these two Remains, and the other Party's ready-Money Price, commit the Work to the *Rule of Three*. Rule.

Example. *A* hath Kerseys at 15 *l.* ready Money, but in *Barter* will have 18 *l.* and besides will have $\frac{1}{3}$ of his *Barter*-price in ready Money: *B* hath Linen at 3 *s.* *per* Ell ready Money: how shall *B* rate his Linen to be equal with *A*? Q. *Of Kersey for Linen.*

Answ. Taking 6, which is $\frac{1}{3}$ of 18, from 15 and 18, the Remains 9 and 12 are the first and second Numbers of the *Rule of Three*, and 3 *s.* the Third; by the Work whereof it appears *B* must rate his Linen at 4 *s.* *per* Ell. Answer.

$\frac{1}{3}$ of 18 = 6   15 − 6 = 9,   18 − 6 = 12

	l.	*l.*	*s.*	*s.*
As	9 .	12 ::	3 .	4
	20	20		
	180 *s.*	240 *s.*		
		3		
		720		

$\frac{720}{180}$ (4 *s.* for 1 Ell of Linen.

Case 6. If both the Price in ready Money and *Barter* of one Party be given, and a Part of his *Barter*-Price in ready Money desired; to know how the other may ballance the *Barter*, and gain a Sum on the 100. 6. *Both Prices, and part Money, to gain a Sum on the 100.*

Take as in the last *Case* the Part desired from both the Prices given, and find, by the *Rule of Three*, the Advance of the other Party's ready-Money Price, according to the Sum to be gained on the 100; this Number found, with the other Remains, commit to another Work of the *Rule of Three*. Rule.

Example. *A* hath Stuffs which he rateth at 25 *s.* the Piece ready Money; but in *Barter* will have 30 *s.* and will have $\frac{1}{4}$ of his *Barter*-Price in ready Money. *B* hath Stockins at 40 *s.* the Dozen ready Money, and would ballance the *Barter*, and gain 10 *l.* *per Cent.* how shall *B* rate the Stockins in *Barter*? Q. *Of Stuffs for Stockins.*

Answ. Substracting 7 *s.* 6 *d.* which is $\frac{1}{4}$ Part of 30 *s.* from 25 and 30, the Remains are 17 *s.* 6 *d.* and 22 *s.* 6 *d.* And finding the Gain of 40 *s.* at the rate of 10 *l.* *per Cent.* to be 4 *s.* it appears by the *Rule of Three*, *B* must rate his Stockins at $2\frac{29}{35}$ *l.* the Dozen, or reduced lower at 2 *l.* 16 *s.* $6\frac{6}{7}$ *d.* Answer.

$\frac{1}{4}$ of 30 *s.* = 7 *s.* : 6 *d.*

25 *s.* − 7 *s.* : 6 *d.* = 17 *s.* : 6 *d.*
30 − 7 : 6 = 22 : 6

	l.	*l.*	*l.*	*l.*
As	100 .	110 ::	2 .	$2\frac{1}{5}$
		2		

100) 220 ($2\frac{1}{5}$

	l.	*l.*	*l.*	*l.*	
As	$\frac{7}{8}$.	$1\frac{1}{8}$ ::	$2\frac{1}{5}$.	$2\frac{29}{35}$	Stockins rated *per* Dozen.

$\frac{7}{8}$) $\frac{99}{40}$ ($\frac{99}{35}$ ($2\frac{29}{35}$
 1   5

Case 7. If besides the different Rates given of one Party, he would gain a Sum on the 100, and have a Part ready Money; to know how the other Party shall ballance the *Barter*. 7. *Rates and ready Money of one, Gain on the 100 for the other.*

Enquire, by the *Rule of Three*, the Gains upon the 100, after the *Barter*-Price; from which take the Part demanded in ready Money, as also from the *Barter*-Price; and with these two Remains, and the other's ready-Money Price, commit the Work to the *Rule of Three*. Rule.

Example. *A* and *B* will *barter*: *A* hath Pease at 2 *s.* *per* Bushel ready Money; but in *Barter* will have 2 *s.* 6 *d.* and will gain 10 *l.* *per Cent.* and have $\frac{1}{3}$ of his over-price in ready Money. *B* hath Flax at 7 *d.* *per* lb. how shall he rate his Flax to ballance the *Barter*? Q. *Of Pease for Flax.*

Answ. At 10 *l.* *per* C. 2 *s.* 6 *d.* will be 2 *s.* 9 *d.* of which $\frac{1}{3}$ is 11 *d.* which taken from both, the other Question will stand thus: If 19 *d.* ready Money make Answer.

22 *d.* in *Barter*; what will 7 *d.* in ready Money? The Answer to which is, $8\frac{2}{19}$ *d.* and so must *B* rate his Flax in *Barter*.

	l.		*l.*		*s.*	*d.*		*s.*	*d.*
As	100	.	110	::	2	: 6	.	2	: 9
	240				12				
	240,00				30				
					110				
					33,00				

$\frac{33}{240}$ ($\frac{11}{80}$ *l.* or 2 : 9 *s. d.*

$\frac{1}{3}$ of 2 *s.* : 9 *d.* or 33 *d.* = 11 *d.*

2 *s.* : 6 *d.* — 11 = 19 *d.*
2 : 9 — 11 = 22

	d.		*d.*		*d.*		*d.*
As	19	.	22	::	7	.	$8\frac{2}{19}$ The Rate in *Barter* of 1 ℔. of Flax.

7

19) 154 ($8\frac{2}{19}$

8. *Rates of one, Mony and Goods of the other.* *Case* 8. If the different Rates of one Party be given, and the other will have some Money and some Goods, to know the Quantity of the latter.

Rule. To the Total Value of the Price in ready Money and *Barter* severally, add the Sum to be paid ready down; by these Totals, with the ready-Money Price of the other, will be found the *Barter*-Price, with which the Quantity of Goods by another Work of the *Rule of Three* will be had accordingly.

Q. *Of Ashes for Allum.* *Example.* *A* hath 20 Tuns of Ashes, at 54 *l.* the Tun ready Money; but in *Barter* rateth them at $55\frac{1}{2}$ *l.* *B* hath Allum at $12\frac{1}{4}$ *l.* the C. ready Money, and he will have 360 *l.* ready Money of *A*: how must *B* rate the Hundred of Allum in *Barter*? and how much Allum must he deliver for the said 20 Tuns of Ashes, and 360 *l.* in Money?

Answer. *Answ.* The ready-Money Price of the Ashes found to be 1080 *l.* and the *Barter*-price 1110 *l.* to both which 360 *l.* added, makes 1440 *l.* and 1470 *l.* And if 1440 *l.* give 1470 *l.* then shall $12\frac{1}{4}$ *l.* give $12\frac{97}{192}$ *l.* the *Barter*-Price of the Allum. Then if $12\frac{97}{192}$ *l.* be for 1 C. of Allum, 1470 *l.* shall be for $117\frac{27}{49}$ C. which is the Quantity desired, and to be delivered for the 20 Tuns of Ashes, with 360 *l.* in Money.

	Tun.	*l.*		*Tun.*	*l.*			
As	1 .	54	::	20 .	1080	Ready Money	Ashes.	1080 + 360 = 1440
As	1 .	$55\frac{1}{2}$	::	20 .	1110	*Barter*-Price		1110 + 360 = 1470

	l.	*l.*	*l.*	*l.*	*Allum.*		*l.*	C.	*l.*	C.
As	1440 .	1470 ::	$12\frac{1}{4}$.	$12\frac{97}{192}$	*Barter*-Price.	As	$12\frac{97}{192}$.	1 ::	1470 .	$117\frac{27}{49}$ Allum.

9. *Time for delivery: how to rate the other.* *Case* 9. If the Proposition include some Time for delivery of the Goods, and propounds the Rates of the one Party, to know how to rate the Goods of the other Party:

Rule. State the Question for Resolution by the *Rule of five Numbers*, and what is gotten thereby add to the given Price.

Q. *Of Wine for Sugar.* *Example.* *A* hath Wine at 24 *l.* *per* Tun in Money, but in *Barter* 27 *l.* to be delivered at 3 Months. *B* hath Sugar at 5 *l.* *per* Hundred in Money, to be delivered at 6 Months: how shall *B* rate his Sugar in *Barter*?

Answer. *Answ.* At $6\frac{1}{4}$ *l.* *per* Hundred, for so it will be when $1\frac{1}{4}$ gotten by the Work is dded to the Price given.

	l.		*Months.*		*l.*		*l.*		*Months.*		*l.*
As	24	.	3	.	3	::	5	.	6	.	$1\frac{1}{4}$
	72						18				(18
							90				90
											72

$1\frac{1}{4}$ *l.* + 5 *l.* = $6\frac{1}{4}$ *l.*

l.			*l.*		
24	Ready-Money Price	of the Wine.	5	Ready-Money Price	of the Sugar.
27	*Barter*-Price		$6\frac{1}{4}$	*Barter*-Price	
3	Difference.		$1\frac{1}{4}$	Difference.	

Case

Case 10. If the Proposition propound a Time for delivery with the *Barter*-Price of both, and the ready-Money Price of one Party, with part of his over-Price; to know the ready-Money Price of the other. — 10. *Time and Price of one, for the Price of the other.*

Proceed according to the State of the Questions by the several Cases needful to the Resolution, as in other-like mixture of Questions, *mutatis mutandis*. — Rule.

Example. *A* hath Hemp at 16 *l.* the C. in Money, in *Barter* at 20 *l.* the C. to be delivered at 4 Months, and will have $\frac{1}{4}$ of his *Barter*-Price in ready Money: *B* hath Saffron in *Barter* 10 *l.* $\frac{2}{17}$ *per* ℔. at 6 Months to be delivered: what is the Saffron worth in ready Money? — Q. *Of Hemp for Saffron.*

Answ. 7 *l. per* ℔. For by the 5th *Case* the $\frac{1}{4}$ of the *Barter*-Price, which is 5 *l.* is taken from 16 and 20: Then the Difference of the two Remains with the Times gets a Number to be added to the ready-Money Price, after the $\frac{1}{4}$ Part is taken from it; which with the *Barter*-Price of the Saffron, the ready-Money Price of the Saffron is gotten by the third *Case*. — Answer.

	l.		*l.*		*l.*
Ready Money	16	—	5	=	11
Barter-Price	20	—	5	=	15
			Difference		4

$\frac{1}{4}$ of 20 = 5

	Mon.		*l.*		*Mon.*	*l.*
As	4	.	4	::	6	. 6

11 + 6 = 17

As 17 . 11 :: $10\frac{2}{11}$. 7

$10\frac{2}{11}$

110

9

17) 119 (7*l.* Ready-Money Price of the Saffron

Exchange of one sort of Money or Merchandise for another, or the same by different Accompts, and the Necessaries thereof, proper for this Place, though placed by some under *Reduction*, as in *Reduction of Geodeticals* was before noted, may be comprised under the two following *Cases*. — Exchange. *Where placed by some. Two Cases thereof.*

Case 1. If the Proposition be single, as to the Exchange of one sort of Money or Merchandise for another; or the same Merchandise by different Weights, Measures, *&c.* — 1. *Proposal single.*

When the Proportion of one to the other is not readily known, or cannot be reduced to Digits or small Numbers, whereby the Parts may be easily taken, commit the Work to the *Rule of Three*. — Rule, *without Reduction.*

Example. A Merchant transporteth from *London* to *Bourdeaux* 13 C. Weight of Copperas, and the same is exchanged at *Bourdeaux* for 13 C. of Prunes their Weight: The Question is, how much the Prunes weighed *London*-weight. — Q. *Of Copperas for Prunes.*

Answ. Because the Proportion between the Weights of *London* and *Bourdeaux* being as 104 to $94\frac{3}{4}$, is not easy to be brought to small Parts, therefore the Question is committed to the *Rule of Three*, by which is gotten $1426\frac{266}{379}$, which divided by 112 the Pounds in 1 C. *Averdupois*, there is found but 12 C. $82\frac{266}{379}$ ℔ at *London*. — Answer.

	Bourdeaux.		*London.*		*Bourdeaux.*
As	$94\frac{3}{4}$ ℔.	.	104 ℔.	::	1300 ℔.

1300

31200

104

135200

$\frac{379}{4}$) $\frac{135200}{1}$ ($\frac{540800}{379}$ (℔. $1426\frac{266}{379}$ *London*.

When the Proportions are easy to be reduced to small Numbers, the Resolution may be had by *Practice*, or the *Rule of Three*. — Rule *with Reduction.*

Examples in Measure. — Examples in Measure.

A Merchant buyeth at *Antwerp* 1440 Ells of Silks, and selleth the same at *London*: how many Ells shall he make out by *London*-Measure? — Q. *Of Silk* Antwerp *Measure sold at* London.

Answ. Because by the first Chapter of *Geodeticals* 100 Ells *Antwerp* make but 60 Ells *London*, the Proportion is soon espied to be as 5 to 3. If I multiply 1440 by 3, and divide the Product by 5; or else add $\frac{2}{3}$ of the Number together; or otherwise substract $\frac{2}{5}$ therefrom; either Way make 864, the Number of Ells at *London*. — Answer.

Antwerp.	*London.*		*Antwerp.*	*London.*
As 5 Ells	. 3 Ells	::	1440 Ells.	. 864 Ells.

$$\frac{3}{5)\,4320\,(864}$$

Otherwise by Practice.
$\frac{1}{5}$ of 1440 = 288
3
Ells 864 *London.*

As 100 . 60 :: 1440 . 864
60
100)864,00(864

$\frac{2}{5}$ = 576
864 Ells *London.*

London *Ells turned into* Antwerp.

On the contrary, to turn Ells *English* into Ells *Antwerp*, add $\frac{2}{3}$ of the Number thereto.

As if it were desired to know how many Ells *Antwerp* 864 Ells *London* would make; $\frac{2}{3}$ thereof added thereto, make 1440 Ells *Antwerp*, as before.

864 Ells *London.*
$\frac{2}{3}$ $\begin{cases} 288 \\ 288 \end{cases}$
1440 Ells *Antwerp.*

Q. *Of Cloth, whether any lost.*
Answer.

A Merchant buyeth at *London* 738 Yards of Cloth, and at *Antwerp* maketh out but 980 Ells? whether did he lose any by the Way?

Answ. The Proportions being as 75 to 100, or 3 to 4 by the *Rule of Three*, or *Practice*, by adding $\frac{1}{3}$ of 738 thereto, there should be 984 Ells made out at *Antwerp*; so as 4 Ells *Antwerp* were lost.

London.	*Antwerp.*		*London.*	*Antwerp.*
As 3 Yards	. 4 Ells	::	738 Yards	. 984 Ells.

4
3)2952(984

As 75 . 100 :: 738 . 984
100
75)73800(984

Otherwise by Practice.
738 Yards *London.*
$\frac{1}{3}$ 246
984 Ells *Antwerp.*

Antwerp *Ells turned into* London *Yards.*

On the contrary, to turn Ells *Antwerp* into Yards *London*, the Proportion being as 4 to 3, take $\frac{3}{4}$ of the *Antwerp*-Measure, either by substracting $\frac{1}{4}$ thereof, or taking half the given Number, and adding thereto half that half.

As if it were desired to know how many Yards *London* 984 Ells *Antwerp* would make: either 246, which is $\frac{1}{4}$ thereof, taken away, leaves the Remain 738; or $\frac{3}{4}$ of the given Number is taken, which is all alike.

984 Ells *Antwerp.*
$\frac{1}{4}$ 246
738 Yards *London.*

984 Ells *Antwerp.*
$\begin{rcases} \frac{1}{2}\ 492 \\ \frac{1}{4}\ 246 \end{rcases}$
738 Yards *London.*

Examples in Money.

Examples in Money.

Q. *Of* Sterling *and* French *Money.*

A Merchant delivereth to the Exchangers at *London* 300 l. *Sterling*, after 54 *d.* the French Crown, (that is 3 *Liures*) and taketh a Bill to receive at *Paris* 60 *Sols Tournois* for every Crown: how much *Tournois* or French Money will pay the said Bill?

Answer.

Answ. By the *Rule of Three* 4000 *Liures*: and so by *Practice*, if 300 *l.* be brought into Shillings, and $\frac{1}{3}$ taken away, because 54 *d.* to 3 *Liures* in less Terms, is as 18 *d.* to 1; that is, $1\frac{1}{2}$ *s.* for 1 *Liure*.

As

Sterl. *Liure.* *Sterl.* *Liures.*
As $1\frac{1}{2}$ *s.* . 1 :: 300 *l.* . 4000
20
6000

1 2000
$\frac{3}{2}$) $\frac{6000}{1}$ ($\frac{4000}{1}$

d. *Liures.* *l.* *Liures.*
As 54 . 3 :: 300 . 4000
240
72000
3
54) 216000 (4000

Otherwise by Practice.
l. *s.* *s.*
300 × 20 = 6000 *Sterling.*
$\frac{1}{3}$ 2000
4000 *Liures.*

If in *Sols* 80000
But in Crowns $1333\frac{1}{3}$

On the contrary, to turn French Money into English: So much as the *Liure* is valued above an English Shilling, convert into Parts one or more of a Shilling, and add thereto, and the Total is Shillings *Sterling*. *French Money turned into English.*

As if 4000 *Liures* at 18 *d.* a piece were to be turned into *Sterling* Money, because 18 *d.* is $1\frac{1}{2}$ *s.* that is $\frac{1}{2}$ above a Shilling, $\frac{1}{2}$ of 4000 is added thereto: So is the Total 6000 *s.* or 300 *l.* as before.

But if the *Liure* be rated at 20 *d.* then 8 *d.* above a Shilling being $\frac{2}{3}$ of a Shilling, $\frac{2}{3}$ of 4000 is to be added thereto; and the Total will be $6666\frac{2}{3}$ *s.* or 333 *l.* 6 *s.* 8 *d.*

4000 *Liures.*
$\frac{1}{2}$ 2000 at 18 *d.* per *Liure.*
600,0 *s.*
l. 300:0 *Sterling.*

4000 *Liures.*
$\frac{2}{3}$ { $1333\frac{1}{3}$ / $1333\frac{1}{3}$ } at 20 *d.* per *Liure.*
$666,6\frac{2}{3}$ *s.*
l. 333:6:8 *d. Sterling.*

A Merchant in *Spain* taketh up 300 Pieces of Eight, at 4 *s.* 4 *d.* a Piece, and payeth for the same in *London* 64 *l.* whether hath he paid his due? *Q. Of Spanish Money paid in London.*

Answ. 300 Pieces of Eight, at 4 *s.* 4 *d.* amount to in *Sterling* Money 65 *l.* by the *Rule of Three*, or *Practice*, multiplying by the Shillings, and adding $\frac{1}{3}$ for the 4 *d.* So it appeareth he paid 20 *s.* too little. *Answer.*

Piece. *Sterl.* *Pieces.* *Sterl.*
As 1 . $4\frac{1}{3}$ *s.* :: 300 . 65 *l.*
$4\frac{1}{3}$
1200
100
130,0 *s.*
l. 65:0 *s. Sterl.*

Piece. *d.* *Pieces.* *l.*
As 1 . 52 :: 300 . 65
300
15600
12
$\frac{15600}{240}$ (65 *l. Sterl.*

On the contrary, to turn English Money into Spanish, commit the Question to the *Rule of Three*, because the Proportions are as 60 to 13, uneasy Parts to take of some Numbers that may be given. *English Money turned into Spanish.*

As if 65 *l. Sterling* were to be turned into Pieces of Eight, at 4 *s.* 4 *d.* the Piece, the Work would be thus.

s. *Piece.* *l.* *Pieces of* 8.
As $4\frac{1}{3}$. 1 :: 65 . 300
20
$\frac{13}{3}$ 1300

1 100
$\frac{13}{3}$) $\frac{1300}{1}$ ($\frac{300}{1}$ Pieces of Eight.

Case 2. If the Proposition be double, as to express or include two Questions therein: *2. Proposal double.*

Rule. Commit the Work of either to the *Rule of Three*, or *Practice*, as before, or both to the *Rule of five Numbers*, as the State of the Queſtion will admit.

Q. *Of Braces and Ells.* *Example.* Seeing by *Geodeticals* 100 Ells of *Antwerp* are equal to 108 Silk Braces at *Venice*, and 100 Ells of *Antwerp* make 60 Ells at *London*: how many Braces *Venice* are in 3648 Ells *London*?

Anſwer. *Anſw.* 6566⅖ Braces, as by the following Works appear.

By the Rule of Three.

London. *Antwerp.* *London.* *Antwerp.*
As 3 Ells . 5 Ells :: 3648 Ells . 6080

$3648 \times 5 = \frac{18240}{3}(6080$

Antwerp. *Venice.* *Antwerp.* *Venice.*
As 100 Ells. 108 Braces :: 6080 Ells. 6566⅖ Braces.

$6080 \times 108 = \frac{656640}{100}(6566\frac{2}{5}$

By the Rule of five Numbers.

Venice. *London.* *Antw.* *Lond.* *Antw.* *Venice.*
As 108 . 3648 . 100 :: 60 . 100 . 6566⅖

$$\begin{array}{r} 108 \\ \hline 29184 \\ 3648 \\ \hline 393984.00 \end{array} \qquad 60,00 \qquad \frac{393984(2}{6\quad 0}(6566\frac{2}{5}$$

Or omitting the ſuperfluous Numbers.

Lond. *Venice.* *Lond.* *Venice.*
As 60 . 108 :: 3648 . 6566⅖

$\frac{393984}{6\quad 0}(6566\frac{2}{5}$

Of Millan *Braces.* But if the Demand had been of *Millan* Braces for Linen, and not for Silk; then becauſe 60 Ells *London*, or 100 Ells *Antwerp* are equal to 120 Braces uſed at *Millan* for Linen Cloth, there needed nothing but to double the 3648 Ells given, and 7296 had been the Braces deſired.

And on the contrary, to take ½ the *Millan* Braces for Linen, you have forthwith the *London* Ells.

Nevertheleſs in the Silk Braces at *Millan*, becauſe 141 of them anſwer to 60 Ells of *London*, which will not be reduced to ſmall Numbers, and ſo in others of like ſort, it is beſt to work by the *Rule of Three*, or *Five Numbers*, as aforeſaid.

Proof of Barter *and* Exchange. All Queſtions of *Barter and Exchange*, receiving their Reſolution by the *Rule of Three*, *Practice*, *Specificks*, or *Rule of five Numbers*, will be accordingly proved: or the Queſtion may be reverſed, which will add to the Demonſtration.

CHAP. XI.

Loſs and Gain.

Loſs and Gain, *the Subject thereof. Four Caſes thereof.* THE Title of this Chapter ſhews the Subject thereof, to converſe with Queſtions reſolving what *Loſs* or *Gain* may accrue by Traffick; and this in one of theſe four Caſes following.

1. *Bare enquiry of either.* *Caſe* 1. When there is a bare enquiry of *Loſs* or *Gain*.

Either the Rates of buying and ſelling are given, to find the *Gain* or *Loſs* in general, or upon the Hundred.

Or on the contrary, to find the Rates of a Quantity, the *Gain* or *Loſs* of the Whole, or upon the Hundred is given.

How reſolved. In all which, Reſolution is to be had by the *Rule of Three*, the *Data* being duly diſpoſed or prepared, according to the State of the Queſtion propounded.

Q. *Of* Loſs. *Example* 1. I have 10 Yards of Cloth that coſt me 8 *l.* 5 *s.* which I ſell again for 15 *s.* the Yard: what do I loſe thereby?

Anſwer. *Anſw.* 15 *s.* For by the *Rule of Three*, every Yard bought in, is found to coſt 16 *s.* 6 *d.* which is 1 *s.* 6 *d.* a Yard more than the ſelling Price, and this in 10 Yards amounts to 15 *s.*

Yards. l. s. Yard. s.
As 10 . 8 : 5 :: 1 . $16\frac{1}{2}$
20
10)165($16\frac{1}{2}$ s.

$16\frac{1}{2}$ s. Buying Price.
15 Selling Price.
$1\frac{1}{2}$ s. Loss in 1 Yard.

10 Yards.
$1\frac{1}{2}$ s.
10
5
15 s. Total Loss.

Example 2. If 1 Yard cost 5 *s.* 4 *d.* and be sold again for 6 *s.* 8 *d.* how much is gained on the 100? — Q. *Of Gain on the* 100.

Answ. 25 *l.* For taking the buying Price from the selling Price, the Gain of 1 Yard is had, the rest appeareth by the Work of the *Rule of Three.* — Answer.

s. d.
6 : 8
5 : 4
1 : 4 Gain.

s. Gain. l. Gain.
As $5\frac{1}{3}$. $1\frac{1}{3}$ s. :: 100 . 25 *l.*
$\frac{16}{3}$
20
2000
$1\frac{1}{3}$
2000
$666\frac{2}{3}$
$2666\frac{2}{3}$

$\frac{16}{3}$)$\frac{8000}{3}$(50,0 / *l.* 25:0 *s.* Gain on the 100 *l.*

Example 3. If I pay 34 *l.* for 560 Yards of Cloth, and would gain 17 *l.* 6 *s.* 8 *d.* thereby: how must I sell 1 Yard? — Q. *Of the Price of a Yard.*

Answ. For 22 *d.* Here adding the Gain to the buying Price, the Total 51 *l.* 6 *s.* 8 *d.* is the second Number of the *Rule of Three*, the other Numbers are the Yards given. — Answer.

l.
34
17 : 6 : 8
51 : 6 : 8

Yards. l. Yard.
As 560 . $51\frac{1}{3}$ :: 1 . $\frac{11}{120}$ *l.* or 22 *d.*
40 11
$\frac{560}{1}$)$\frac{154}{3}$(*l.* $\frac{11}{120}$ Price of 1 Yard.

Example 4. If I buy Cloth at 7 *s.* 6 *d.* the Ell, and it proving worse than expected, I am resolved to lose 5 in the 100: how must I rate the Cloth an Ell? — Q. *Of the Price of an Ell.*

Answ. 7 *s.* $1\frac{1}{2}$ *d.* For if 100 *l.* lose 5 *l.* then 7 *s.* 6 *d.* shall lose $4\frac{1}{2}$ *d.* which taken from 7 *s.* 6 *d.* leaves 7 *s.* $1\frac{1}{2}$ *d.* — Answer.

l. Loss. l. Loss.
As 100 . 5 *l.* :: $\frac{3}{8}$. $\frac{3}{160}$ *l.* in 1 Yard.
20 3 *d.* (8
$\frac{100}{1}$)$\frac{15}{8}$($\frac{3}{160}$ × 240=72(0 / 16 0 (*d.* $4\frac{1}{2}$

s. d.
7 : 6 Buying Price.
$4\frac{1}{2}$ Loss.
7 : $1\frac{1}{2}$ Selling Price.

Case 2. When the Enquiry is of *Loss* or *Gain* with Time. — 2. Loss or Gain *with Time, how resolved.*

The *Data* duly disposed, Resolution is to be had by the *Rule of five Numbers.*

Example 1. If 1 Ell of Holland cost 2 *s.* 6 *d.* and it be sold for 2 *s.* 8 *d.* to be paid at the end of four Months: what after that rate is gained upon the 100 in 12 Months? — Q. *Of Gain in* 12 *Months.*

Answ. 20 *l.* — Answer.

s. d.
2 : 8 Selling Price.
2 : 6 Buying Price.
0 : 2 Gain.

l. Mon. d. l. Mon. l.
As $\frac{1}{8}$. 4 . 2 :: 100 . 12 . 20
$\frac{1}{2}$
24
$\frac{1}{2}$)$\frac{2400}{1}$($\frac{4800}{240}$(*l.* 20

Example 2. If a parcel of Goods that cost 15 *l.* be sold again for 14 *l.* 16 *s.* $10\frac{1}{2}$ *d.* to be paid at the end of 3 Months: whether at that rate is there lost 6 in the 100 for 12 Months? — Q. *Of Loss in* 3 *Months.*

Answ.

Answer. *Answ.* No, but 4 *l.* 3 *s.* 4 *d.*

l.	*s.*	*d.*	
15	00	00	Buying Price.
14	16	$10\frac{1}{2}$	Selling Price.
00	3	$1\frac{1}{2}$	Loss.

l. *Mon.* *s.* *l.* *Mon.* *l.*
As 15 . 3 . $3\frac{1}{8}$ :: 100 . 12 . $4\frac{1}{6}$
45
$37\frac{1}{2}$
45)3750($8,3\frac{1}{3}$ *s.*
l. 4:3:4 *d.*

3. Loss or Gain, with Allowance or Rebatement, how resolved. *Case* 3. When the Enquiry is of *Loss* or *Gain* with Allowance, or Rebatement in the Weight; both passing commonly by the Names of *Tare*, *Tret* and *Cloff*.

Addition for the Allowance, and *Substraction* for the Rebatement being made, the residue of the Work is by the *Rule of Three*, or *Practice*.

Q. Of the Hundred with Tret. *Example* 1. At 14 *l.* the 100 Suttle, what will 890 ℔. Suttle be worth, giving 6 ℔. weight upon every 100 for Tret?

Answer. *Answ.* $117\frac{29}{53}$ *l.* For so is the Resolution, adding the Allowance, and working by the *Rule of Three*.

℔.	
100	Suttle.
6	Tret allowed.
106	Sum.

℔. *l.* ℔. *l.*
As 106 . 14 :: 890 . $117\frac{29}{53}$
890
1260
112
12460
(5
~~280~~(8
~~12460~~($117\frac{29}{53}$ *l.*
~~106~~

Q. Of the Hundred with Tret. *Example* 2. At 2 *s.* 6 *d.* the Pound, what shall 780 ℔. be worth, allowing 4 ℔. Tret on the gross Hundred?

Answer. *Answ.* $94\frac{4}{29}$ *l.* For by the first Work of the *Rule of Three*, or *Practice*, the Value of 1 C. at the rate of 2 *s.* 6 *d.* the ℔. is found to be 14 *l.* And then by another Work of the same Rule is gotten $94\frac{4}{29}$ *l.*

℔.	
112	Great Hundred.
4	Tret allowed.
116	Sum.

At 2 *s.* 6 *d.* *per* ℔. what costs 112 ℔. / 8 (14 *l.*
$\frac{1}{8}$

℔. *l.* ℔. *l.*
As 116 . 14 :: 780 . $94\frac{4}{29}$
14
3120
780
116)10920($94\frac{4}{29}$ *l.*

Q. Of the Hundred rebating for Tare and Cloff. *Example* 3. If 100 ℔ be worth 38 *s.* what will 860 ℔ be worth, rebating 5 ℔ upon every 100 for Tare and Cloff?

Answer. *Answ.* 15 *l.* $10\frac{23}{50}$ *s.* For according to that Abatement there is but 817 ℔ to be accompted for; which at the rate of 38 *s.* for 100, gives the said Sum.

℔.	
100	Suttle.
5	Tare rebated.
95	Remain.

℔. ℔. ℔. ℔.
As 100 . 95 :: 860 . 817
95
4300
7740
100)81700(817 ℔.

℔. *s.* ℔. *l.* *s.*
As 100 . 38 :: 817 . 15 : $10\frac{23}{50}$
38
6536
2451
100)31046($31,0\frac{23}{50}$
$15:10\frac{23}{50}$

Example

Example 4. If a Merchant sell two Baskets of Raisins weighing 183 ℔, and 159 ℔, for 26 *s.* the Hundred, allowing 3 ℔ Tare on each Basket: what do the Raisins amount to?

Q. Of Baskets of Raisins with Tare.

Answ. 3 *l.* 18 *s.* For so it appears by the *Rule of Three*, the Tare being first deducted.

Answer.

℔.	
183	
159	
342	Weight in Gross.
6	Tare rebated.
336	Weight *Netto.*

$$\begin{array}{l} \text{℔.}\quad s.\quad\ \ \text{℔.}\quad l.\quad s. \\ \text{As } 112 \,.\, 26 :: 336 \,.\, 3 : 18 \\ \qquad\qquad\quad 26 \\ \qquad\qquad\ \ 2016 \\ \qquad\qquad\ \ 672 \\ 112)\,8736\,(\,78\ s. \\ \qquad\qquad\quad l.\,3:18\ s. \end{array}$$

Case 4. When the Enquiry is of *Loss* or *Gain*, with different Allowance or Abatement.

4. Loss or Gain with different Allowance or Rebatement, how resolved.

The *Gain* or *Loss* of one Side being gotten, by some or other of the Ways foregoing, compare it with the other.

Example 1. Whether doth he lose more that giveth 5 ℔ upon the 100, or he that rebateth 5 ℔ upon the 100?

Q. Of the most Loss, by giving or abating 5 on 100.

Answ. Because he that giveth 5 ℔ upon the 100, giveth 105 ℔ for 100 ℔; and he that rebateth 5 ℔ upon the 100, giveth 100 ℔ for 95 ℔: The Question therefore may be set thus; If 105 ℔ be given for 100 ℔, what shall 100 ℔ be delivered for? The Answer to which being $95\frac{5}{21}$ ℔, it appeareth that he who rebateth 5 ℔ on the Hundred is the Loser, by so much as $95\frac{5}{21}$ ℔ is greater than 95 ℔, that is $\frac{5}{21}$ for the other makes $95\frac{5}{21}$ ℔ of the Hundred.

Answer.

100 ℔	
5	Allowance.
105	Sum.
100 ℔	
5	Rebatement.
95	Remain.

$$\begin{array}{l} \quad\ \ \text{℔.}\quad \text{℔.}\quad \text{℔.}\quad \text{℔.} \\ \text{As } 105 \,.\, 100 :: 100 \,.\, 95\tfrac{5}{21} \\ \qquad\qquad\quad 100 \\ 105)\,10000\,(\,95\tfrac{5}{21} \\ \qquad\qquad\quad 95\ \text{Hundred with Rebate.} \\ \qquad\qquad\quad \tfrac{5}{21}\ \text{Difference.} \end{array}$$

Example 2. *A* oweth *B* 600 *l.* to be paid in 3 Months; and *B* oweth *A* 500 *l.* to be paid in 4 Months: But if *A* will clear the Score presently, *B* offereth to take 98 *l.* Which of them, considering the Interest, loseth, and how much?

Q. Of the Sum lost in paying two Debts.

Answ. *B* loseth 3 *l.* For accounting the Interest of 600 *l.* for 3 Months, it is 9 *l.* And the Interest of 500 *l.* for 4 Months is 10 *l.* accounting the Interest at 6 *per Cent.* which abated from the respective Sums, 591 *l.* is left for *A* to pay, and 490 *l.* for *B* to pay; the Difference is 101 *l.* due from *A* to *B*: For which if *B* take 98 *l.* he loseth 2 *l.* of the Principal, and 1 *l.* of the Interest.

Answer.

$$\begin{array}{l} \qquad\qquad\qquad\qquad\quad l.\quad Mon.\quad l. \\ l.\quad Mon.\quad l.\qquad :: 600 \,.\, 3 \,.\, 9\ \ A\ \text{Interest.} \\ \text{As } 100 \,.\, 12 \,.\, 6 \qquad\ \ l.\quad Mon.\quad l. \\ \qquad\qquad\qquad :: 500 \,.\, 4 \,:\, 10\ \ B\ \text{Interest.} \end{array}$$

$$\begin{array}{l} \qquad\qquad\quad l.\quad l.\quad l. \\ A\ \text{oweth } 600 - 9 = 591 \\ B\ \text{oweth } 500 - 10 = 490 \qquad \textit{Paid.}\quad \textit{Lost.} \\ \qquad\qquad \text{Due to } B\ 101 - 98 = 3 \end{array}$$

According to the Resolution of the Propositions in *Loss* or *Gain*, either by the *Rule of Three*, *Practice*, *Specificks*, or *Rule of five Numbers*, so shall be the Proof: And if any Doubt arise, work the Question reversed in any of the Cases.

Proof of Loss and Gain.

CHAP. XII. *Equation of Paiment.*

Equation of Paiment in 3 *Cases.*

SOmetime, as well on the Sale of Goods as Loan of Monies, divers Days or Times of *Paiment* are appointed; and upon a new Agreement, the Debtor willing to be discharged, desireth to know when the whole Sum to be paid may be paid all at once, the Interest duly accounted: How therefore the several Days of paiment may be equated and brought all into one, is the Work of this Chapter, and the Contents thereof may be reduced to three *Cases*.

1. *Time singly propounded.* Rule.

Case 1. When the Time of Paiment is singly propounded.

Multiply the Sums to be paid by the several Times of Paiment, and the Total of the Products divide by the whole Sum to be paid.

Q. *Of* 90 *l. to be paid at once, tho due at several Times.*

Example 1. A Debtor owing 90 *l.* agreed with his Creditor to pay it at three several Days, *viz.* 45 *l.* at 4 Months end, 30 *l.* at 8 Months end, and the remaining 15 *l.* at 10 Months end; but receiving Money unexpectedly, was willing to pay all at one Paiment: what Time must then be given him?

Answer. *Answ.* $6\frac{1}{3}$ Months.

	l.		*l.*		*l.*		
Sums to be paid	45	+	30	+	15		
Times of Paiment	4		8		10		
Products	180	+	240	+	150	=	57(0 (3
					Whole Debt		9 0 ($6\frac{1}{3}$ Months.

Resolved by Fractions.

Or if the several Sums to be paid be propounded in Fractions, or else be reduced into Fractionary Parts of the whole Debt, then multiply after the manner of Fractions, and add the Products into one Total.

As because 45 *l.* is $\frac{1}{2}$ of 90, and 30 *l.* is $\frac{1}{3}$, and 15 *l.* is $\frac{1}{6}$; therefore $\frac{1}{2}$ multiplied by 4, and $\frac{1}{3}$ by 8, and $\frac{1}{6}$ by 10, shall be together $6\frac{1}{3}$ as before.

$$\frac{45}{90} = \frac{1}{2} \times \frac{4}{1} = \frac{4}{2}\ (2$$
$$\frac{30}{90} = \frac{1}{3} \times \frac{8}{1} = \frac{8}{3}\ (2\tfrac{2}{3}$$
$$\frac{15}{90} = \frac{1}{6} \times \frac{10}{1} = \frac{10}{6}\ (1\tfrac{2}{3}$$
$$6\tfrac{1}{3} \text{ Months.}$$

Q. *Of* 500 *l. some due present, and some at other Times, when to be paid at once.* Answer.

Example 2. *A* is indebted to *B* 500 *l.* to be paid as followeth, *viz.* 100 *l.* present, 260 *l.* at 6 Months, and 140 *l.* at 9 Months: when are these Paiments on Time to be paid at once? or when shall the whole Debt of 500 *l.* be paid together?

Answ. At $7\frac{1}{20}$ Months, if 100 *l.* be paid present; but otherwise at the end of $5\frac{16}{25}$ Months the whole 500 *l.* shall be paid.

By Integers.

	l.		*l.*		*l.*		
Sums to be paid	100	+	260	+	140		
Times of Paiment.	Present.		6		9		
			1560	+	1260	=	282|0 ($7\frac{1}{20}$ Months.
			Remaining Debt				40|0
							(3 28(2|0 ($5\frac{16}{25}$ Months.
			Whole Debt				5 0|0

By Fractions.

Present 100 *l.*

$$\frac{260}{400} = \frac{13}{20} \times \frac{6}{1} = \frac{78}{20}\ (3\tfrac{18}{20} \qquad \frac{260}{500} = \frac{13}{25} \times \frac{6}{1} = \frac{78}{25}\ (3\tfrac{3}{25}$$
$$\frac{140}{400} = \frac{7}{20} \times \frac{9}{1} = \frac{63}{20}\ (3\tfrac{3}{20} \qquad \frac{140}{500} = \frac{7}{25} \times \frac{9}{1} = \frac{63}{25}\ (2\tfrac{13}{25}$$
$$\text{Months } 7\tfrac{1}{20} \qquad \text{Months } 5\tfrac{16}{25}$$

Example

Example 3. A Debt of 600 *l.* was to be paid thus; 200 *l.* present, $\frac{1}{5}$ at 8 Months, and $\frac{1}{5}$ at 10 Months, and the Residue at the Year's End: when may this Money be all paid together?

Q. Of 600 l. due, some present, some afterward, when to be paid at once.

Answ. At $6\frac{4}{15}$ Months.

Answer.

By Integers.

$$
\begin{array}{lll}
\text{Whole Debt} & 600\ l. & \\
\frac{1}{3} & 200 & 200 \times 8 = 1600 \\
\frac{1}{5} & 120 & 120 \times 10 = 1200 \\
\text{Rest } \frac{2}{15} & 80 & 80 \times 12 = 960 \\
 & & 3760 \;(\; 6\tfrac{4}{15} \text{ Months.}
\end{array}
$$

$$200 + 200 + 120 + 80 = 600$$

By Fractions.

Present 200 *l.*

$$\frac{200}{600} = \frac{1}{3} \times \frac{8}{1} = \frac{8}{3}\;(2\tfrac{2}{3}$$
$$\frac{120}{600} = \frac{1}{5} \times \frac{10}{1} = \frac{10}{5}\;(2$$
$$\frac{80}{600} = \frac{2}{15} \times \frac{12}{1} = \frac{24}{15}\;(1\tfrac{3}{5}$$
$$6\tfrac{4}{15} \text{ Months.}$$

Case 2. When the Time of Paiment is propounded with *Loss* or *Gain*.

2. Time with Loss or Gain.

Equate the Times of Paiment as before, and then by the *Rule of five Numbers* enquire for the *Gains* or *Loss*.

Rule.

Example 1. A Merchant buyeth Silks at 10 *s.* the Yard, and selleth the same at 12 *s.* the Yard, giving 2 Days of Paiment; *viz.* 4 Months for the one Half, and 8 Months for the other: what doth he gain on the 100 in 12 Months?

Q. Of Silks sold on Time, what Gain.

Answ. 40 *l.* For the Times of Paiment equated make 6 Months: And then if 10 *s.* in 6 Months gain 2 *s.* an hundred Pounds in 12 Months shall gain 40 *l.*

Answer.

$$\frac{1}{2} \times \frac{4}{1} = \frac{4}{2}\;(2$$
$$\frac{1}{2} \times \frac{8}{1} = \frac{8}{2}\;(4$$
$$6 \text{ Months.}$$

$$
\begin{array}{cccccc}
l. & Mon. & s. & l. & Mon. & l. \\
\text{As } \frac{1}{2} & 6 & 2 & :: 100 & 12 & 40
\end{array}
$$

$$3 \qquad 24$$

$$3)\,2400\,(\,80|0\ s.$$
$$40{:}0\ s.$$

Example 2. A Merchant buyeth 20 Cloths, at 6 *l.* the Cloth ready Money; and afterwards selleth 5 of them for 7 *l.* the Cloth to be paid at 4 Months; and the other 15 he selleth for 8 *l.* the Cloth, and giveth 6 Months Time for the Paiment: how much is gained thereby on the 100 in 12 Months?

Q. Of Cloth, some for ready Money, and some on Time, what Gain.

Answ. $63\frac{7}{86}$ *l.* For the Money paid for the Cloths bought being 120 *l.* and to be paid for them sold being 155 *l.* there is 35 *l.* difference; and the Times equated being $5\frac{17}{31}$ Months; by the *Rule of five Numbers* is obtained $63\frac{7}{86}$ *l.*

Answer.

$$
\begin{array}{llllll}
\text{Cloths bought} & 20 & \text{Cloths sold} & 5 & + & 15 \\
\text{Price of one} & 6\ l. & \text{Price of one} & 7\ l. & & 8\ l. \\
 & 120 & & 35 & + & 120 = 155
\end{array}
$$

$$
\begin{array}{llll}
\text{Sums to be paid} & 35 & + & 120 \\
\text{Times of Paiment} & 4 & & 6 \\
 & 140 & + & 720 = \frac{860}{155}\;(\,5\tfrac{17}{31} \text{ Months.}
\end{array}
$$

$(85$

$$
\begin{array}{cccccc}
l. & Mon. & l. & l. & Mon. & l. \\
\text{As } 120 & 5\frac{17}{31} & 35 & :: 100 & 12 & 63\frac{7}{86}
\end{array}
$$

$$420$$

$$\frac{20640}{31}\,)\,\frac{42000}{1}\,(\,\frac{130200}{2064}\,(\,63\tfrac{7}{86}\ l.$$

Example

Q. Of Cloves sold on Time, Gain on the 100.

Example 3. A Merchant buyeth Cloves at 8 *s.* the Pound ready Money: how shall he sell the hundred Weight thereof to gain, after the Rate of 10 *l.* on the 100 for a Year, and be paid $\frac{1}{2}$ at 3 Months, and the rest at 6 Months end?

Answer. *Answ.* For 46 *l.* 9 *s.* $7\frac{1}{5}$ *d.* where the Times being equated for one Paiment, make $4\frac{1}{2}$ Months; And the Rate of 1 C. at 8 *s.* the ℔, being found to be 44 *l.* 16 *s.* the Ready-money Price, Then by the *Rule of five Numbers* is found 1 *l.* 13 *s.* $7\frac{1}{5}$ *d.* to be added to the Ready-money Price.

Paiments $\frac{1}{2} \times \frac{3}{1} = \frac{3}{2}$ ($1\frac{1}{2}$
$\frac{1}{2} \times \frac{6}{1} = \frac{6}{2}$ (3
$4\frac{1}{2}$ Months

	℔.	*s.*	℔.	*l.*	*s.*
As	1 .	8 ::	112 .	44 :	16

4
l. 44|8 Primes.

	l.	*Mon.*	*l.gain.*	*l.*	*Mon.*	*l.*	*s.*	*d.*
As	100 .	12 .	10 ::	$44\frac{4}{5}$.	$4\frac{1}{2}$.	1 :	13 :	$7\frac{1}{5}$

1200

1200) $\frac{45}{2016}$ ($1\frac{17}{25}$ *l.* × 20 = $\frac{340}{25}$ ($13\frac{3}{5}$ *s.* × 12 = $\frac{36}{5}$ ($7\frac{1}{5}$ *d.*

	l.	*s.*	*d.*	
Ready-money Price	44 :	16 :	00	
Gain to be added	1 :	13 :	$07\frac{1}{5}$	
	46 :	09 :	$07\frac{1}{5}$	Selling Price of the Hundred.

3. Time sooner for Part, and later for other Part. Rule.

Case 3. When the Time of Paiment is anticipated for Part of the Debt, and procrastinated for the Residue.

Substract the Paiment or Paiments from the whole Debt, and if the Times of Paiment be more than one, equate them; and this equated Time take from the Time of Paiment, and commit the Work to the *Rule of Three:* But if one Time be propounded, substract the Time of Paiment from the Time when to be paid, and the Sum paid from the whole Debt, and work with the Remainers. Or instead of the Difference of the Time equated or not, if the Times of Paiment be substracted from the Time when the Whole was to be paid, and the Remains be equated, this may be taken to work with.

Q. Of 480 l. if 280 paid before the Day, when the rest.

Example 1. *A* is indebted to *B* 480 *l.* to pay it in $5\frac{1}{2}$ Months; and at 4 Months end he will pay 280 *l.* upon Condition that *B* will stay (accounting by the Interest) for the other 200 *l.* as long after the Time as this 280 *l.* is paid before the Time: when shall the 200 *l.* be paid?

Answer. *Answ.* $2\frac{1}{10}$ Months after the Expiration of $5\frac{1}{2}$ Months.

	l.		*Months.*
Whole Debt	480	to be paid in	$5\frac{1}{2}$
Paiment	280	made in	4
Residue	200	Difference	$1\frac{1}{2}$

	l.	*Months.*	*l.*	*Months.*
As	200 .	$1\frac{1}{2}$ ::	280 .	$2\frac{1}{10}$

$1\frac{1}{2}$
280
140
20) 42,0 ($2\frac{1}{10}$ Months.

Q. Of 240 l. some paid before due, when the rest.

Example 2. A Merchant oweth 240 *l.* to be paid in 6 Months; but $1\frac{1}{2}$ Month past he payeth 60 *l.* and 3 Months after that he payeth 80 *l.* more: in what Time shall he pay the Rest, considering his Time was 6 Months?

Answer. *Answ.* $3\frac{9}{14}$ Months after the end of 6 Months.

	l.		*Months.*		*Months.*
Whole Debt	240	to be paid in	6		6
Paiments	60	in	$1\frac{1}{2} \times 60 = 90$	(3	
	80		$4\frac{1}{2} \times 80 = 360$	45(0	$3\frac{3}{14}$ Time equated.
Residue	100		Paiments	140	
					$2\frac{11}{14}$ Difference.

	l.		*Months.*		*l.*		*Months.*
As	100	.	$2\frac{1}{4}$	::	140	.	$3\frac{9}{10}$

$$\begin{array}{r} 140 \\ 2\frac{1}{4} \\ \hline 280 \\ 110 \\ \hline 10)39,0(3\frac{9}{10} \text{ Months.} \end{array}$$

Example 3. A Merchant is indebted 300 *l.* to be paid the 24*th* of *May*; whereof he payeth the 29*th* of *April* 80 *l.* and the 9*th* of *May* after 120 *l.* upon what Day shall he pay the remaining 100 *l*? — *Q. Of* 300 *l. paid, part before due; when the rest.*

Answ. 38 Days after the 24*th* of *May*, which will be the first Day of *July*. — Answer.

Whole Debt 300 *l.* to be paid in 25 Days. 25 Days.

Paiments { 80 Present, 120 in } $10 \times 120 = 1200$

Residue 100 — Paiments 200 (6 Time equated.

19 Difference.

	l.		*Days.*		*l.*		*Days.*
As	100	.	19	::	200	.	38

$$\begin{array}{r} 200 \\ 19 \\ \hline 100)38,00(38 \text{ Days.} \end{array}$$

Or because 80 *l.* was paid *April* 29, which was 25 Days before *May* 24, when the Whole was to be paid; and 120 *l.* being paid *May* the 9*th*, which was 15 Days before *May* 24; if 25 and 15 be equated with the Paiments, the Time equated will be 19 Days, and the Resolution as above. — *How otherwise wrought.*

Whole Debt 300 *l.* to be paid in 25 Days, that is from *April* 29, to *May* 24.

Paiments { 80 before $25 \times 80 = 2000$, 120 before $15 \times 120 = 1800$ } 3800 (*Days*

Paiments 200 (19 before the Time.

As 100 . 19 :: 200 . 38.

The Operations of the first *Case* are to be proved thus: Rate the Interest of the several Sums to be paid, according to the Times of Paiment, and the Total thereof shall be equal to the Interest of the Total Sum accounted to the Time equated. — *Proof of Equation of Paiment.*

As in the first *Example* of the first *Case*, the Interest reckoned at 6 *l. per Cent. per Annum*, in both is found 57 *s.* — *Of the first Case.*

	l.		*Mon.*		*l.*		*l.*		*Mon.*		*s.*	
							45	.	4	.	18	Interest of the several Paiments in their Times.
As	100	.	12	.	6	::	30	.	8	.	24	
							15	.	10	.	15	
As	100	.	12	.	6	::	90	.	$6\frac{1}{3}$	.	57	Interest of the Total Paiment at the Time equated.

In the Operations of the second *Case*, so far as concerns the Equations of the Paiments, is to be proved as the first. And what further refers to the *Rule of Three*, or *Rule of five Numbers*, admits of their Proof. — *Of the second Case.*

The Operations of the third *Case* are to be proved much like those of the First: For account the Interest of the whole Sum till the Time of Paiment thereof to be made, and this shall be equal with the Interest of the several Sums paid, reckoned till the Times of their Paiments. — *Of the third Case.*

As in the first *Example* of the third *Case*, rating the Interest at 6 *l.* in the Hundred, the Sum found for both is 13 *l.* 4 *s.*

l. *Mon.* *l.*	*l.* *Mon.* *l.* *s.* 280 . 4 . 5 : 12	}	Interest of 280 *l.* for 4 Months, and 200 *l.* for $7\frac{1}{2}$ Months (that is $5\frac{1}{2}$ and $2\frac{1}{10}$) when paid.
As 100 . 12 . 6 ::	200 . $7\frac{1}{2}$. 7 : 12		
As 100 . 12 . 6 ::	480 . $5\frac{1}{2}$. 13 : 4		Interest of 480 *l.* the whole Debt to $5\frac{1}{2}$ Months.

CHAP. XIII. *Factorship.*

Factorship. Questions relating to Merchants and Factors, close up the third Sort of Derivatives, and are treated of in this Chapter under the Title of *Factorship*, but *How placed by some.* with some placed under *Fellowship*.

What to be noted therein. In *Factorship* are to be minded two things.

First, The Estimation of the Factor.

Secondly, The Partition of the Gains between the Merchant and the Factor, according to their Agreement or Bargain.

Estimation of the Factor, what. Estimation of the Factor, is an Allowance to the Factor for his Pains or Imploiment, to countervail some part of the Merchant's Stock. And when the Factor layeth *How to be accounted, with or without Stock.* in no Stock, his Estimation is in such *Ratio*, or Proportion to the Merchant's Stock, as the Gains of the said Factor are to the Gains of the Merchant. But when the Factor layeth in Stock with the Merchant, then, after the whole Estimation of the Person of the Factor with his Stock is valued according to the Merchant's Stock, the Factor's Stock is to be deducted, and the Remain is the Estimation of his Person.

Examples of both.

Q. Of the Factor's Estimation without Stock. 1. A Merchant delivereth unto his Factor 900 *l.* to govern in the Trade of Merchandise, upon Condition that he should have $\frac{1}{3}$ of the Gain: what is his Estimation esteemed at?

Answer. *Answ.* 450 *l.* For that Sum beareth the same *Ratio* to 900 *l.* which $\frac{1}{3}$ doth to $\frac{2}{3}$, the Residue of the Gains belonging to the Merchant.

$$\text{As } \tfrac{2}{3} \;.\; \overset{l.}{900} \;::\; \tfrac{1}{3} \;.\; \overset{l.}{450}.$$

$$\tfrac{2}{3}\Big)\frac{300}{1}\Big(\frac{900}{2}\Big(\overset{l.}{450}\ \text{Estimation.}$$

Q. Of the Factor's Estimation with Stock. 2. A Merchant delivered unto his Factor 600 *l.* and the Factor layeth in Stock therewith 250 *l.* besides his Personal Estimation; therefore it is agreed he shall have $\frac{2}{5}$ of the Gain: what is the Estimation of his Person?

Answer. *Answ.* 150 *l.* For the whole Estimation of his Stock and Person is 400 *l.* from which 250 *l.* the Factor's Stock deducted, leaves 150 *l.* for the Estimation of his Person.

$$\text{As } \tfrac{3}{5} \;.\; \overset{l.}{600} \;::\; \tfrac{2}{5} \;.\; \overset{l.}{400}$$

$$\tfrac{3}{5}\Big)\frac{240}{1}\Big(\frac{1200}{3}\Big(\begin{array}{ll} \overset{l.}{400} & \text{Whole Estimation.} \\ 250 & \text{Factor's Stock.} \\ \hline 150 & \text{Factor's Personal Estimation.} \end{array}$$

Partition of the Gains how accounted at, or out of the sending of the Merchant. Partition of the Gains enquired, propounds the Estimation or Agreement between the Merchant and his Factor: And sometimes accounts the Estimation according to the Merchant's Stock, called *Estimation* upon, or at the sending of the Merchant. And sometimes the Gain is to be parted proportionally, according to the Merchant's Stock and the Factor's Estimation added together, called *Estimation* out of the sending of the Merchant.

Examples

Examples of both.

1. A Merchant putteth into his Factor's Hands to improve in the way of Merchandise 800 *l.* upon condition that the said Factor shall have $\frac{1}{4}$: And after certain Time they found in Profit 135 *l.* 6 *s.* 8 *d.* how much shall the Factor have thereof? *Q. Of Gain parted to the Merchant and Factor.*

Answ. Because $\frac{1}{4}$ is understood to be $\frac{1}{4}$ of the Gains; the Gains shall be divided, $\frac{3}{4}$ to the Merchant, and $\frac{1}{4}$ to the Factor: That *Answer.*

		l.	*s.*	*d.*
$\frac{3}{4}$	To the Merchant	101	10	00
$\frac{1}{4}$	To the Factor	33	16	08
	Total Gains	135	06	08

But then his Estimation is but $\frac{1}{3}$, because $\frac{1}{4}$ of the whole Gains is but $\frac{1}{3}$ of $\frac{3}{4}$ the Merchant's Part.

And if the Factor's Estimation were $\frac{1}{4}$, then must he have but $\frac{1}{5}$ of the Gains, and the Merchant $\frac{4}{5}$; and so must the Gains be parted thus,

	l.	*s.*	*d.*
To the Merchant	108	05	04
To the Factor	27	01	04
	135	06	08

The Reason whereof is, because if his Estimation had been but $\frac{1}{4}$, then the $\frac{1}{4}$ of 800 *l.* being 200 *l.* had made the 800 *l.* to be 1000 *l.* of which $\frac{1}{5}$ is equal to $\frac{1}{4}$ of 800 *l.* the Merchant's Stock.

2. A Merchant delivereth to his Factor 800 *l.* upon Condition that his Factor shall have the Profit of 160 *l.* as though he laid in so much ready Money: what Portion of the Gains shall the said Factor take up for himself at the Reckoning? *Q. Of Gains how parted to the Merchant and Factor.*

Answ. If it be intended that the said 160 *l.* shall be reckoned as Stock to the other 800 *l.* then the whole Stock maketh 960, of which 160 is $\frac{1}{6}$, and such part of the Gains shall the Factor have. *Answer.*

But if the Intent of their Covenants between them were, that the Factor should have the Gains of 160 *l.* of the 800 *l.* then shall the Factor take $\frac{1}{5}$ of the Gains; for 160 to 800, is as 5 to 1.

Examples mixt. Mixt Examples.

1. A Merchant puts into his Factor's Hands 390 *l.* to trade with, and the Factor's Estimation is accounted 60 *l.* how much Money must the Factor put in Stock, that he may have $\frac{1}{4}$ of the Gain? *Q. Of Stock laid in by the Factor.*

Answ. 70 *l.* For $\frac{1}{4}$ of the Gain is understood of the whole Gain, Estimation and all accounted thereto: So as $\frac{3}{4}$ Gain for the Merchant coming of 390 *l.* the $\frac{1}{4}$ Gain for the Factor must arise of 130 *l.* that is 60 *l.* Estimation, and 70 *l.* Stock. Answer.

		l.			*l.*	
As	$\frac{3}{4}$	390	::	$\frac{1}{4}$	130	
					60	Factor's Estimation.
					70	Factor's Stock.

2. A Merchant hath 400 *l.* to trade with in Merchandise; and agreeth with a Factor, that if he put in 90 *l.* to the Stock, then he shall have $\frac{1}{5}$ for his Pains. Afterward another Merchant desireth to be a Copartner, and putteth in Stock 350 *l.* and promiseth to observe the same Agreement made with the Factor: when the Trade is over they find 250 *l.* gained; how must the Gains be divided? and what is the Factor's Pains esteemed at? *Q. Of Gains how to be parted, and of the Factor's Estimation.*

Answ. The Factor's Estimation being valued according to the first Merchant's Stock, is 110 *l.* his Stock 90 *l.* which together with the Stock of both Merchants, make 950 *l.* by which the Gains are to be divided as in *Fellowship.* Answer.

As

			l.			*l.*	
		As $\frac{2}{3}$	400	::	$\frac{1}{3}$	200	
						90	Factor's Stock.
	Stock.					110	Factor's Estimation.
A	400 *l.*						
B	350				*l.*		
Factor	200	*l. gain.*	400	.	$105\frac{5}{19}$ *A*		
As	950	. 250 ::	350	.	$92\frac{2}{19}$ *B*		Gain divided.
			200	.	$52\frac{12}{19}$ Factor		

But if the Factor's Estimation had been valued according to both the Merchants Stocks at $\frac{1}{3}$, then his 90 *l.* Stock and Estimation had been together 375 *l.* and his Stock substracted, his Estimation would have been 285 *l.* and the Gain divided by this Account, the Factor will have almost as much as the First, and more than the second Merchant.

			l.			*l.*	
		As $\frac{2}{3}$	750	::	$\frac{1}{3}$	375	
						90	Factor's Stock.
	Stock.					285	Factor's Estimation.
A	400 *l.*						
B	350				*l.*		
Factor	375	*l. gain*	400	.	$88\frac{8}{9}$ *A*		
As	1125	. 250 ::	350	.	$77\frac{7}{9}$ *B*		Gain divided.
			375	.	$83\frac{1}{3}$ Factor		

Proof of Factorship. The Proof of nothing here need be spoken to, except the finding of the Estimation; forasmuch as all the other Operations of this Chapter depend on *Proportions*, as the *Rule of Three*, *Fellowship*, &c. the Proof whereof hath been set forth at large where they have been handled.

In particular of finding the Estimation. To prove the finding of the Estimation, add the Estimation to the whole Stock, as well of the Factor, if any be, as of the Merchant's; and take the Part thereof which the Factor was to have, and it will agree with the Estimation found: But where the Factor had Stock, this Part taken shall include his Stock with his Estimation, otherwise his bare Estimation only.

As in the first and last *Examples* of this Chapter.

	First.			*Last.*	
Stock	900 *l.*	Merchant.	Stock	400 *l.*	Merchant.
Estimation	450	Factor.	Stock	90	Factor.
$\frac{1}{3}$ Part)	1350	(450 *l.*	Estimation	110	Factor.
			$\frac{1}{3}$ Part)	600	(200 *l.* Stock and Estimation.

CHAP. XIV. *Falshood or Position.*

Falshood or Position. THE Comparative Elements of the third Sort of Derivatives done with, yet remains for this Chapter the handling of the fourth Sort, (which have some peculiar Operation requisite to their Resolution) called *Falshood* or *Position*.

Why so called. *Falshood* is not so called, because it teacheth any Deceit, but for that by false Positions, or erroneous Numbers taken to work with, the Truth is found out. And though in some other of the foregoing Elements, the Numbers wrought with were not the true, yet were they in just Proportion to the True; but here they are taken at adventure; and if examining thereby the Question, according to the Tenor thereof, your Position fall true, the Question is answered: but because sometime false, it gave the Rule the Name of the *Rule of false Position*.

Of two sorts. This *Rule of false Position*, or *Falshood*, is twofold, *viz.* *Single* and *Double*.

Single Position. The first is called *Position Single*, because sometimes at one working the Question propounded may be resolved, which happens when there is in the Proposition

How known. some partition of Numbers in Parts proportional.

The

The Process herein is thus, Imagine a Number at pleasure, and proceed to work therewith according to the Purport of the Question, as if it were the true Number; and what Proportion there is between the false Conclusion, and the false Position, such Proportion hath the given Number to the Number sought: Therefore the Number found by Argumentation shall be the first Number of the *Rule of Three*, and the Hypothetical or supposed Number shall be set in the second Place, and in the third Place shall be the given Number. *What to be done.*

Example 1. *A*, *B*, and *C*, consent together to buy a Ship for 220 *l.* so that *B* must pay twice so much as *A*, and *C* four times so much as *B*: How much must each Man pay? *Q. Of a Ship bought, what each paid.*

Answ. I suppose *A* paid 8 *l.* then must *B* pay 16 *l.* and *C* four times so much, which is 64 *l.* But all these Numbers added together make no more than 88 *l.* and there should be 220 *l.* yet by this Number supposed I proceed to work, saying, Answer.

If 88 *l.* come of 8 *l.* of what comes 220 *l*?

Where in the Work is gained 20 *l.* for *A*; then must *B* pay 40 *l.* and *C* 160 *l.* all which added together, produce 220 *l.* the Number propounded.

			l.	*Position false.*	*l.*	*Conclusion true.*	
		As	88 .	8 *l.* ::	220 .	20 *l.*	*A*
Position	8			220		40 .	*B*
Double	16			88)1760(20		160 .	*C*
Quadruple	64					220	Proof.
	88						

Example 2. A Mercer buyeth 30 Yards of Taffety, and 40 Yards of Sattin for 330 *s.* but every Yard of Sattin cost twice so much as a Yard of Taffety: what did a Yard of Taffety cost? *Q. Of Taffety and Sattin, the Price.*

Answ. Suppose a Yard of Taffety cost 4 *s.* then must a Yard of Sattin cost 8 *s.* at which Rates 30 Yards of Taffety will cost 120 *s.* and 40 Yards of Sattin 320 *s.* but 120 and 320 make 440, which should be but 330. I therefore say, As 440 . 4 :: 330 . 3 . So is 3 *s.* gotten for a Yard of Taffety, and then the Yard of Sattin shall cost 6 *s.* Answer.

s. *Position.* *s.* *Conclusion.*
If 440 come of 4 *s.* of what comes 330? *facit* 3 *s.*

Position 4 Doubled 8 — 4 — 440)1320(3 *s.* doubled 6 *s.*

Taffety 30 Sattin 40 — Taffety 30 Sattin 40

120 + 320 = 440 Proof 90 + 240 = 330 *s.*

If the Question hath a Fraction or more therein, it is best, for more facility in proceeding, to take such a Number for the *Position* as may be equally parted by the Parts exprest in the Question. *If the Question have Fractions.*

Example 1. A Captain ordered upon Service with a Party, being demanded how many Souldiers he had in his Party? answered, that $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{4}{5}$ of them added together, made 245: how many Men had he? *Q. Of a Captain and his Party, how many.*

Answ. I suppose 30, (a Number that will be equally parted by 2, 3, and 5) and adding the $\frac{1}{2}$ which is 15, with the $\frac{1}{3}$ which is 10, and $\frac{4}{5}$ which is 24, find but 49, and it should be 245: Therefore tho I have erred in supposing, yet I commit the Work to the *Rule of Three*, and obtain 150 the Number sought. Answer.

Men. *Position.* *Men.* *Conclusion.*
If 49 come of 30: of what comes 245? *facit* 150.

		245	$\frac{1}{2}$	75
Position	30	49)7350(150 Souldiers.	$\frac{1}{3}$	50
Half	15		$\frac{4}{5}$	120
Third	10			245 Proof.
4 Fifths	24			
	49			

Q. Of Sheep left when some gone.

Example 2. A Man having 1000 Sheep sold at one Time, $\frac{1}{2}$ so many as he now hath, and at another Time $\frac{1}{3}$ so many; and at another Time lost $\frac{1}{4}$ so many: The Question is, how many Sheep he hath yet remaining?

Answer.

Answ. Suppose he had 12 Sheep left, the Half which is 6, added to $\frac{1}{3}$ which is 4, and $\frac{1}{4}$ which is 3, make with the 12 remaining but 25 instead of 1000; then by the Analogy is found 480 Sheep to be left.

As 25 . 12 :: 1000 . 480 . Sheep left.

Position	12
$\frac{1}{2}$	6
$\frac{1}{3}$	4
$\frac{1}{4}$	3
	25

$$25 \overline{)\,12000\,}(480 \qquad \text{(12 × 1000 = 12000)}$$

	480	.	Sheep left.
$\frac{1}{2}$	240	.	Sold.
$\frac{1}{3}$	160	.	Sold.
$\frac{1}{4}$	120	.	Lost.
	1000	.	Proof.

If a Number propounded may be left or taken out of the Whole.

When in the Question some Number is propounded to be left or taken out of the Whole, then the Parts proposed may be added together as Fractions, and taken out of the whole Integer: Or the whole Numbers by Argumentation substracted out of the *Position*, and the rest serve for the Analogy as before; only in the former Way the Number so proposed to be left or taken out of the Whole, shall be the second Number.

Q. Of a Man's Estate.

Example 1. A Man having spent 50 *l.* had yet $\frac{3}{4}$ and $\frac{1}{5}$ of his Estate remaining: what was his Estate at first?

Answer.

Answ. 1000 *l.* For if I add $\frac{3}{4}$ and $\frac{1}{5}$, they make $\frac{19}{20}$, which taken from $\frac{20}{20}$, the whole Substance, there remaineth $\frac{1}{20}$; which if it be 50 *l.* shall make the Whole 1000 *l.*

Or if I suppose 20 *l.* then $\frac{3}{4}$ is 15, and $\frac{1}{5}$ is 4; which added are 19, and substracted from 20, the *Position* leaves 1; And if 1 come of 20, then 1000 shall come of 50.

$$\frac{3}{4} + \frac{1}{5} = \frac{19}{20}$$

$$\frac{20}{20} - \frac{19}{20} = \frac{1}{20}$$

If $\frac{1}{20}$ be 50 *l.* what shall 1? *facit* 1000 *l.*

$$\frac{1}{20}\Big)\frac{50}{1}\Big(\frac{1000}{1}$$

	l.
	1000
$\frac{3}{4}$	750
$\frac{1}{5}$	200
Spent	50
	1000 Proof.

Otherwise.

Position	20
$\frac{3}{4}$	15
$\frac{1}{5}$	4
Total	19
Remain	1

As 1 . 20 *l.* :: 50 . 1000 *l.*

50 × 20 = 1000 *l.*

Q. Of Stock, what it was.

Example 2. One having spent $\frac{2}{3}$ and $\frac{1}{5}$ of his Stock, had only 36 *l.* remaining; what was his Stock?

Answer.

Answ. 270 *l.* Here the Fractions added make $\frac{13}{15}$, which taken from the whole $\frac{15}{15}$, leaves $\frac{2}{15}$; and this being equal to 36, makes the whole 270.

Or by supposing 15, then $\frac{2}{3}$ is 10, and $\frac{1}{5}$ is 3, which make 13; that taken from 15 leaves 2: therefore as 2 to 15, so is 36 to 270.

$$\frac{2}{3} + \frac{1}{5} = \frac{13}{15}$$

$$\frac{15}{15} - \frac{13}{15} = \frac{2}{15}$$

If $\frac{2}{15}$ be 36 *l.*: what shall 1? *facit* 270 *l.*

$$\frac{\overset{1}{2}}{15}\Big)\frac{\overset{18}{36}}{1}\Big(\frac{270}{1}\ l.$$

	l.
	270
$\frac{2}{3}$	180
$\frac{1}{5}$	54
Left	36
	270 Proof.

Otherwise.

Otherwise.

Position	15
$\frac{2}{3}$	10
$\frac{1}{5}$	3
Total	13
Remain	2

As 2 . 15 :: 36 . 270 (*l.*)

$$\begin{array}{r} 15 \\ 36 \\ \hline 90 \\ 45 \\ \hline 2)540(270\ l. \end{array}$$

Sometime a Number in the Question propounded abideth unalterable by the Fractions given, and so may be substracted from the Sum given, and set by for a Time till Operation be made with the rest, and then restored again. *If a Number may be set by.*

Example. A Grasier said of his Stock, If I had as many more Head of Cattel as I have with the half, third and fourth Parts thereof, and 1 overplus, I should have just 630: how many Cattel had he? *Q. Of Cattel, what Number.*

Answ. I set by the 1, and so remaineth 629; when I have found therefore a Number, which being twice taken with the $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ Parts thereof make 629, I add the 1 thereto: As supposing 12, the Parts whereof accordingly taken, and added to the double thereof, make 37, by which is found 204 the Number desired. Answer.

Position	12
Double	24
$\frac{1}{2}$	6
$\frac{1}{3}$	4
$\frac{1}{4}$	3
	37

As 37 . 12 (*Position.*) :: 629 . 204 Cattel.

37)7548(204

204	
102	$\frac{1}{2}$
68	$\frac{1}{3}$
51	$\frac{1}{4}$
1	added.
630	Proof.

Sometime two Numbers are demanded, and in the Work two Numbers are taken, yet the Resolution by *Single Position*, because one of the Numbers so taken proves true, and not suppository only. *Two Numbers, yet single Position.*

Example. What two Numbers are they, whose $\frac{2}{3}$ of the one is $\frac{3}{4}$ of the other? *Q. Of two Numbers.*

Answ. I take 12, a Number that hath such Parts in it; $\frac{2}{3}$ of 12 is 8: Then seeking what Number 8 is $\frac{3}{4}$ of, I suppose 20; but $\frac{3}{4}$ of 20 is 15, and I should have but 8: Nevertheless the *Analogy* will hold; If 15 come of the *Position* 20, then will 8 come of $10\frac{2}{3}$ the other Number desired: So shall 12 and $10\frac{2}{3}$ be the Numbers demanded. Answer.

$\frac{2}{3}$ of 12 = 8

Position	20
$\frac{3}{4}$	15

As 15 . 20 :: 8 . $10\frac{2}{3}$

$$\begin{array}{r} 8 \\ \hline 15)160(10\frac{2}{3} \end{array}$$

Proof.

$\frac{2}{3}$ of 12 = 8 = $\frac{3}{4}$ of $10\frac{2}{3}$

$\frac{1}{3}$	4	$\frac{1}{2}$	$5\frac{1}{3}$
$\frac{1}{3}$	4	$\frac{1}{4}$	$2\frac{2}{3}$
	8		8

If there be no Partition in the Numbers to make a Proportion, then must be used the *Rule of Double Position.* Double Position, *how known.*

In *Double Position* make a Supposition twice, proceeding therein according to the State of the Question; and if either Number of them supposed happen to resolve the Question, the Work is done: But if not, observe the Errors, and whether they be greater or lesser than the Resolution requireth, and mark the Errors accordingly with the Signs + or —. *What to be done.* *Errors to be marked.*

Then multiply contrarywise the one *Position* by the other Error: And if the Errors be both too great or both too little, substract the one Product from the other, and the one Error from the other, and divide the Difference of the Products by the Difference of the Errors. *Multiplied crosswise into the Positions.*

But if the Errors be unlike, as the one + and the other —, add the Products, and divide the Sum thereof by the Sum of the Errors added together: For the Proportion of the Errors, is the same with the Proportion of the Excesses or Defects of the Numbers supposed, to the Numbers sought. *Errors like or unlike, and then what to do.*

Example

Q. *Of the Number of Tenements.*

Example 1. *A* and *B* discoursing of their Tenements; *A* says to *B*, If I had 2 of your Tenements I should have double the Number you have: To whom *B* replies, If I had 2 of your Tenements I should be equal with you: How many Tenements had each of them?

Answer.

For Answer, suppose *A* had 16, to which 2 added makes 18, which is double the Number of 9; but having taken 2 from thence, it appeareth by this Supposition *B* had 11; wherefore 2 taken from 16 and added to 11, make *B* 13, and leave *A* 14. But they should be equal, therefore the *Position* is erroneous, and the Error too much by 1.

Again, suppose *A* had 20, to which 2 being put, it's 22, and double the Number of 11; but from thence 2 being taken, *B* by this Supposition must have 13. Now take 2 from 20 and put to 13, it's 15 for *B*, and leaves 18 for *A*, which is not equal; therefore I have erred again by 3 too much.

Errors both +.

Then multiplying 16 the first *Position* by 3 the second Error, and likewise 20 the second *Position* by 1 the first Error, the Product 20 is taken from the Product 48, because the Errors are both +, and the Remainer 28 is the Dividend; and abating 1 the lesser Error from 3 the Greater, there is 2 for the Divisor; the Quotient of which Division will be 14, the Number sought for *A*, and then by Consequence must *B* have 10: for 2 taken from 10, and added to 14, makes 16, which is double to 8; and 2 taken from 14, and put to 10, makes both 12 alike.

	First.		*Second.*	
Position	16	X	20	Position.
Error	1 +		3 +	Error.
	20		48	

Products 48 — 20 = 28 (14 *A.*
Errors 3 — 1 = 2 (10 *B.*

Proof.
14 + 2 = 16
10 — 2 = 8 Half.
14 — 2 = 12
10 + 2 = 12 Equal.

Resolution by other Positions, where the Errors are both —.

If the Suppositions had been 12 and 10, supposing 12 for *A*, then must *B* have 9; and taking 2 from 9 to put to 12, makes *A* 14, which is double to 7 for *B*; but taking 2 from 12 to put to *B*, makes *B* 11, and leaves but 10 for *A*, which is an Error too little by 1. And supposing 10 for *A*, then must *B* have 8; and taking 2 from 8 to put to 10, makes 12 for *A*, double to 6 for *B*; but then taking 2 from 10 to put to 8 makes *B* 10, and leaves *A* but 8, which is an Error too little by 2, for they should be equal: And because the Errors are both alike, the Work is as before.

	First.		*Second.*	
Position	12	X	10	Position.
Error	1 —		2 —	Error.
	10		24	

Products 24 — 10 = 14 (14 *A.*
Errors 2 — 1 = 1 (10 *B.*

Resolution by other Positions, where the Errors are + and —

But if 20 and 10 had been the *Positions*, then the Errors as before being found unlike, that is 3 + and 2 —, the Total of the Products must be the Dividend, and the Total of the Errors the Divisor.

	First.		*Second.*	
Position	20	X	10	Position.
Error	3 +		2 —	Error.
	30		40	

Products 30 + 40 = 70 (14 *A.*
Errors 3 + 2 = 5 (10 *B.*

Q. *Of 3 Debts, what they were.*

Example 2. *A*, *B*, and *C*, were indebted to *D*, who hath forgotten their Particular Debts, but remembreth the Debt of *A* and *B* added together made 50 *l.* and *C* and *B* together owed 80 *l.* and the Debt of *A* and *C* together was 70 *l.* The Question is, what each Man's particular Debt was?

Answer.

Answ. Suppose *A* did owe 15 *l.* then must *B* owe 35 *l.* (for both their Debts made 50 *l.*) And if *B* owe 35 *l.* then did *C* owe 45 *l.* (because both their Debts made 80 *l.*) But then the first Man's 15, and the third Man's 45, make together but 60 *l.* and they should be 70 *l.* So is the Error 10 —.

Then

Then suppose *A* did owe 26 *l.* it will follow that *B* owed 24 *l.* and *C* 56 *l.* But now 56 and 26 arise to 82 *l.* and it should be but 70 *l.* so here is an Error of 12 +.

Now multiplying crosswise as before, the Products 260 and 180 added, the Errors being unlike, make 440 to be divided by 22 the Sum of the Errors. So is 20 *l.* found the Debt of *A*, and consequently *B* 30 *l.* and *C* 50 *l.*

First Position. *A* & *B.*	*Consequence.* *B* & *C.*	*Consequence.* *A* & *C.*
A 15 + *B* 35 = 50	*B* 35 + *C* 45 = 80	*A* 15 + *C* 45 = 60
		True Debts 70 Error 10 —

Second Position. *A* & *B.*	*Consequence.* *B* & *C.*	*Consequence.* *A* & *C.*
A 26 + *B* 24 = 50	*B* 24 + *C* 56 = 80	*A* 26 + *C* 56 = 82
		True Debts 70 Error 12 +

	First.		*Second.*	
Position	15	X	26	Position.
Error	10 —		12 +	Error.
	260		180	

Products 260 + 180 = 440 (*l.* 20 *A.*
Errors 10 + 12 = 22

Proof.

A 20 + *B* 30 = 50   *B* 30 + *C* 50 = 80   *A* 20 + *C* 50 = 70

Example 3. A Parcel of Linen Cloth, *viz.* Lockram and Canvas, was sold to the Number of 30 Ells for 51 *s.* The Canvas was rated at 18 *d.* the Ell, and the Lockram at 2 *s.* how many Ells of each sort were there? *Q. Of Lockram and Canvas, how much of each.*

Answ. By supposing 10 Ells of Canvas, the Error will be 4 +: And by supposing 6 Ells, the Error will be 6 +; the Products therefore of 6, the *Position* into 4 and of 6, the Error into 10, taken one from the other, leave 36 the Dividend, and the Difference of the Errors 2 is the Divisor: So is there found to be 18 Ells of Canvas, and by Consequence 12 Ells of Lockram. Answer.

First Position.			*s.*	*d.*	*s.*
Canvas	10 Ells	×	1	: 6 =	15
Lockram	20 Ells	×	2	: 0 =	40
	30				55
				Paiment	51
				Error	4 +

Second Position.			*s.*	*d.*	*s.*
Canvas	6 Ells	×	1	: 6 =	9
Lockram	24 Ells	×	2	: 0 =	48
	30				57
				Paiment	51
				Error	6 +

	First.		*Second.*	
Position	10	X	6	Position.
Error	4 +		6 +	Error.
	24		60	

Products 60 — 24 = 36 (18
Errors 6 — 4 = 2

Proof.

			s.	*d.*	*s.*
Canvas	18 Ells	×	1	: 6 =	27
Lockram	12 Ells	×	2	: 0 =	24
	30				51

Example 4. A Carpenter was to build a Piece of Work in 30 Days, and by Agreement was to receive for every Day he wrought 3 *s.* but for every Day he did not work within that Time, he was to be amerced 2 *s.* When the Work was done he received but 25 *s.* for his Work: The Question is, how many Days he worked, and how many Days he plaid? *Q. Of a Carpenter's Work and Play.*

Answ. By the Suppositions of 20 and 12, the Errors will be 15 + and 25 —; the Products added make 680, and the Errors added 40: So by Division is found he worked 17 Days and plaid the rest. Answer.

First Position.				*s.*		*s.*	*Second Position.*				*s.*		*s.*
Wrought	20 Days	×	3	=		60	Wrought	12 Days	×	3	=		36
Played	10 Days	×	2	=		20	Played	18 Days	×	2	=		36
	30					40		30					00
			Received			25				Received			25
			Error			15 +				Error			25 −

First.		*Second.*						
Position 20	×	12 Position.	Products	180 + 500	=	680		
Error 15 +		25 − Error.	Errors	15 + 25	=	40	(	17
180		500						

Proof.

			s.
Working Days	17 at 3*s.* a Day	=	51
Playing Days	13 at 2*s.* a Day	=	26
	30		25 Received.

Notes. *Note* 1. As in *Single*, so in *Double Position*, though the Number supposed be never so false, Resolution may be had thereby: Yet for more ease in the Work, suppose a Number likely or apt to be parted equally into so many Parts as are necessary to the Resolution of the Question.

1. Suppose Number easy to be parted.

2. Second Position Homogeneal to the first. 2. Let the second *Position* always be *Homogeneal* to the first, that is, belong both to one Man, one Thing, *&c.* For though the Operator is at liberty to suppose for which he will of the Positives sought in the Question; yet if he suppose first for one, and then for another, the Resolution will be confused.

3. Both Errors if equal & unlike in Signs. 3. If both the Errors be equal in Numbers, and yet their Signs unlike, half of both the *Positions* is the Sum desired.

Q. Of Apples given 3 Maids, how many. *Example.* A Man having a certain Number of Apples, met three Maids who desired some of his Apples; whereupon he giveth *A* $\frac{1}{4}$ of his Apples, and she giveth him three Apples again. To *B* he giveth $\frac{1}{3}$ of his remaining Apples, and she giveth him two Apples again. To *C* he giveth $\frac{1}{7}$ of his remaining Apples, and she giveth him one again; so had he 13 Apples left: how many Apples had he at first?

Answer. *Answ.* 20: For $\frac{1}{4}$ thereof is 5, and 3 returned again leaves 18; of which $\frac{1}{3}$ is 6, and 2 returned leaves him 14; of which $\frac{1}{7}$ is 2, and 1 returned leaves 13.

And if the Suppositions made for Resolution be 16 and 24, the Errors will be $1\frac{5}{7}$ — and $1\frac{5}{7}$ +: So 20 the half of 40 (which is the Total of 16 and 24) is the Number sought.

4. Double Position most useful. 4. All the Propositions resolved by *Single Position*, will be resolved by *Double*; and several by other of the Comparative Elements foregoing.

Example by Specificks *of the fourth Sort, as well as Position.*

Q. Of filling a Cistern, the Time. At *Bronze* is a Statue in form of a Lion standing upon a Fountain, with this *Epigram*; If I let the Water pass out of my right Eye, I can fill the Cistern (holding 732 Gallons) in 2 Days: If I let it pass out of my left Eye, I can fill it in three Days: If it pass out of my Feet, the Cistern will be filled in 4 Days: But if it pass out of my Mouth, I can fill the Cistern in 6 Hours: In what time then shall I fill it, if I pour forth Water by all the Passages at once?

Answer. *Answ.* In 4 Hours and 44 Minutes *ferè*: For in that time cometh out of his Mouth 576 Gallons, out of his right Eye 72, out of his Left 48, and out of his Feet 36.

By Position.

			Hours.	*Gallons.*		*Hours.*	*Gallons.*
1. Position 3 Hours.	Then as	Right Eye	48				$45\frac{3}{4}$
Error $266\frac{7}{8}$ —		Left Eye	72	732	::	3	$30\frac{1}{2}$
		Feet	96				$22\frac{7}{8}$
		Mouth	6				366
							$465\frac{1}{8}$

$732 - 465\frac{1}{8} = 266\frac{7}{8}$

2. Position

			Hours.		Gallons.		Hours.		Gallons.
2. Position 4 Hours.	Then as	Right Eye	48	:	732	::	4	.	61
Error $111\frac{5}{6}$ —		Left Eye	72						$40\frac{2}{3}$
		Feet	96						$30\frac{1}{2}$
		Mouth	6						488
									$620\frac{1}{6}$

$$732 - 620\frac{1}{6} = 111\frac{5}{6}$$

$$266\frac{7}{8} - 111\frac{5}{6} = 155\frac{1}{24}$$

$$\begin{array}{lll} 266\frac{7}{8} \times 4 = 1064, \; +3\frac{1}{2} = 1067\frac{1}{2} & 111\frac{5}{6}\times 3 = 333, \; +2\frac{1}{2} = 335\frac{1}{2} & 1067\frac{1}{2} - 335\frac{1}{2} = 732 \end{array}$$

$$\begin{array}{l} 732 \times 24 = 17568 \\ 155 \times 24 + 1 = 3721 \end{array} \quad \frac{17568}{3721} = 4\frac{44}{61} \text{ Hours. } (\;|2684)$$

By Ratio's *after the manner of* Specificks: *Thus*, By Specificks.

$$\begin{array}{ccccccccc} \textit{Right Eye.} & & \textit{Left Eye.} & & \textit{Feet.} & & \textit{Mouth.} & & \\ 48 & \times & 72 & \times & 96 & \times & 6 & = & 1990656 \\ 1 & & 1 & & 1 & & 1 & & \\ \hline 41472 & + & 27648 & + & 20736 & + & 331776 & = & 421632 \end{array} \quad \frac{1990656}{421632} = 4\frac{44}{61} \text{ Hours.}$$

Example by Position *and* Equation of Paiment.

A is indebted to *B* 800 *l.* to be paid at 4 Months; 1 Month being past, he payeth 200 *l.* and $1\frac{1}{2}$ Month after the first Month he payeth 300 *l.* more; when shall the other 300 *l.* be paid? Q. *Of a Debt paid, part before due, when the rest.*

Answ. 5 Months after the last Paiment, that is, $3\frac{1}{2}$ Months after the end of the 4 Months. Answer.

	l.			
Debt	800	multiplied by the Time of Paiment	$800 \times 4 =$	3200
Paid	200	multiplied by the Time of Paiment	$200 \times 1 =$	200
Rest	600	multiplied by the Time of Paiment	$600 \times 2\frac{1}{2} =$	1500
Paid	300	more, so remaineth yet 300, which suppose *A* kept 2 Months	$300 \times 2 =$	600
				2300

By Position.

Suppose *A* kept the 300 *l.* 4 Months; Error 900—
Then instead of the 600 above, must be
1200 added to 1500 and 200, which ═══ 2900
Error 300—

$$900 - 300 = 600$$

$$\begin{array}{l} 900 \times 4 = 3600 \\ 300 \times 2 = 600 \end{array} \quad 3600 - 600 = 3000 \qquad \frac{3000}{600} = 5 \text{ Months after } 2\frac{1}{2} \text{ Months.}$$

By Equation of Paiment *thus*. By Equation of Paiment.

	l.				
Debt	800	in 4 Months.			4 Months.
Paid	200×1	$= 200$	950		$1\frac{9}{10}$ Time equated.
	$300 \times 2\frac{1}{2}$	$= 750$			
Rest	300	Paid	500		$2\frac{1}{10}$ Difference.

$$\text{As } 300 \,.\, 2\frac{1}{10} :: 500 \,.\, 3\frac{1}{2}$$

$$\begin{array}{r} 500 \\ \times\, 2\frac{1}{10} \\ \hline 1000 \\ 50 \\ \hline 300)\,1050\,(3\frac{1}{2} \text{ Months after 4 Months.} \end{array}$$

Endless were the Questions that might be propounded for Resolution by *Position*; and several Authors are found stored with much variety of such Propositions, *Other Questions chosen out of other Books.*

ons, besides those already answered, some of the choice Ones here selected may serve instead of the rest.

1. Of the Travel of two Posts. *Example* 1. There are two Towns distant one from the other 200 Miles, from whence two Posts that depart upon one Day from the one Town unto the other, and one goeth two Miles a Day more than the other, they meet in five Days; how many Miles doth each Post travel in a Day?

Answer. *Answ.* The one went 19, and the other 21 Miles in a Day; which in 5 Days made the One to have gone 95, and the Other 105, and together 200 Miles.

2. Of 3 Silver Cups, with a Cover. *Example* 2. A Goldsmith hath three Silver Cups with a Cover of 18 Ounces, and the second Cup weigheth half as much as the First and third Cups. If the Cover be put to the first Cup, it weighs as much as all the three Cups; and if joined to the Second, it will weigh as much as the second Cup twice, and the third Cup once: But if put to the third Cup, it weigheth twice as much as the first and second Cups: what then was the Weight of each Cup?

Answer. *Answ.* The first Cup weighed 6 ℥, the Second 8 ℥, and the Third 10 ℥, that is together 24 ℥: And so much is the Cover and first Cup 18 + 6. The Cover and second Cup is 26, that is 18 + 8, which is equal to the third Cup, and double the Second 10 + 8 + 8. And the Cover and third Cup is 28 the double of 14, the Weight of the first and second Cups.

3. Of Crowns caught up in a Difference. *Example* 3. Two Partners had in Accompt between them 400 French Crowns, whereof one should have 230, and the other 170: But in parting them they fell so at Variance, that he had most that could catch most. Yet afterward being reconciled, they agreed that he that had most should lay down $\frac{1}{2}$ of them he had, and he that had least should lay down $\frac{1}{3}$, and the Sum of both should be equally divided between them, and so should each Man have his due: The Question is, how many of the Crowns each Man caught up?

Answer. *Answ.* One caught up 280, and the other 120: for $\frac{1}{2}$ of 280 is 140, and there remaineth 140; and $\frac{1}{3}$ of 120 is 40, and there remaineth 80; this 40 and that 140 added are 180, the Half whereof is 90 for each; and this added to the Remains, makes for one 230, that is 90 + 140, and for the other 170, that is 90 + 80.

4. Of Money and Cloth, how much. *Example* 4. A Merchant buying Cloth, findeth if he take 12 Cloths, he shall want 42 *l.* to pay for them; but if he take 9 Cloths, then he hath 84 *l.* too much: How much Money had he, how much did a Cloth cost, and how many Cloths bought he?

Answer. *Answ.* He had 462 *l.* which will pay for 11 Cloths at 42 *l.* a Cloth.

5. Of the Money Partners had. *Example* 5. *A*, *B*, and *C*, buy a Ship for 200 *l.* If *A* have $\frac{1}{2}$ of what *B* pays, then he can pay for the Ship alone: And if *B* have $\frac{1}{4}$ of *C*, or *C* have $\frac{1}{3}$ of *A*, then they respectively can pay for the Ship: what Monies had each of them?

Answer. *Answ.* *A* had 120 *l.* which lacking 80 *l.* of 200 *l.* *B* must be double to 80, that is 160 *l.* this wanting 40 *l.* of 200 *l.* *C* must have 4 times 40, that is 160 *l.* as *B* had. And then 160 *l.* and 40 *l.* which is $\frac{1}{3}$ of *A*, will make up 200 *l.* also.

6. Of Hiero's *Crown.* *Example* 6. *Hiero* King of *Syracuse* in *Sicily*, had caused to be made a Crown of Gold to be offered for his good Success in Wars; in making whereof his Goldsmith fraudulently took out a Portion of the Gold, and put in Silver for it, yet so that there was nothing thereof to be seen or abated of the full Weight. The King suspecting the Fraud, propounds the Doubt to *Archimedes*; *viz.* How he might discover the Fraud without breaking the Crown. *Archimedes* not knowing presently how to answer the King's Desire; a while after as he chanced to enter into a Bath, he observed, as his Body entred into the Bathing-Vessel, the Water ran over; and thereupon apprehending a Reason of Solution to the King's Question, as the Story reports, was so rejoiced, that forgetting he was naked, ran Home, crying, ἑύρηκα, ἑύρηκα, *I have found, I have found:* And caused two massy Pieces, one of Gold and the other of Silver, to be prepared, of the same Weight with the Crown; and considering that Gold being heavier, and more compact by Nature than Silver, and so occupying less room, if it were put into a Vessel brim full of Water, would cause less Water to run over than a Mass of the same Weight of Silver would do: Whereupon trying both, he noted the Quantities of Water at each Time so run over, and learned thereby what Proportion in Quantity is between Gold and Silver of equal Weight. And then putting the Crown into the Vessel brim full of Water, (as before) marked how much Water ran over then; and comparing it with the Water that ran over when the Gold was

put

put in, noted how much it did exceed that; and likewise comparing it to the Water that ran over when the Silver was put in, marked how much it was less than that; and by these Proportions found out the just Quantity of Gold that was taken out of the Crown, and how much Silver was put in instead thereof.

Vitruvius who writeth the History, doth not declare the Particulars: But suppose the Crown to weigh 8 ℔, and so the other Masses, and imagine when the Gold was put in, there ran over 2 ℔ of Water; and when the Silver was put in, there ran over $3\frac{1}{2}$ ℔; and when the Crown was put in, there ran over but $2\frac{1}{4}$ ℔. Now supposing there was 2 ℔ of Silver in the Crown, then must there be 6 ℔ of Gold: And accordingly by the *Rule of Three*,

	℔ *Gold*	℔ *Water.*		℔ *Gold*	℔ *Water.*
As	8	2	::	6	$1\frac{1}{2}$

$$\begin{array}{r} 6 \\ 8\,\overline{)\,12\,}\,(1\frac{1}{2} \end{array}$$

	℔ *Silver.*	℔ *Water.*		℔ *Silver.*	℔ *Water.*
As	8	$3\frac{1}{2}$	::	2	$\frac{7}{8}$

$$\begin{array}{r} 2 \\ \hline 6 \\ 1 \\ 8\,\overline{)\,7\,}\,(\frac{7}{8} \end{array}$$

$2\frac{3}{8}$ Sum.
$2\frac{1}{4}$ when the Crown was put in.
$\frac{1}{8}$ Error +

Supposing again the Silver 1 ℔, then must the Gold be 7 ℔; and the Proportions by the *Rule of Three*, thus:

	℔ *Gold.*	℔ *Water.*		℔ *Gold.*	℔ *Water.*
As	8	2	::	7	$1\frac{3}{4}$

$$\begin{array}{r} 7 \\ 8\,\overline{)\,14\,}\,(1\frac{3}{4} \end{array}$$

	℔ *Silver.*	℔ *Water.*		℔ *Silver.*	℔ *Water.*
As	8	$3\frac{1}{2}$	::	1	$\frac{7}{16}$

$\frac{8}{1})\,\frac{7}{2}\,(\frac{7}{16}$

$2\frac{3}{16}$ Sum.
$2\frac{1}{4}$ when the Crown was put in.
$\frac{1}{16}$ Error −

Products.

$\frac{2}{1} \times \frac{1}{16} = \frac{1}{8}$

$\frac{1}{1} \times \frac{1}{8} = \frac{1}{8}$

$\frac{1}{8}$ + $\frac{1}{8}$ = $\frac{1}{4}$ (Products)

Products $\frac{1}{8} + \frac{1}{8} = \frac{1}{4}$
Errors $\frac{1}{8} + \frac{1}{16} = \frac{3}{16}$ $\frac{3}{16})\,\frac{1}{4}\,(\frac{4}{3}\,(1\frac{1}{3}$ ℔ Silver.

So had the Crown, if 8 ℔ in all $\left\{\begin{array}{l} 1\frac{1}{3} \text{ ℔ Silver} \\ 6\frac{2}{3} \text{ ℔ Gold} \end{array}\right\}$ By the Suppositions abovesaid.

Yet *Archimedes* needed not to have the Masses of equal Weight with the Crown; for by other Quantities of the same Metal, though of unequal Weight, the Solution may be had, as *Record* hath well observed.

Proof of Position of both sorts.

The Proof of all the Works wrought by *Position*, is when the Number desired is found, to add, substract, multiply, or divide thereby according to the tenor of the Question: and if in Conclusion there be Agreement in all things with the *Data* in the Proposition, the Work is right; as by the Proof of several of the Works before in this Chapter is fully approved.

CHAP. XV. *Proportions doubled.*

Figural Proportions.

Hitherto this second Part of the fourth Book hath spoken of plain *Disjunct Proportions*; now therefore a little is to be seen of *Figural*, which are so named, because the Proportion between the Numbers given and required, lies pro-

perly between them as they are, or are to be Figurate, or to the Resolution make use of Figural Numbers: So as being Figurate to the Square Quantity, they are called *Doubled Proportions*; if cubed, *Tripled Proportions*, &c.

Doubled Proportions how taken. *Doubled Proportions*; are not here to be strictly taken only for the Proportions about Squares, but take in among them other Plain or Superficial Figures, as Circles, Triangles, *&c.* and consider as well the *Proportions* between the *Parts* of such Figures one to another, as the *Proportions* between the *Data* and *Quæsita* in Propositions concerning them.

Circle, the Principal Parts, and Analogies thereof. The principal Parts of a Circle, wherein an Analogy is requisite, are three; The *Diameter*, the *Circumference* (called also *Peripherie* and *Perimeter*) and the *Area*: And because the Product of half the Circumference into half the Diameter is the *Area*, accompting the Diameter to the Circumference, according to the aforementioned *Archimedes*, or *Oughtred* from *Ludolph van Ceulen*, mentioned before also in *Figural Numbers*, Chap. 2.

The Analogies *are:*

As 7, to 22 . or 1, to 3,1416 } So is the Diameter to the Circumference.

As 14, to 11 . or 1, to 0,7854 } So is the Square of the Diameter to the *Area*.

As 88, to 7 . or 1, to 0,0795775 } So is the Square of the Circumference to the *Area*.

Propositions.

Q. Of the Circumferance and Area. A Circle whose Diameter is 21: What shall be the Circumference, and what the *Area* by *Archimedes*?

Answer. *Answ.* 66 Circumference, and $346\frac{1}{2}$ *Area*.

As 7 . 22 :: 21 . 66 Circumference.
As 14 . 11 :: 441 (Q. 21) . $346\frac{1}{2}$ *Area*.

Q. Of the Diameter & Area. A Circle whose Circumference is 66: what shall be the Diameter, and what the *Area* by *Archimedes*?

Answer. *Answ.* 21 Diameter, and $346\frac{1}{2}$ *Area.*

As 22 . 7 :: 66 . 21 . Diameter.
As 88 . 7 :: 4356 (Q. 66) . $346\frac{1}{2}$ *Area*.

Q. Of the Diameter and Circumference. A Circle whose *Area* is $346\frac{1}{2}$: what shall be the Diameter, and what the Circumference by *Archimedes*?

Answer. *Answ.* 21 Diameter, and 66 Circumference.

As 11 . 14 :: $346\frac{1}{2}$. 441 (Q. 21) . Diameter.
As 7 . 88 :: $346\frac{1}{2}$. 4356 (Q. 66) . Circumference.

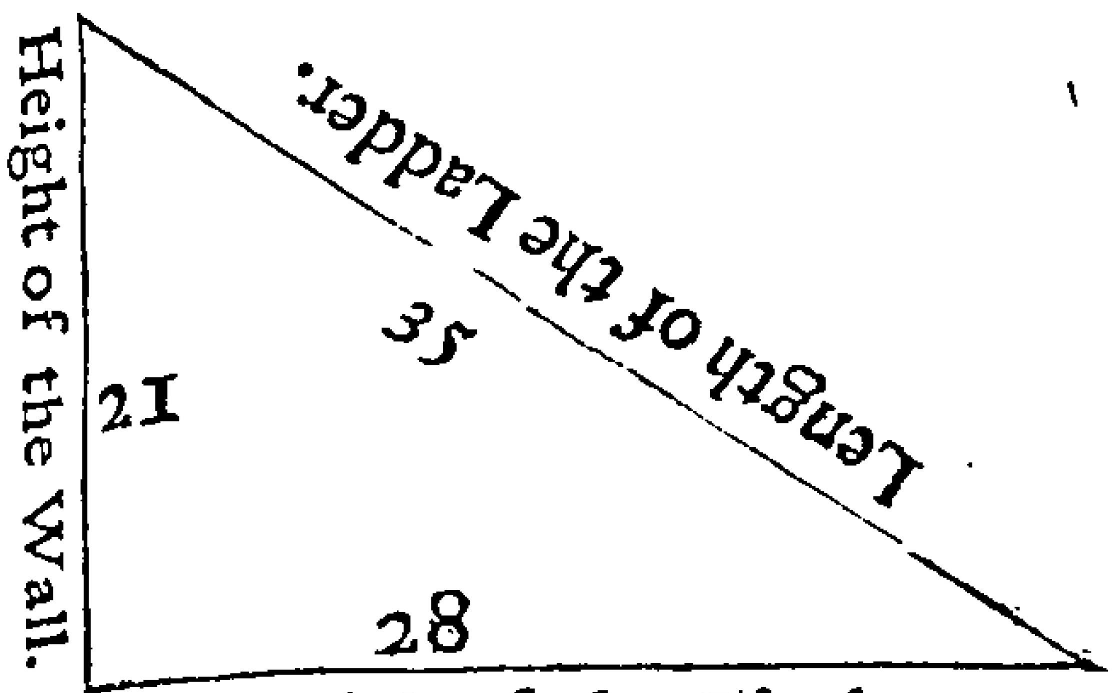
Triangles, *their Sides and Angles*, &c. Touching *Triangles*, their principal Parts are their Sides, Angles and *Areas*; but (that *Geometry* be not too far intruded upon, to which properly these Speculations belong) it will be enough to say something here of the Sides and Angles of Right-angled Triangles only.

Of a Right Angle. *The Doctrine of Triangles* acquaints us, that in all plain Right-angled Triangles, the Square of the Side that subtendeth the Right-angle, is equal to the two Squares of both the other Sides. Wherefore in such Propositions, where the *Data* and Number sought represent the three Sides of such a Triangle, the Resolution of either in question is accordingly to be found.

Q. Of the length of Scaling Ladders. *Example* 1. The Walls of a City 21 Feet high, which hath a Ditch at the bottom 28 Feet broad, are to be scaled; and the scaling Ladders are commanded to be made a Foot longer than will reach from the outermost Brink of the Ditch, to the Top of the Wall: how long must the Ladders be?

Answer. *Answ.* 36 Feet long: For the Squares of 21 and 28, which are 441, and 784 added, make 1225, whose Square Root is 35 the length, that will reach from the Brink of the Ditch to the Top of the Wall; to which 1 Foot added, because the Ladders were to be so much longer, makes the whole length of the Ladder 36 Feet.

Height



Breadth of the Ditch.

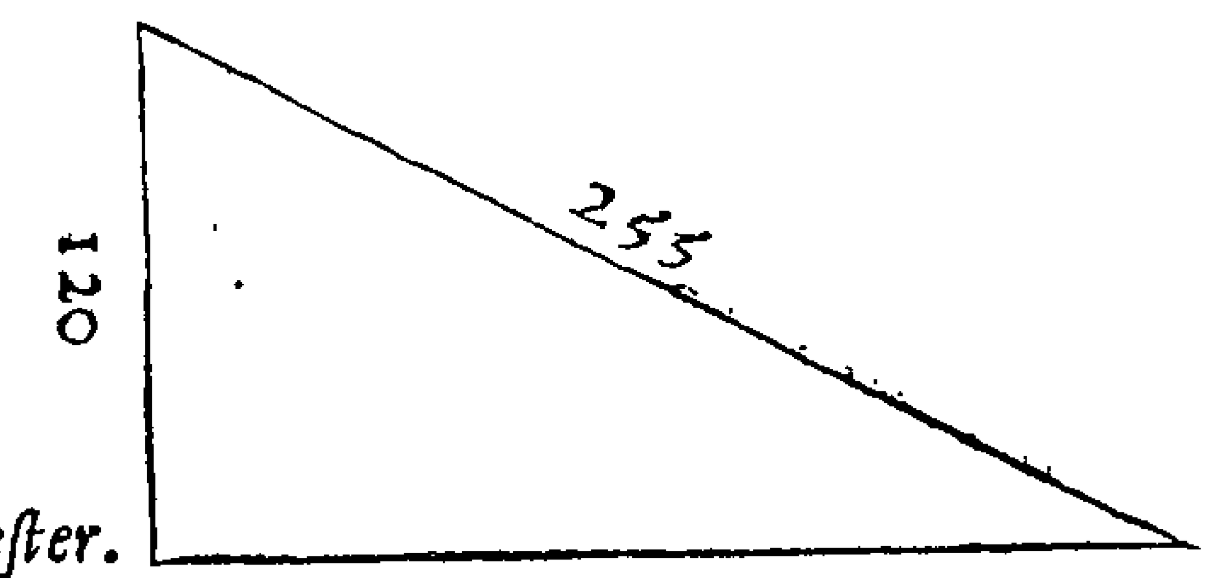
$21 \times 21 = 441$
$28 \times 28 = 784$ } $= 1225$ (35
9
325 — 1 added.

Length of the Ladders 36

Example 2. There are two Towns, as suppose *Chichester* and *York*, which lie South and North, distant one from the other 225 Miles; a third Town, as *Exeter* lying plain West from *Chichester*, is distant from *York* 255 Miles: how far is *Chichester* distant from *Exeter* in a right Line? *Q. Of the Distance of two Cities.*

Answ. 120 Miles: For here the number sought being not represented by the *Hypotenusal* Line, but by one of the Sides, containing the Right Angle; the Square of the other Side containing the same Angle, is to be taken from the Square of the subtending Side, that is, 50625, the Square of 225 from 65025 the Square of 255; so is the Remain 14400, the Square of 120. Answer.

Exeter.



Chichester. York.

225

$255 \times 255 = 65025$
$225 \times 225 = 50625$
14400 (120 Miles.
144

The *Area* of such Triangles being always half the Product of the 2 Sides, containing the Right Angle: If it were inquired how many Acres of Ground there were in the Triangle last mentioned, considering that one English Mile contains 5280 Feet or 320 Perches, and one Acre 160 Square Perches, the Sides 120 and 225 multiplied by 320, and the Products one into another, and the Half divided by 160, make such a Triangle to contain 8640000 Acres of Land. *Q. Of the Area of such a Triangle.* Answer.

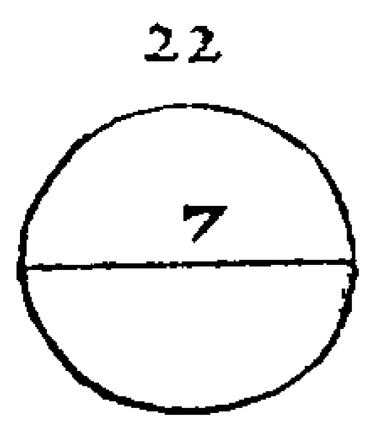
Sides of the Triangle	120		225 Miles.	
Perches in an English Mile	320		320	
	2400		4500	
	360		675	Product.
Perches in the Sides	38400	×	72000	= 2764800000
			Half	1382400000

$\dfrac{1382400000}{160}$ (8640000 Acres.

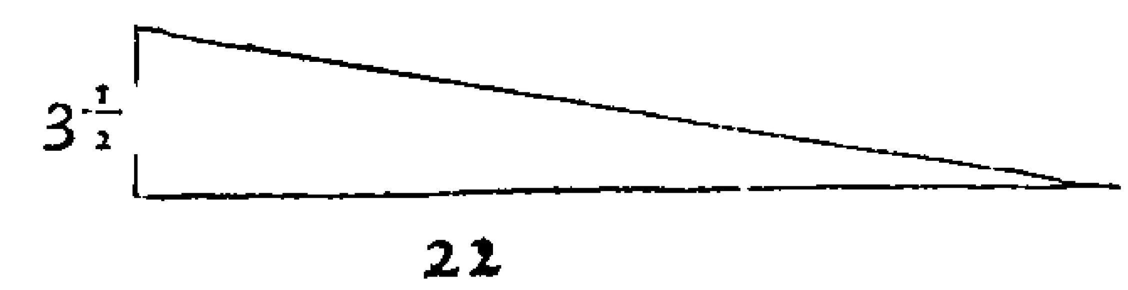
Sometime it is needful to turn a Circle into a Rectangle-Triangle, which is commonly performed thus; Make the Perpendicular of the Triangle equal to the Semidiameter of the Circle, and the Ground or Base-Line of the Triangle equal to the *Peripherie* of the Circle. *To turn the Triangle into a Circle.*

Example. If a Circle have the Diameter 7, and *Peripheric* 22: what shall the Sides of a Rectangle-Triangle be, that is equal in *Area* to the Circle, according to *Archimedes*? *Example.*

Answ. The Perpendicular $3\frac{1}{2}$ and the Base 22. Answer.



11
$3\frac{1}{2}$
33
$5\frac{1}{2}$
Area of $38\frac{1}{2}$ the Circle.



22
$3\frac{1}{2}$
66
11
77
Area of $38\frac{1}{2}$ the Triangle.

Quadrangles, *what considered in them.* *Quadrangles*, whether exact Squares, or other Rectangle Figures, (so far as concerns *Arithmetick*) have considered, their Sides, their *Area's*, Values, and Alteration of their Forms: But because all these, more or less, have been touched before, as among *Figural Numbers* in the second Part of the second Book, or in the *Indirect Rule of Three*, or *Specificks*, in this second Part of the fourth Book, they must be more sparingly remembred now.

Of their Sides and Area's. Touching the Sides and *Area's* of *Quadrangles*.

Square. *Of the* Square.

If the *Data* be the { Side / *Area* } and the *Quesita* the { *Area*. / Side.

For the Area. To resolve the First, square the Side, and the Product is the *Area*.

For the Side. To resolve the Second, extract the Square Root of the *Area*, as before in *Figural Numbers*.

If another Denomination be sought. But if the Proposition require the Number sought in another Denomination to that given; Then if the Denomination be greater than that given, divide the Number found by so many of the Lesser, as are contained in one of the Greater. And if the Denomination required be lesser than that given, multiply the Number given or found accordingly.

Q. *Of the Acres in a Field.* *Example* 1. A Field is 36 Rods Square every way: how many Acres doth that Field contain?

Answer. *Answ.* $8\frac{1}{10}$ Acres: Here, because Acre is an higher Denomination than Rod, after 36 the Side is multiplied into it self, the Product 1296 being the *Area* in Rods, is divided by 160 the Rods in an Acre, and the Quotient is $8\frac{1}{10}$ as above.

Side $36 \times 36 = 1296$ ($8\frac{1}{10}$ Acres.
Perches in an Acre 160

Ex. *If the Yards or Feet therin be sought.* But if the Proposition had demanded, how many Yards or Feet, *&c.* there had been, then these Denominations being lesser than Rods, either 36 must be multiplied by $5\frac{1}{2}$ for Yards, and by $16\frac{1}{2}$ for Feet, and the Products squared; or 1296 must be multiplied by $30\frac{1}{4}$ for Yards, and by $272\frac{1}{4}$ for Feet, because so many make a Square Rod.

Answer.

36	1296	36	1296
$5\frac{1}{2} \times 5\frac{1}{2} =$	$30\frac{1}{4}$	$16\frac{1}{2} \times 16\frac{1}{2} =$	$272\frac{1}{4}$
180	38880	216	2592
18	324	36	90724
$198 \times 198 =$	39204 Yards.	18	25922
			3
		$594 \times 594 =$	352836 Feet.

Q. *Of the Rods in one Side.* *Example* 2. A Square Field containeth 39204 Yards: how many Rods doth one Side contain?

Answer. *Answ.* 36 Rods: For Rod being an higher Denomination than Yard, 198 the Square Root of 39204 extracted, is divided by $5\frac{1}{2}$, or 5,5 the Yards in one Rod, and the Quotient is 36 Rods as before.

39204

```
. . .   3 3
39204 ( 19 8,0 ( 36 Rods.
1        5,5
Gnomon  261     5,5
Gnomon   3104
```

But if the Proposition had been to know how many Rods or Yards in the Side of a Field that is $8\frac{1}{10}$ Acres: Then, because Rods and Yards are lesser Denominations, $8\frac{1}{10}$ is multiplied by 160 for Rods, and 4840 for Yards, because so many are in 1 Acre, and the Root of each shall be the Side in those Denominations. *Ex. If the Rods or Yards of the Side be sought, and Acres given.* Answer.

```
 160                          4840
   8 1/10                        8 1/10
 1280                         38720
   16    √                      484    √
 1296 ( 36 Rods.              39204 ( 198 Yards.
```

Of Oblongs.

Oblongs.

Such Rectangle Figures as have their opposite Sides parallel.

If the *Data* be { Both Sides / *Area* and Side } and the *Quæsita* { *Area*. / Other Side.

In the first Case multiply the Sides together. *For the Area.*

But if the Proposition require the Number sought in another Denomination to that given: then, as before in Squares, divide or multiply accordingly by so many of the Lesser as are contained in one Greater. *If another Denomination be sought.*

Example 1. A Board is 1,17 Feet broad, and 16,32 Feet long: how many square Feet doth it contain? *Q. Of the Feet in a Board.*

Answ. Almost 19,1 Feet, as the Product of 16,32 by 1,17 makes appear. Answer.

```
      16,32 Length.
       1,17 Breadth.
     11424
     1632
    1632
Area 19,0944 Square Feet.
```

Example 2. A tiled Roof hath the Breadth $16\frac{1}{4}$ Feet, and the Length 47 Feet: how many Squares of Tiling doth the whole Roof contain? *Q. Of the Squares of Tiling in a Roof.*

Answ. 15,275 Squares, or $15\frac{11}{40}$: Here the Length 47 being doubled (for both Sides) is 94; which multiplied by $16\frac{1}{4}$, or 16,25, the Product 1527,5 is divided by 100 (or 10 × 10) the Feet in a Square. Answer.

```
  16,25 Breadth.
     94 Double length.
   6500                    1527,500
  14625                    -------- ( 15,275 Squares.
1527,50 Feet.                 100
```

Example 3. A Pavement broad 17,35 Feet, and long 30,5: how many Square Yards doth it contain? *Q. Of the Yards in a Pavement.*

Answ. A small matter above 58,797 Yards: For the Product of the Length into the Breadth 529,175 divided by 9, (or 3 × 3) the square Feet in 1 Yard give 58,797, and 2 is left remaining on the Division. Answer.